ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E2 - 8126

R.N.Faustov, V.R.Garsevanishvili,
A.N.Kvinikhidze, V.A.Matveev, A.N.Tavkhelidze

RELATIVISTIC FORM FACTORS OF COMPOSITE PARTICLES

1974

ЛАБОРАТОРИЯ
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

E2 - 8126

R.N.Faustov, V.R.Garsevanishvili,
A.N.Kvinikhidze, V.A.Matveev, A.N.Tavkhelidze

RELATIVISTIC FORM FACTORS OF COMPOSITE PARTICLES

Submitted to *ТМФ*

1. Introduction

Recently various composite models, such as quark model, parton model and others, have extensively been used in the elementary particle theory. In this connection the problem of a self-consistent relativistic description of interactions of composite particles is of much importance. The effective method of describing the properties of relativistic composite systems is the quasipotential approach in quantum field theory /1/.

In the present paper we develop a theory of the form factors of composite systems on the basis of the relativistically covariant quasipotential equations /2/. The relativistically covariant quasipotential approach is based on a relativistic generalization of the notion of equal time in the description of a particle system.

We recall the main aspects of this approach in the simplest example of two scalar particles. The Bethe-Salpeter amplitude

$$\chi_P(x_1, x_2) = \langle 0 \,|\, T(\phi_1(x/2)\phi_2(-x/2)) \,|\, P \rangle \qquad (I.1)$$

is known to contain an "unphysical" (from the point of view of quantum mechanics) variable of the relative time of two particles. In refs./2/ one suggested the following definition of the equal time wave function

$$\psi_P(x) = \chi_P(x)|_{Px=0} \,. \qquad (I.2)$$

Thus, the relativistic wave function of two particles is determined by the values of the Bethe-Salpeter amplitude (I.4) on the space-like surface $Px = P(x_1 - x_2) = 0$.

In the c.m.s. ($\vec{P}=0$) the equal time condition (2) means that the particle times coincide $t_1 = t_2$, while in an arbitrary system this equality holds for generalized invariant "times" of particles $\tau_i = \frac{1}{\sqrt{P^2}}(Px_i)$.

In the momentum representation the definition (1.2) for the relativistic wave function of composite particles takes the form

$$\psi_P(\pi) = \int_{-\infty}^{\infty} d\alpha\, \chi_P(\alpha n + \pi), \tag{I.3}$$

where the variables α and π_μ are determined in the following way

$$p_\mu = \frac{1}{2}(p_1 - p_2)_\mu = \alpha n_\mu + \pi_\mu;$$
$$n_\mu = \frac{P_\mu}{\sqrt{P^2}}; \quad (P\pi) = 0. \tag{I.4}$$

As is shown in refs.[2] the wave function (I.3) obeys the relativistically covariant quasipotential equation

$$[P^2 - (\omega_1 + \omega_2)^2]\psi_P(\pi) = \frac{\omega_1 + \omega_2}{2\omega_1\omega_2}\int U_P(\pi,\pi')\psi_P(\pi') \times$$
$$\times \delta(n\pi')d\pi', \tag{I.5}$$

where $U_P(\pi,\pi')$ is the integral operator defined for $(P\pi) = (P\pi') = 0$; $\omega_i = \sqrt{m_i^2 - \pi^2}$.

The following normalization condition holds:

$$i(2\pi)^4 \int \psi_P^+(\pi)\frac{\partial \tilde{G}^{-1}(P;\pi,\pi')}{\partial P^2}\psi_P(\pi')\delta(n\pi)\delta(n\pi')d\pi d\pi' = 1, \tag{I.6}$$

where

$$\tilde{G}^{-1}(P;\pi,\pi') = \frac{1}{2\pi i}\left\{\frac{2\omega_1\omega_2}{\omega_1+\omega_2}\delta_P(\pi-\pi')[P^2 - (\omega_1+\omega_2)^2] - U_P(\pi,\pi')\right\} \tag{I.7}$$

is the inverse Green function of a system of interacting particles. The problem is to express the vertex functions and the related form factors of composite particles in terms of the relativistically covariant quasipotential wave functions.

In refs.[3-6] one suggested a general representation for the matrix element of the local current operator between bound states

$$\langle P | J(0) | Q \rangle = \int d\pi d\pi' \delta(n\pi)\delta(n\pi')\psi_P^+(\pi)\tilde{\Gamma}(P,\pi;Q,\pi')\psi_Q(\pi') \tag{I.8}$$

with a certain vertex integral operator $\tilde{\Gamma}(P,\pi;Q,\pi')$. A detailed study of the currents of composite particles in the method of coherent states is given in ref.[7]. The present paper is based on the results of refs.[1-6] and is a further development of them.

