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1. Introduction

In paper [1] devoted to the relativistic-covariant description of a system of interacting particles a problem was raised to find the quasipotential Green functions [2,3] for composite particles with correct projective properties.

Note should be made that the hypothesis on projective properties of the Green functions has been used essentially in a series of papers [4] in studying various properties of composite particles within the parton model in the infinite-momentum frame ($p_z \to \infty$). In this connection the investigation of projective properties of the Green functions within the quasipotential formalism, without using additional assumptions, is of a fundamental significance. Results of the paper [1] clearly show that this problem can be considered in a consistent way within the framework of the relativistic-covariant quasipotential equations without appealing to the limit $p_z \to \infty$.

Hence it follows that the quasipotential approach is thus an adequate realization of concepts of the parton model. We would remind that this method is based on the relativistic generalization of the concept of equal time in describing a system of particles.

The present paper deals with studying the structure of perturbation theory expansion for the quasipotential Green functions of the system of two scalar particles. It is shown that for many various types of diagrams the two-particle quasipotential Green function has the required projective properties.

2. Definition of the Concept of Equal Time

In quantum field theory information on an interaction process of two particles can be extracted from the Green function

$$G\,(\vec r_1, t_1; \vec r_2, t_2 \,/\, \vec r_1^{\,\prime}, t_1^{\prime}; \vec r_2^{\,\prime}, t_2^{\prime}) .$$

However, as is known, the Green function in the four-dimensional formalism depends on relative times which have no direct physical interpretation and besides result in extra mathematical difficulties. To remove the above difficulty, A.A.Logunov and A.N.Tavkhelidze have suggested a quasipotential method /2/ based on the consideration of equal-time procedure. Within the framework of the approach it is possible to formulate the theory in terms of the two-time Green functions $G(t_1 - t_2 = t_1^{\prime} - t_2^{\prime} = 0)$ all the merits of quantum field theory being conserved. In doing so, wave functions become functions of equal time, i.e., they have a probabilistic quantum-mechanical interpretation.

In paper /1/ it has been suggested to perform the removing of the relative times within the quasipotential approach via the transition to the space-like surface

$$(t_1 - t_2) + (z_1 - z_2) = 0 . \qquad (1)$$

For this method of "equating of times" it is convenient to introduce the following relative coordinates

$$x_{\pm} = x_0 \pm x_3 \qquad (2)$$

and their conjugate momenta

$$p_{\pm} = p_0 \pm p_z .$$

The four-dimensional vectors are parametrized as follows

$$x = (x_+, x_-, \vec x_\perp) ,$$
$$p = (p_+, p_-, \vec p_\perp) . \qquad (3)$$

In this parametrization the scalar products of 4-vectors p and q have the form

$$p^2 = 2p_+p_- - \vec p_\perp^{\,2} ,$$
$$(pq) = p_+q_- + p_-q_+ - (\vec p_\perp \vec q_\perp) . \qquad (4)$$

In terms of the introduced variables the operation of "equating of times" (2) can be written as

$$x_+ = 0 . \qquad (5)$$

In this momentum space condition (5) or transition to the equal-time functions is realized through integrating over the variable p_- .

Thus, our consideration starts from the two-time quasipotential Green function

$$\tilde G\,(p_+, \vec p_\perp;\; q_+, \vec q_\perp)\,\delta(P-Q) =$$
$$= \int dp_-\, dq_-\, G\,(p_+, p_-, q_+, q_-, \vec q_\perp, \vec p_\perp)\,\delta(P-Q), \qquad (6)$$

where P and Q are the total momenta of the system.

As has been shown in /1/ a specific feature of the consideration is that the function (6) has definite projective properties when there is no interaction. These are as follows:

$$\tilde G_0(x, y, P_+)\,\delta(P-Q) = \frac{i\pi\,\theta(x-1)\,\theta(x)\,\delta(\vec p_\perp - \vec q_\perp)\,\delta(x-y)}{x(1-x)P_+^2\left(M^2 - \frac{\vec p_\perp^{\,2} + m_1^2}{x} + \frac{\vec p_\perp^{\,2} + m_2^2}{1-x}\right)} \qquad (7)$$

where

$$x = \frac{1}{2} + \frac{p_+}{P_+}$$
$$y = \frac{1}{2} + \frac{q_+}{Q_+} \qquad (8)$$

and $M^2 = P^2$.

