

# ОбъедМНенНы KHCTHTYT ндерных 

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## CORRELATION ANALYSIS

IN THE FRAMEWORK OF THE MANY-FOLD KNO-SCALING

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In ref. ${ }^{1 /}$, starting with the KNO-distribution for the two types of particles it is confirmed that in the problem of the neutral-charged particle correlation must take place the following relation

$$
\begin{equation*}
\frac{\left\langle n_{i}\left(n_{j}\right)\right\rangle}{\left\langle n_{i}\right\rangle}=\Phi_{3}^{i}\left(z_{j}\right) \tag{1}
\end{equation*}
$$

where $\left\langle n_{i}\right\rangle\left(\left\langle n_{i}\left(n_{j}\right)\right\rangle\right.$ is the mean number of $i-t y p e$ hadrons (as a function of the number of $n_{j}$-sort hadrons); $z_{j}=\frac{n_{j}}{\left\langle n_{j}\right\rangle} ; \Phi_{3}^{i}=$ $=\Phi_{2 / 3 / 1}^{i}{ }^{i}$ is a ratio of two scaling forms suggested in refs. ${ }^{\text {in }}$ and ${ }^{13 / 1}$ respectively.

In the confirmation of that result the plots for the correlated pairs $\left(K n^{c}\right),\left(\pi^{\circ} n^{c}\right),(\Lambda n 9)$ and $\left(\bar{\Lambda} n^{c}\right)$ (data for $\pi^{-p}$ and pp collision in the ranges $100-400 \mathrm{GeV} / \mathrm{c}$ ) are given, but the comparison with theory is not presented. According to ref. ${ }^{/ 4 /}$ the function $\Phi_{3}^{i}$ must be dependent not only on $z_{j}$ but also on the number of correlated sorts of hadrons $\nu$, $i . e$. , the scaling relation (1) must be broken. This statement, as can be seen below, is confirmed by the analysis of experimental data on the correlation of neutral and charged particles $/ 1,5 /$ and also of different types of particles $\left(\pi^{-} p\right),\left(n_{s} n_{p}\right),\left(n_{s} N_{n}\right)$, $\left(\mathrm{V}^{\circ} \pi^{--}\right)^{16 /}$.

The purpose of this article is to establish the character of the scaling (1) violation and to determine the number of correlating systems of particles of different sorts in the reactions $a+b \rightarrow n_{1} c_{1}+\ldots+n_{\nu} c_{\nu}+\ldots+n_{\nu_{\max }} c_{\nu_{\max }}$, when $\nu_{\max } \geq \nu \geq 2$ and $n_{1}, \ldots, n_{\nu_{\text {max }}}$ are the multiplicities of $c_{1}, \ldots, c_{\nu_{\max }}$ types of particles.

It is not difficult to be convinced that $\Phi_{1}{ }^{i}$ and $\Phi_{2}^{i}$ are determined as $(\nu-1)$-fold multiple integrals of the $\left(\nu_{\max }-1\right)$ fold KNO-function with the following structure

$$
\mathrm{F}\left(\mathrm{z}_{1}, \ldots, \mathrm{z}_{\nu}, \ldots, \mathrm{z}_{\nu_{\max }}\right)=\left[\prod_{\mathrm{i}=1}^{\nu_{\mathrm{max}}} \mathrm{~F}_{\mathrm{i}}\left(\mathrm{z}_{\mathrm{i}}\right)\right] \overline{\mathrm{F}}_{\nu}\left(\mathrm{z}_{1}, \ldots, \mathrm{z}_{\nu}\right)
$$

Here the function $F_{i}\left(z_{i}\right)$ corresponds to the noncorrelated creation of $i=1, \ldots, \nu_{\max }$ sorts and $\overline{\mathrm{F}}_{\nu}\left(z_{1}, \ldots, z_{\nu}\right)$ is the correlated creation of $i=1, \ldots, \nu$ sorts of particles. After simplified assumptions

$$
\begin{aligned}
& \overline{\mathrm{F}}_{\nu}\left(\mathrm{z}_{1}, \ldots, \mathrm{z}_{\nu}\right)=\stackrel{\rightharpoonup}{\mathrm{F}}_{\nu}\left(\mathrm{t}=\mathrm{z}_{1}+\ldots+\mathrm{z}_{\nu}\right),{\underset{\substack{\mathrm{i}=1 \\
\mathrm{i} \neq \mathrm{j}}}{\nu} \mathrm{~F}_{\mathrm{i}}\left(\mathrm{z}_{\mathrm{i}}\right)=1}_{\text {have }}=\text {, }
\end{aligned}
$$

$$
\begin{align*}
& \Phi_{3}^{i}\left(z_{j}, \nu\right)=\frac{1}{(\nu-1)} \frac{\int_{j}^{\infty}\left(t-z_{j}\right)^{\nu-1} \tilde{F}_{\nu}(t) d t}{\int_{0}^{\infty}\left(t-z_{j}\right)^{\nu-2} \tilde{F}_{\nu}(t) d t} \tag{2}
\end{align*}
$$