Depending on a specific character of the problem in consideration a choice of other surfaces is possible, which differs from that made in definition (I.2). As long ago as 1949 Dirac had pointed out the advantages which emerge in using the so-called "light-front" variables:

$$x_\pm = \frac{x_0 \pm x_3}{2}; \quad p_\pm = p_0 \pm p_3. \tag{I.9}$$

In particular herewith "linearization" of the square root in the expression for a particle energy takes place and the negative energy values vanish. Variables (I.9) appear to be especially convenient for investigation of the behaviour of form factors and scattering processes of composite particles at high energies and momentum transfers. In fact as the momentum values in the mentioned case are much larger than the particle masses, so these particles to a certain extent may be considered as ''light-like'' objects.

As a result we come to the following definition of the quasipotential wave function

$$\psi_P(x_-, \vec{x}_\perp) = \chi_P(x)|_{x_+=0} \tag{I.10}$$

thus given on the hyperplane of the ''light-front''
$x_0 + x_3 = 0$.

This approach will be discussed at greater length in Section 3.

2. Construction of Relativistically Covariant Form Factors of Composite Particles. The Vertex Function of a Composite System

The reaction of a composite systems on a weak external perturbation corresponding to the local field $A(x)$ is described in quantum field theory[8] by the expression

$$< P,\alpha| \frac{\delta S}{\delta \tilde{A}(k)} |Q,\beta>|_{A=0} = (2\pi)^4 \delta(P-Q-k) <P,\alpha| \ J(0)|Q,\beta>. \tag{2.1}$$

Here $J(x)$ is the local current of the system

$$J(x) = i \frac{\delta S}{\delta A(x)} S^+, \tag{2.2}$$

$|P,\alpha>$ and $|Q,\beta>$ are the state vectors of composite particles with momenta P and Q and the sets of additional quantum numbers α and β, normalized in a relativistically invariant manner

$$<P,\alpha|Q,\beta> = 2P_0 (2\pi)^3 \delta(\vec{P}-\vec{Q}) \delta_{\alpha\beta} \,. \tag{2.3}$$

Below we suggest a method of constructing relativistically covariant form factors of composite particles in terms of equal time quasipotential wave functions

We consider the quantity R defined by the vacuum expectation value of the chronologically ordered product of Heisenberg field operators of the scalar particles $\phi_i(x_i)$ and a certain local current $J(x)$

$$R(x_1,x_2,y_1,y_2) = <0|T(\phi_1(x_1)\phi_2(x_2)J(0)\phi_1^+(y_1)\phi_2^+(y_2))|0> =$$

$$= \frac{1}{(2\pi)^{16}} \int dp_1 dp_2 dq_1 dq_2 \, e^{-i\sum_{j=1}^{2}(p_j x_j - q_j y_j)} \quad R(p_1,p_2;q_1,q_2). \tag{2.4}$$

Introducing new variables, relative coordinates and momenta

$$X = \frac{x_1+x_2}{2}, \quad x = x_1 - x_2$$

$$Y = \frac{y_1+y_2}{2}; \quad y = y_1 - y_2$$

$$P = p_1 + p_2 ; \quad p = \frac{p_1 - p_2}{2} \tag{2.5}$$

$$Q = q_1 + q_2 ; \quad q = \frac{q_1 - q_2}{2}$$

we rewrite expression (2.4) in the form

$$R(X,x;Y,y) = \frac{1}{(2\pi)^{16}} \int dPdpdQdq \, e^{-i(PX-QY)-i(px-qy)} \times$$

$$\times R(P,p;Q,q). \tag{2.6}$$

As is known[9], the quantity R can be presented in the form

$$R = G\Gamma G, \tag{2.7}$$

i.e.,

$$R(X,x;Y,y) = \int G(X-X';x,x')\Gamma(X',x';Y',y') \times$$

$$\times G(Y'-Y,y',y) \quad dX'dx'dY'dy' \tag{2.7a}$$

or

$$R(P,p;Q,q) = \int Q(P;p,p')\Gamma(P,p',Q,q')G(Q,q',q)\,dp'dq'. \quad (2.7b)$$

Here

$$G(X-X';x,x') <0|T(\phi_1(x_1)\phi_2(x_2)\phi_1^+(x_1')\phi_2^+(x_2'))|0> =$$

$$(2.8)$$

$$= \frac{1}{(2\pi)^8}\int dP\,dp\,dp'\,e^{-iP(X-X')-i(px-p'x')}\,G(P,p,p')$$

is the two-particle Green function of scalar fields $\phi_i(x_i)$ and the vertex function Γ is the sum of all two-particle irreducible diagrams for 5-point Green function (2.4) (see Fig.1).