Following a standard procedure we construct the quasipotential

$$V = G_0^{-1}\,\widetilde{G_0 K\ G_0}\,G_0^{-1}\ , \qquad (9)$$

where K is the interaction kernel.

Due to definite projective properties of the operator G_0 in /1/ a transition is made to the subspace

$$0 < x < 1 \qquad (10)$$

when constructing the quasipotential.

The operators given in this subspace have the inverse ones, i.e., they possess all properties necessary for defining the quasi-potential (9).

It will be shown below that at least for some class of the Feynmann diagrams the projective properties of the total Green function naturally follow from the above defined procedure of removing the relative times.

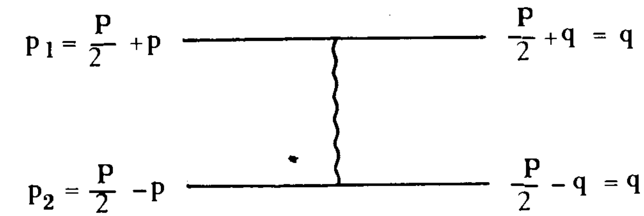
3. Construction of the Quasipotential for One-Meson Exchange

Proceeding from the quasipotential equation for composite systems written in terms introduced in Sect. 1 (see ref. /1/)

$$\left(P^2 - \frac{\vec{p}_\perp^{\,2} + m_1^2}{x} - \frac{\vec{p}_\perp^{\,2} + m_2^2}{1-x}\right)\psi(x,\vec{p}_\perp) = \qquad (11)$$

$$= \int dy\ d\vec{q}_\perp\, V(y,x,\vec{p}_\perp,\vec{q}_\perp)\,\psi(y,\vec{p}_\perp)\ P_+^{-1}\ x^{-1}\ (1-x)^{-1}$$

we consider the kernel K in (9) corresponding to the scalar-meson exchange:



In expression (9) we calculate $\widetilde{G_0 K\ G_0}$ (tilde means the operation (6)):

$$\widetilde{G_0 K\ G_0}(p_1,p_2;q_1,q_2)\ \delta(p_1+p_2-q_1-q_2) =$$

$$= \int dp_-\,dq_-\Big\{2P_+x\Big[p_- + \frac{P_-}{2} - \frac{\vec{p}_\perp^{\,2}+m_1^2}{2P_+x} + \frac{i\epsilon}{2P_+x}\Big]\cdot$$

$$\cdot 2P_+(1-x)\Big[-p_- + \frac{P_-}{2} - \frac{p_\perp^2+m_2^2}{2P_+(1-x)} + \frac{i\epsilon}{2P_+(1-x)}\Big]\cdot$$

$$\cdot 2P_+(x-y)\Big[p_- - q_- - \frac{(\vec{p}_\perp-\vec{q}_\perp)^2+\mu^2}{2P_+(x-y)} + \frac{i\epsilon}{2P_+(x-y)}\Big]\cdot$$

$$\cdot 2P_+y\Big[q_- + \frac{P_-}{2} - \frac{\vec{q}_\perp^{\,2}+m_1^2}{2P_+y} + \frac{i\epsilon}{2P_+y}\Big]\cdot \qquad (12)$$

$$\cdot 2P_+(1-y)\Big[-q_- + \frac{P_-}{2} - \frac{\vec{q}_\perp^{\,2}+m_2^2}{2P_+(1-y)} + \frac{i\epsilon}{2P_+(1-y)}\Big]\Big\}^{-1}.$$

Integrating over p_- and q_- it is easy to show that a nonzero contribution comes only from the region $0 < x < 1$.