The function $\tilde{F}_{\nu}(t)^{z}$ according to ref. ${ }^{/ 4 /}$ is given by *

$$
\begin{equation*}
\overrightarrow{\mathrm{F}}_{\nu}(\mathrm{t}) \sim \mathrm{t}^{\mathrm{a}-\nu} \exp \left(-\frac{\mathrm{a}}{\nu} \mathrm{t}\right), \tag{3}
\end{equation*}
$$

where an a-parameter is determined by the normalization condition

$$
\begin{equation*}
\sum_{\mathrm{k}=1}^{\nu} \frac{\left\langle\mathrm{n}_{\mathrm{k}}^{2}\right\rangle}{\left\langle\mathrm{n}_{\mathrm{k}}\right\rangle^{2}}+2 \sum_{\mathrm{k}\rangle \mathrm{m}=1}^{\nu} \frac{\left\langle\mathrm{n}_{\mathrm{k}} \mathrm{n}_{\mathrm{m}}\right\rangle}{\left\langle\mathrm{n}_{\mathrm{k}}\right\rangle\left\langle\mathrm{n}_{\mathrm{m}}\right\rangle}=\nu^{2}\left(\frac{1}{\mathrm{a}}+1\right) . \tag{4}
\end{equation*}
$$



Fig. 1 shows the shapes of ( 2 ) as function of $z_{j}$ for the following notation of parameters $(a, \nu)=(3.2),(3.3),(3.5)$ and (3.6) (solid lines). The dashed curve corresponds to the limit $\nu \gg 1$, where the function (2) degenerates into the following one-parameter function
*In ref. ${ }^{/ 4 /}$ function $\tilde{F}_{\nu}$ is defined by the argument $t=\sum_{i=1}^{\nu} \gamma_{i} n_{i} / \sum_{i=1}^{\nu} \gamma_{i}\left\langle n_{i}\right\rangle$, where multipliers $\gamma$ are anomalous dimensions of the particle fields. The conditions $\gamma_{i}\left\langle n_{i}\right\rangle=\gamma_{k}\left\langle n_{k}\right\rangle$ take place because $\bar{t}=\mathrm{t} / \gamma$.

$$
\begin{equation*}
\Phi_{3}^{i}\left(z_{j}, \nu \gg 1\right)=\left(\frac{z_{j}}{a}\right)^{1 / 2}\left[K_{a}\left(2 \sqrt{a z_{j}}\right) / K_{a-1}\left(2 \sqrt{a z_{j}}\right)\right], \tag{5}
\end{equation*}
$$

where $K_{a}$ is the modified Bessel function.
At large $z_{j} \gg 1$ function (4) increases as $z_{j}^{1 / 2}$, where expression (2) becomes constant.


The approximation of the experimental data ${ }^{1,5,6 /}$ by (2) and (5) gives satisfactory results. In Fig. 2 the functions (2) (solid line) and (5) (dashed line) are compared with the data for $\pi^{-}$p-interactions at $5 \mathrm{GeV} / \mathrm{c}$ in the case of ( $\pi^{\circ} \mathrm{n}^{\mathrm{c}}$ )correlation $/ 5 /$ and CTa -interactions at $4.2 \mathrm{GeV} /(\mathrm{c}$. nucl.) in the case of $\left(\pi^{-} p\right)^{/ 6 /}$, respectively. The parameters equal: $\nu=1.99+0.40 ; \quad \mathrm{a}=$ $=5.45+3.02 ; \quad \bar{x}^{2} / \mathrm{N}=0.1 / 3$ for $\pi \bar{p}$ and $a=1.22+0.14$; $x^{2} / \mathrm{N}=1.4 / 4$ for CTa.

It is important to note that in the positive correlation region $\nu>$ a func-
tions $\Phi_{3}^{i}\left(z_{j}, \nu\right)$ and $\Phi_{3}^{i}\left(z_{j}, \nu \gg 1\right)$ with $20 \%$ accuracy give the same results as the functions (5) (the corresponding range in Fig. 1 is shaded). For examples, in Fig. 2 the dash-dotted line is an approximation of the data for CTa by (2) with the following values of the parameters: $\nu=5.51+0.52, a=0.54+0.15$ and $\chi^{2} / N_{=s}$ $=1.8 / 4$. We see that $x^{2}$ confidence level is satisfactory in both the cases.

Because of this saturation phenomenon for rather large experimental errors the scaling relation (1) takes place in the form (5), for example, in the $\pi^{-} p$ and pp-interactions at $100-400 \mathrm{GeV} / \mathrm{c}$ range of energy $/ 1 /$. For negative $\nu<$ a correlations (or absent correlations $\nu=\mathrm{a}$ ) the scaling (1) is strongly broken, but the meaning of $\nu$ is determined explicitly.

For a more exact definition of $\nu$ for the positive correlations it is necessary that the experimental errors do not exceed $1 \%$.

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Коррепяционный анализ в рамках многомерного КНО скейлинга
Предлагается способ определения чнсла коррелированных сортов вторичных частиц $\nu$ на основе анализа экспериментальных данных множественных процессов в рамках многомерного КНО скейлинга. Получен монотонный рост величины $\nu$ в интервале энергии $5-400$ ГэВ/с. Соответствующие корреляции переходят с отрицательного к положительному режиму, достигая в пределе $\nu \gg 1$ насыщения.

Работа выполнена в Јаборатории теоретической физики оили.

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Amaglobeli N.S.et al. E2-82-107 Correlation Analysis in the Framework of the Many-Fold KNO-Scaling

A method for determining the number of the correlated types of hadrons $\nu$ is suggested in the framework of the many fold KNO-scaling. The monotonous increase of $v$ is obtained in the range of energy $5-400 \mathrm{GeV} / \mathrm{c}$. The corresponding corralations pass from negative to positive regime, reaching the saturation in the limit $\nu \gg 1$.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR