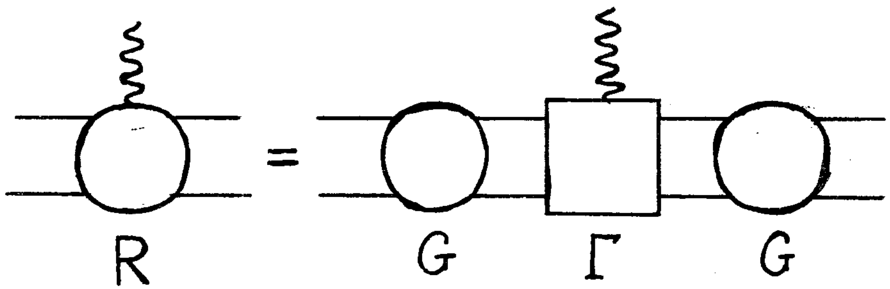

Fig. 1.

Passing to the relativistically covariant equal time description of a two-particle system we consider the quantity

$$\tilde{R}(P,\pi;Q,\pi') = \int_{-\infty}^{\infty} d\alpha\, d\alpha'\, R(P,p;Q,q), \quad (2.9)$$

where

$$p = \alpha n + \pi; \quad n = \frac{P}{\sqrt{P^2}}; \quad (P\pi) = 0$$

$$(2.10)$$

$$q = \alpha' n' + \pi'; \quad n' = \frac{Q}{\sqrt{Q^2}}; \quad (Q\pi') = 0.$$

The quantity $\tilde{R}$ can be presented in the following form

$$\tilde{R} = \tilde{G}\tilde{\Gamma}\tilde{G}, \quad (2.11)$$

i.e.

$$\tilde{R}(P,\pi;Q,\pi') = \int \tilde{G}(P;\pi,\pi'')\tilde{\Gamma}(P,\pi'',Q,\pi''') \times$$

$$\times \tilde{G}(Q;\pi''',\pi')(d\pi'')_P\,(d\pi''')_Q, \quad (2.12)$$

where $(d\pi)_P$ is an invariant volume element

$$(d\pi)_P = d^4\pi\,\delta(n\pi); \quad (2.13)$$

and $\tilde{\Gamma}(P,\pi;Q,\pi')$ is the vertex integral operator. Let us show that the quantity $\tilde{\Gamma}$ defines the form factors of composite particles. Starting from the spectral properties of the 5-point Green function (2.4), it is possible to show that the quantity $\tilde{R}$ has double pole singularities

$$\tilde{R}(P,\pi;Q,\pi') \underset{\substack{P^2\to M_\alpha^2\\ Q^2\to M_\beta^2}}{\simeq} [i(2\pi)^4]^2\,\frac{\psi_{P,\alpha}(\pi)<P,\alpha|J(0)|Q,\beta>\psi^+_{Q,\beta}(\pi')}{(P^2-M_\alpha^2)(Q^2-M_\beta^2)}, \quad (2.14)$$

where M_α and M_β are the masses of composite particles with quantum numbers α and β, respectively. One the other hand, owing to the knowledge of the pole singularities of the two-particle Green function

$$\tilde{G}(P;\pi,\pi') \underset{P^2\to M_\alpha^2}{\simeq} i(2\pi)^4\,\frac{\psi_{P,\alpha}(\pi)\psi^+_{P,\alpha}(\pi')}{P^2-M_\alpha^2} \quad (2.15)$$

we find from (2.12)

$$\tilde{R}\underset{\substack{P^2\to M_\alpha^2\\ Q^2\to M_\beta^2}}{(P,\pi;Q,\pi')} \simeq [i(2\pi)^4]^2 \frac{\psi_{P,\alpha}(\pi)\psi^+_{Q,\beta}(\pi')}{(P^2-M_\alpha^2)(Q^2-M_\beta^2)}\times \tag{2.16}$$

$$\times \int \psi^+_{P,\alpha}(\pi)\tilde{\Gamma}_{\alpha\beta}(P,\pi;Q,\pi')\psi_{Q,\beta}(\pi')(d\pi)_P(d\pi')_Q\,,$$

where

$$\tilde{\Gamma}_{\alpha\beta}(P,\pi;Q,\pi') = \tilde{\Gamma}(P,\pi;Q,\pi')\,\Big|_{\substack{P^2=M_\alpha^2\\ Q^2=M_\beta^2}} \,. \tag{2.17}$$

Comparing equations (2.14) and (2.16) we get a relativistically covariant expression for the form factor

$$\langle P,\alpha|J(0)|Q,\beta\rangle = \int \psi^+_{P,\alpha}(\pi)\tilde{\Gamma}_{\alpha\beta}(P,\pi;Q,\pi')\times$$

$$\times\psi_{Q,\beta}(\pi')(d\pi)_P(d\pi')_Q \tag{2.18}$$

in terms of equal time wave functions and the generalized vertex operator (2.17).