Finally, the required function has the form:

$$\widetilde{G_0 K\ G_0}(p_1, p_2; q_1, q_2)\,\delta(p_1 + p_2 - q_1 - q_2) =$$

$$= (-2\pi i)^2 \{ 2P_+ x(1-x)\,[M^2 - \frac{\vec{p}_\perp^{\,2} + m_1^2}{x} - \frac{\vec{p}_\perp^{\,2} + m_2^2}{1-x}] \cdot$$

$$\cdot 2P_+(x-y)\,[M^2 - \frac{m_1^2 + \vec{q}_\perp^{\,2}}{y} - \frac{m_2^2 + \vec{p}_\perp^{\,2}}{1-x} - \frac{(\vec{p}_\perp - \vec{q}_\perp)^2 + \mu^2}{x-y}] \cdot$$

$$\cdot 2P_+ y(1-y)\,[M^2 - \frac{\vec{q}_\perp^{\,2} + m_1^2}{y} - \frac{\vec{q}_\perp^{\,2} + m_2^2}{1-y}] \}^{-1} .$$

$$\cdot \theta(x)\,\theta(1-x)\,\theta(y)\,\theta(1-y)\,\theta(x-y) + (x \leftrightarrow y) . \tag{13}$$

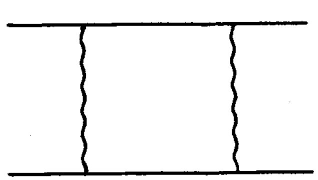
From eqs. (9), (7) and (13) we obtain for the quasipotential the following expression

$$V(x,y,P_+) = \theta(x)\,\theta(1-x)\,\frac{\theta(x-y)\,\theta(y)\,\theta(1-y)}{(x-y)[M - \frac{m_1^2 + \vec{q}_\perp^{\,2}}{y} - \frac{m_2^2 + \vec{p}_\perp^{\,2}}{1-x} - \frac{(\vec{p}_\perp^{\,2} - \vec{q}_\perp^{\,2}) + \mu^2}{x-y}} + \tag{14}$$

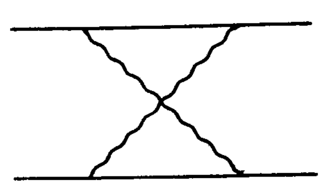
$$+ (x \leftrightarrow y) .$$

4. Projective Properties of the Quasipotential for Ladder-Type Diagrams

Consider the second-order diagrams

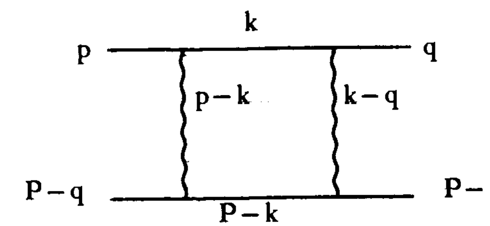


and



Examining the functions $\widetilde{G_0 K\ G_0}$ corresponding to these diagrams by the method given in the previous section one can easily show that only the regions $0 < x < 1$ and $0 < y < 1$ contribute there.

Indeed, introducing the following notation for momenta



the function $\widetilde{G_0 K\ G_0}$ is written as follows:

$$\widetilde{G_0 K\ G_0}(x,y,z,P) = \int d^4k \int dp_- dq_- (2P\ )^{-8} \cdot$$

$$\cdot [\,x(1-x)(x-z)\,z(1-z)(z-y)\,y(1-y)\,]^{-1} \cdot$$

$$\cdot (p_- - A_1 + \frac{i\epsilon}{x})^{-1} (P_- - p_- - A_2 + \frac{i\epsilon}{1-x})^{-1} (p_- - k_- - A_3 + \frac{i\epsilon}{x-z})^{-1} \cdot$$

$$\cdot (k_- - A_4 + \frac{i\epsilon}{z})^{-1} (P_- - k_- - A_5 + \frac{i\epsilon}{1-z})^{-1} \cdot \tag{15}$$

$$\cdot (k_- - q_- - A_6 + \frac{i\epsilon}{z-y})^{-1} (q_- - A_7 + \frac{i\epsilon}{y})^{-1} (P - q_- - A_8 + \frac{i\epsilon}{1-y})^{-1} ,$$

where A_{1-8} are the terms independent of integration momenta, and

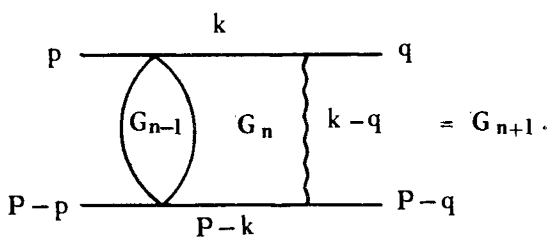
$$x = \frac{p_+}{P_+}, \qquad y = \frac{q_+}{P_+}, \qquad z = \frac{k_+}{P_+} .$$

Let $x < 0$, then poles in the first and second denominators have the same direction of their contours, and from the third denominator we obtain the condition $x > z \ (z < 0)$ for the integral over p_- be nonzero. In this case the poles in the third, fourth and fifth denominators again have the same direction of their contours, and in order that the integral over k_- differ from zero the condition $z > y \ (y < 0)$ is taken. However, then the integral over q_- necessarily becomes zero. Thus, the region $x < 0$ gives no contribution.