The Ward Identity and Relativistic Normalization of the Wave Functions of Composite Particles

The Ward identity for the vertex functions corresponding to the conserved vector current is tightly connected with the relativistic normalization condition of the wave functions of composite particles.

For a system of two scalar particles the generalized Ward identity is of the form

$$(P-Q)_\mu\,\tilde{\Gamma}_\mu(P,\pi;Q,\pi') = i(2\pi)^4 Z[\tilde{G}^{-1}(P;\pi,\pi') - $$

$$-\tilde{G}^{-1}(Q;\pi,\pi')] \tag{2.19}$$

or

$$(P-Q)_\mu\,\tilde{R}_\mu(P,\pi;Q,\pi') = i(2\pi)^4 Z[\tilde{G}(P;\pi,\pi')-\tilde{G}(Q,\pi,\pi')]\,. \tag{2.20}$$

The quantity Z is the charge of the two-particle system corresponding to the conserved vector current J_μ

$$\langle P,\alpha|\int_{nx=0} d\sigma_\mu\,J_\mu(x)|Q,\beta\rangle = Z\langle P,\alpha|Q,\beta\rangle\,. \tag{2.21}$$

It is not difficult to verify relation (2.20) comparing the residues at the pole singularities $P^2 = M_\alpha^2$ and $Q^2 = M_\beta^2$ in the left- and right-hand sides of this relation.

Rewritting identity (2.19) in the form

$$n_\mu\tilde{\Gamma}_\mu(P,\pi;Q,\pi') = i(2\pi)^4 Z\left[\frac{\tilde{G}^{-1}(P;\pi,\pi')-\tilde{G}^{-1}(Q;\pi,\pi')}{\sqrt{P^2}-\sqrt{Q^2}}\right] \tag{2.22}$$

and passing to the limit $P^2-Q^2\to 0$ we find the relation

$$n_\mu\tilde{\Gamma}_\mu(P,\pi;P,\pi') = i(2\pi)^4\,2\sqrt{P^2}Z\,\frac{\partial\tilde{G}^{-1}(P,\pi,\pi')}{\partial P^2} \tag{2.23}$$

which establishes the connection with the normalization condition (I.6) of the relativistically covariant wave functions of composite particles.

Perturbation Theory for Vertex Operator

Formulas (2.7) and (2.11) give an explicit expression for the vertex operator of composite particles

$$\tilde{\Gamma}_{\alpha\beta}(P,\pi;Q,\pi') = \lim_{\substack{P^2 \to M^2_\alpha \\ Q^2 \to M^2_\beta}} \int \tilde{G}^{-1}(P;\pi,\pi'') \times \tag{2.24}$$
$$\times [G\Gamma G](P,\pi'';Q,\pi''')\tilde{G}^{-1}(Q;\pi''',\pi')(d\pi'')_P\,(d\pi''')_Q$$

in terms of 4- and 5-point Green functions, G and R. Using the perturbation methods of finding the functions G and R it is possible to get by means of (2.24) a power series in the coupling constant for the vertex operator of composite particles.

To demonstrate this method we consider the so-called "impulse" approximation for the vertex operator $\tilde{\Gamma}$ which corresponds to the limit of "weakly coupled" (noninteracting) particles (see Fig. 2).

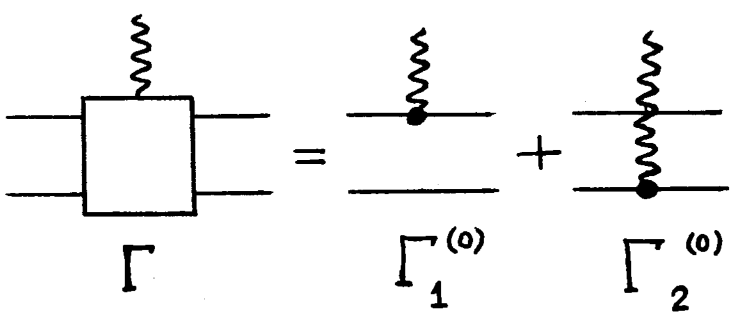

Fig. 2

For the vertex operator corresponding to the conserved vector current we find from (2.24)

$$\tilde{\Gamma}_\mu = \tilde{\Gamma}^{(0)}_{1\mu} + \tilde{\Gamma}^{(0)}_{2\mu} \tag{2.25a}$$

$$\tilde{\Gamma}^{(0)}_{i\mu} = [\tilde{G}^{(0)}]^{-1}[\widetilde{G^{(0)}\Gamma^{(0)}_{i\mu}G^{(0)}}][\tilde{G}^{(0)}]^{-1}, \tag{2.25b}$$

where

$$\Gamma^{(0)}_{i\mu} = (2\pi)^4 Z_i\,(p_i + q_i)_\mu\,\delta(p_j - q_j)[\tilde{G}^{(0)}_j(p_j)]^{-1}_{i \neq j}. \tag{2.26}$$