In a completely analogous way it can be proved that both for the diagram under consideration and for the crossed diagram the regions $x > 1$, $y < 0$, $y > 1$ also make no contribution.

A proof of vanishing of contributions from the regions $x<0$, $x > 1$, $y < 0$ will be given below for a ladder type diagram by mathematical induction.

Consider an arbitrary ladder diagram in the form



Let G_n reduce to zero for $x < 0$

$$G_n(x,z) = \int G'_{n-1}(P, x, p, z, k) \cdot \tag{16}$$

$$\cdot \left(k_- + B_1 + \frac{i\epsilon}{z}\right)^{-1} \left(P_- - k_- + B_2 + \frac{i\epsilon}{1-z}\right)^{-1} dp_- dk_- ,$$

where B_i do not depend on k_-, z, p_-, x, and the prime means that in G'_{n-1} times are not equating and this function contains all the coefficients of the latter denominators, noncontributing to the pole structure.

Then we shall prove that the function also

$$G_{n+1}(x,y,z) = \int G'_n(P, x, p, z, k) \cdot$$

$$\cdot \left(k_- - q_- + B_3 + \frac{i\epsilon}{z-y}\right)^{-1} \left(q_- + B_4 + \frac{i\epsilon}{y}\right)^{-1} \cdot \tag{17}$$

$$\cdot \left(P_- - q_- + B_5 + \frac{i\epsilon}{1-y}\right)^{-1} dp_- dk_- dq_-$$

becomes zero for $x < 0$.

Consider the first case, when

$$\int dp_- G'_{n-1}(P, x, p, z, k) \qquad \text{for} \quad x < 0$$

but then obviously, $G_{n+1} = 0$ as well, for $x < 0$.

The second case, when

$$\int dp_- G'_{n-1}(P, x, p, z, k) \neq 0 \qquad \text{for} \quad x < 0 .$$

In this case we take the z variable in a region such that the integral over k_- be zero as here this is a unique possibility to satisfy the initial assumption.

There are two possible versions. The first one, when

$z < 0$, $1 - z > 0$

and then, in order that $G_{n+1} \neq 0$ as the integral over k_- it should be supposed that

$z - y > 0$, $y < 1$.

But hence it immediately follows that $G_{n+1} = 0$ in integrating over q_-.

The second version, when

$z > 0$, $1 - z < 0$.

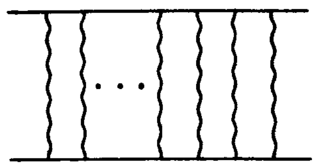
Hence, in order that $G_{n+1} \neq 0$ as the integral over k_- one should assume that

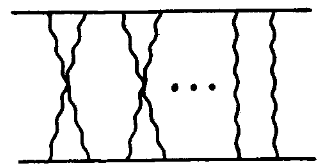
$z - y < 0$, $y > 1$.

However, in this case integrating over q_- reduces G_{n+1} to zero.

Thus, for $x < 0$ from the assumption that $G_n = 0$ it follows that $G_{n+1} = 0$ as well.

Analogously, it can be shown that also in the regions $x > 1$, $y < 0$, $y > 1$ arbitrary diagrams of the type





do not contribute to $\widetilde{G_0 K G_0}$.

5. Conclusion

The very fact of a possibility to establish the projective properties of the Green functions in the framework of the quasipotential approach, without using any additional assumptions is rather interesting. We would like to emphasize that the projective properties of the Green functions are very important in numerous investigations of the parton model or in considerations of composite particles in the frame $p_z \to \infty$, but they are only postulated there.

This note presents an argument in favour of further developing the quasipotential approach to the study of composite particles, as this approach clearly reflects the basic properties of elementary systems both in the case when these really consist of partons and even if the parton language is simply a suitable guide for describing their behaviour.

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