Here

$$G^{(0)}(P,p) = G^{(0)}_1(p_1)G^{(0)}_2(p_2) = i^2 \prod_{i=1}^{2}(p_i^2 - m_i^2)^{-1}$$
$$G^{(0)}(Q,q) = G^{(0)}_1(q_1)G^{(0)}_2(q_2) = i^2 \prod_{i=1}^{2}(q_i^2 - m_i^2)^{-1} \tag{2.27}$$

are the two-particle Green functions for free particles with masses m_i and charges Z_i.

Then the vertex operator takes the form

$$\tilde{\Gamma}_\mu(P,\pi;Q,\pi') = i^3(2\pi)^4 Z_1\,[\tilde{G}^{(0)}(P,\pi)]^{-1}[\tilde{G}^{(0)}(Q,\pi')]^{-1} \times$$

$$\times \int_{-\infty}^{\infty} \frac{d\beta\, d\beta'\,(P + n\beta + \pi + Q + n'\beta' + \pi')_\mu\,\delta^{(4)}(n'\beta' + \pi' - n\beta - \pi)}{[(n\beta+\pi)^2 - m_2^2 + i\epsilon][(P + n\beta + \pi)^2 - m_1^2 + i\epsilon][(Q + n'\beta' + \pi')^2 - m_1^2 + i\epsilon]} +$$

$$+ (1 \to 2). \tag{2.28}$$

Next, dividing the argument of the 4-dimensional δ-function into longitudinal and transverse components with respect to the direction of the four-vector Q and integrating over β' by means of the one-dimensional δ-function of the longitudinal component we have

$$\int_{-\infty}^{\infty} d\beta'\,\delta^{(4)}(n'\beta' + \pi' - n\beta - \pi) = \delta^{(3)}_Q(\pi' - (\pi - n'(n'\pi)) -$$
$$- \beta(n - n'(nn'))) \tag{2.29}$$

$$\beta' = (nn')\beta + (n\pi),$$

where, according to (2.9), the three-dimensional δ-function is defined in the following manner

$$\int \psi(\pi')\delta_Q^{(3)}(\pi-\pi')(d\pi')_Q=\psi(\pi). \qquad (2.30)$$

The remaining integral over β can be taken with the account of only pole singularities in the denominator (2.28). To simplify the expression (2.28) within the framework of the "impulse" approximation it is possible to neglect "small" quantities proportional to

$$(\sqrt{P^2}-\sqrt{m_1^2-\pi^2}-\sqrt{m_2^2-\pi^2})$$

and

$$(\sqrt{Q^2}-\sqrt{m_1^2-\pi^2}-\sqrt{m_2^2-\pi^2})$$

which obviously is equivalent to rejecting terms of the order of the interaction potential between particles in the r.h.s. of equality (2.18). Then for the invariant form factor F of a composite system defined by the relation

$$\langle P|J_\mu(0)|Q\rangle=(P+Q)_\mu F(\Delta^2),\quad \Delta=P-Q \qquad (2.31)$$

we finally get

$$F(\Delta^2)=2(2\pi)^3 Z_1\int \psi_P^+(\pi)\psi_Q(\pi-n'(\pi n')-\omega_2(n-n'(nn')))\times$$

$$\times\frac{\omega_2[\omega_1\sqrt{P^2}-\frac{1}{4}(P-Q)^2]}{P^2-\frac{1}{4}(P-Q)^2}(d\pi)_P;\quad \omega_i=\sqrt{m_i^2-\pi^2}. \qquad (2.32)$$

Using the explicit invariance of this expression we pass to the Breit frame of reference, in which

$$\vec{P}=-\vec{Q}=\frac{1}{2}\vec{\Delta};\quad P_0=Q_0=E=\sqrt{M^2+\frac{1}{4}\vec{\Delta}^2} \qquad (2.33)$$

$$(P-Q)^2=-\vec{\Delta}^2.$$

Introduce a new three-dimensional integration variable

$$L_P^{-1}\pi=(0,\vec{p});\quad (P\pi)=0, \qquad (2.34)$$

where L_P is the matrix of a Lorentz transformation such that

$$L_P^{-1}P=(M,\vec{0}). \qquad (2.35)$$

Employing the transformation properties of the wave function by the Lorentz transformations, for eq. (2.32) we get finally

$$F(-\vec{\Delta}^2)=2(2\pi)^3 Z_1\int \psi^+_{M,\vec{0}}(\vec{p})\psi_{M,\vec{0}}(\vec{p}+\vec{\Delta}(\frac{\vec{\Delta}\vec{p}}{2M^2}-\frac{p_0\omega_2}{M^2}))\times \qquad (2.36)$$

$$\times\frac{\omega_2(\omega_1 M+\frac{1}{4}\vec{\Delta}^2)}{M^2+\frac{1}{4}\vec{\Delta}^2}d\vec{p}+(1\leftrightarrow 2)$$

which is in agreement with the result obtained in refs [6].

From a comparison with the normalization condition we find the required value for the form factor at the zero transfer

$$F(0)=Z_1+Z_2. \qquad (2.37)$$

3. Quasipotential Method in the Variables of the "Light Front"

In this section we develop quasipotential formalism using "light front" variables which have been introduced by Dirac [10]. Consider the Bethe-Salpeter amplitude

$$\chi_{P,a}(x_1,x_2)=\langle 0|T(\phi_1(x_1)\phi_2(x_2))|P,a\rangle= \qquad (3.1)$$

$$=e^{-iPX}\chi_{P,a}(x).$$

The variables P, X and x are defined by the formulas (2.5). Let us introduce the "light front" variables

$$x_{\pm} = \frac{x_0 \pm x_3}{2}, \quad p_{\pm} = p_0 \pm p_3, \quad P_{\pm} = P_0 \pm P_3 \tag{3.2}$$

and pass to the momentum representation

$$\chi_{P,a}(x) = \chi_{P,a}(x_+, x_-, \vec{x}_\perp) = \int d^4 p\, e^{-ipx} \chi_{P,a}(p) =$$
$$= \frac{1}{2} \int dp_+ dp_- d\vec{p}_\perp\, e^{-i(p_+ x_- + p_- x_+ - \vec{p}_\perp \vec{x}_\perp)} \chi_{P,a}(p_-, p_+, \vec{p}_\perp). \tag{3.3}$$

Determine the function

$$\psi_{P,a}(p_+, \vec{p}_\perp) = \int_{-\infty}^{\infty} dp_- \chi_{P,a}(p_-, p_+, \vec{p}_\perp). \tag{3.4}$$

It is easy to see that the function $\psi_{P,a}(p_+, \vec{p}_\perp)$ depends on the values of the Bethe-Salpeter amplitude on the hyperplane

$$x_0 + x_3 = 0. \tag{3.5}$$

In fact, using definition (3.4) and the Fourier transformation (3.3) we get

$$\psi_{P,a}(p_+, \vec{p}_\perp) = \frac{2}{(2\pi)^3} \int dx_+ dx_- d\vec{x}_\perp\, \delta(x_+) e^{i(p_+ x_- - \vec{p}_\perp \vec{x}_\perp)} \times$$
$$\times \chi_{P,a}(x_+, x_-, \vec{x}_\perp). \tag{3.6}$$

We now consider the two-particle Green function (2.8) and determine the Fourier transform of the "two-time" Green function

$$\overline{G}(P; p_+, \vec{p}_\perp; p'_+, \vec{p}\,'_\perp) = \int_{-\infty}^{\infty} dp_- dp'_- \, G(P; p, p'). \tag{3.7}$$

For free particles we have

$$G^{(0)}(P; p, p') = \frac{-\delta^{(4)}(p-p')}{[(\frac{P}{2}+p)^2 - m_1^2 + i\epsilon][(\frac{P}{2}-p)^2 - m_2^2 + i\epsilon]} \equiv$$
$$\equiv \tilde{G}^{(0)}(P, p)\, \delta^{(4)}(p-p'), \tag{3.8}$$

Performing an integration according to the definition (3.7) we obtain

$$\overline{G}^{(0)}(P; p_+, \vec{p}_\perp; p'_+, \vec{p}\,'_\perp) = \theta(x)\theta(1-x)\tilde{G}^{(0)}(P; p_+, \vec{p}_\perp; p'_+, \vec{p}\,'_\perp), \tag{3.9}$$

where

$$\tilde{G}^{(0)}(P; p_+, \vec{p}_\perp; p'_+, \vec{p}\,'_\perp) = \frac{4\pi i \delta(p_+ - p'_+)\delta^{(2)}(\vec{p}_\perp - \vec{p}\,'_\perp)}{P_+ x(1-x)[P_+^2 + \vec{P}_\perp^2 - \frac{(\vec{P}/2+\vec{p})_\perp^2 + m_1^2}{x} - \frac{(P/2-p)_\perp^2 + m_2^2}{1-x}]} \equiv$$
$$\equiv \tilde{G}^{(0)}(P; p_+, \vec{p}_\perp)\, \delta(p_+ - p'_+)\, \delta^{(2)}(\vec{p}_\perp - \vec{p}\,'_\perp). \tag{3.10}$$

In (3.9) the variable x is introduced according to the formula

$$x = \frac{1}{2} + \frac{p_+}{P_+}. \tag{3.11}$$

It is obvious that when the variable x changes in the limits

$$0 < x < 1 \tag{3.12}$$

the variable p_+ changes in the interval

$$-P_+/2 < p_+ < P_+/2. \tag{3.13}$$

We pay attention to the projection properties (the presence of θ -functions) of the function G . It has been shown that such a kind of projection properties is attributed to the Green functions calculated for a wide class of perturbation theory graphs /16/ .

Let us define now the inverse operator by the relation

$$\int_{-P_+/2}^{-P_+/2} dp''_+ \int d\vec{p}''_\perp \tilde{G}^{-1}(P;p_+,\vec{p}_\perp,p'_+,\vec{p}''_\perp)\tilde{G}(P;p''_+,\vec{p}''_\perp;p'_+,\vec{p}'_\perp) = \delta(p_+-p'_+)\delta^{(2)}(\vec{p}_\perp-\vec{p}'_\perp). \tag{3.14}$$

For the free particles we have

$$\tilde{G}^{(0)-1}(P;p_+,\vec{p}_\perp;p'_+,\vec{p}'_\perp) = \tilde{G}^{(0)-1}(P;p_+,\vec{p}_\perp)\delta(p_+-p'_+)\delta^{(2)}(\vec{p}_\perp-\vec{p}'_\perp).$$

Introduce a quasipotential (3.15)

$$\tilde{G}^{-1}(P;p_+,\vec{p}_\perp;p'_+,\vec{p}'_\perp) = \tilde{G}^{(0)-1}(P;p_+,\vec{p}_\perp)\delta(p_+-p'_+)\times$$

$$\times\delta^{(2)}(\vec{p}_\perp-\vec{p}'_\perp) - \frac{1}{4\pi i}V(P;p_+,\vec{p}_\perp;p'_+,\vec{p}'_\perp). \tag{3.16}$$

After simple transformations the equation for the wave function $\psi_P(x,\vec{p}_\perp)$ takes the form

$$\left[P^2 - \frac{(\vec{p}_\perp+(1/2-x)\vec{P}_\perp)^2}{x} - \frac{(\vec{p}_\perp+(1/2-x)\vec{P}_\perp)^2}{1-x}\right]\psi_P(x,\vec{p}_\perp) =$$

$$= \frac{1}{x(1-x)}\int_0^1 dx' \int d\vec{p}'_\perp V(P;x,\vec{p}_\perp;x',\vec{p}'_\perp)\psi_P(x',\vec{p}'_\perp). \tag{3.17}$$

The obtained equation defines the wave function of a bound state with arbitrary transverse components of the total momentum. Comparing it with the equation in the

frame $\vec{P}_\perp = 0$ and taking into account the transformation properties of the quasipotential it is not difficult to show that

$$\psi_P(x,\vec{p}_\perp) = \psi_{\vec{P}_\perp=0}(x,\vec{p}_\perp+(1/2-x)\vec{P}_\perp). \tag{3.18}$$

Applying the methods similar to those suggested in Section 2, for the matrix elements of currents of composite particles we obtain

$$\langle P,\alpha|J(0)|Q,\beta\rangle = \int_{-P_+/2}^{P_+/2} dp_+dp'_+ \int d\vec{p}_\perp d\vec{p}'_\perp \psi^+_{P,\alpha}(x,\vec{p}_\perp)\times$$

$$\times\overset{\frown}{\Gamma}(P,x,\vec{p}_\perp;Q,x',\vec{p}'_\perp)\psi_{Q,\beta}(x',\vec{p}'_\perp), \tag{3.19}$$

where the vertex operator $\Gamma(P,x,\vec{p}_\perp;Q,x',\vec{p}'_\perp)$ is connected with the function R by the ralation

$$\overset{\frown}{R} = \overset{\frown}{G}\overset{\frown}{\Gamma}\overset{\frown}{G}, \tag{3.20}$$

$$\overset{\frown}{R} = \int_{-\infty}^{\infty} dp_-dq_-R(P,p;Q,q). \tag{3.21}$$

An analogue of formula (2.32) is

$$\langle P,\alpha|J_+(0)|Q,\beta\rangle = \frac{(2\pi)^3}{2}Q_+^2\{Z_1\int_0^1 dx(1-x)\int d\vec{p}_\perp\times$$

$$\times\psi^+_{P,\alpha}\left((p+\frac{P-Q}{2})_+,\vec{p}_\perp+\frac{\vec{P}_\perp-\vec{Q}_\perp}{2}\right)(2p+P)_+\psi_{Q,\beta}(p_+,\vec{p}_\perp)+$$

$$+\text{ analogous term with } Z_2\}. \tag{3.22}$$

Passing to the frame of reference where

$$P_+ = Q_+ \;;\; (P-Q)^2 = -\vec{\Delta}_\perp^2 = -(\vec{P}_\perp-\vec{Q}_\perp)^2$$

and accounting for the transformation properties of the wave functions (3.18) we get

$$F(-\vec{\Delta}_\perp^2) = \frac{Z_1(2\pi)^3}{2}\int_0^1 \frac{dx}{x(1-x)}\int d\vec{p}_\perp \Phi^+_{\vec{P}_\perp=0}(x,\vec{p}_\perp+(1-x)\vec{\Delta}_\perp)\times \tag{3.23}$$

$$\times\Phi_{\vec{P}_\perp=0}(x,\vec{p}_\perp) + \text{analogous term with } Z_2 ,$$

$$\Phi = P_+ x(1-x)\psi$$

The formula of this kind was derived in ref. /10/ by the methods derived in ref. /11/.

We now consider a somewhat other choice of "light front" variables when the light front normal is oriented in the direction of the total momentum of the system. This choice is more tightly related with the variables (I.4) of the covariant quasipotential method. To this end, we introduce the following parameterization of the relative momentum

$$p = L_P \check{p} = a n + \sum_{i=1}^{3} e_i \check{p}^i \quad ; \qquad n = \frac{P}{\sqrt{P^2}} , \tag{3.24}$$

where the Lorentz transformation L_P is defined by equality (2.37) and, consequently, in the c.m.s.

$$\check{p} = (a, \check{\vec{p}})$$

and three linearly independent four-vectors e_i possess the following properties

$$(e_i n) = 0; \quad (e_i e_j) = -\delta_{ij} \tag{3.25}$$

and in the explicit form

$$e_i^\mu = (L_P)^\mu_{\ i} = \{ \frac{P_i}{\sqrt{P^2}} , \delta_{ij} + \frac{P^i P^j}{\sqrt{P^2}(P_0+\sqrt{P^2})} \} .$$

We define the new basis vectors

$$e_\pm = n \pm e_3 \tag{3.26}$$

with the properties

$$(n e_\pm) = 1; \quad e_\pm^2 = 0; \quad e_+ e_- = 2$$

and expand in them the vector of the relative momentum

$$p = \check{p}_+ e_+ + \check{p}_- e_- + \sum_{i=1}^{2} (\check{p}_\perp)^i e_i ,$$

where the new variables

$$\check{p}_\pm = (e_\mp p) = a \pm \check{p}_3 ; \quad a = \frac{1}{2}(\check{p}_+ + \check{p}_-). \tag{3.27}$$

An interesting property of this choice of variables is that the total momentum vector has no "normal" component

$$\vec{P}_\perp = 0, \quad P_\pm = (e_\mp P) = \sqrt{P^2} .$$

Further it will also be convenient to use the variable η defined by

$$\eta - \frac{1}{2} = \frac{\check{p}_+}{P_+} . \tag{3.28}$$

Now by analogy with (3.4) and (3.7) we define the quasipotential function

$$\psi_P(\check{p}_+, \check{\vec{p}}_\perp) = \int_{-\infty}^{\infty} d\check{p}_- \chi_P(\check{p}_+, \check{p}_-, \check{\vec{p}}_\perp) \tag{3.29}$$

and the Green function

$$\overset{\frown}{G}(P; p_+, p_\perp; p'_+ p'_\perp) = \int_{-\infty}^{\infty} d\check{p}_- d\check{p}'_- G(P; p, p'). \tag{3.30}$$

In particular, for the free Green function we find the expression

$$\overset{\frown}{G}^{(0)}(P; \check{p}_+, \check{\vec{p}}_\perp; \check{p}'_+, \check{\vec{p}}'_\perp) = \frac{4\pi i\,\theta(\eta)\theta(1-\eta)\delta(\eta-\eta')\delta^{(2)}(\check{\vec{p}}_\perp - \check{\vec{p}}'_\perp)}{\eta(1-\eta)[P^2 - \frac{\check{\vec{p}}_\perp^2 + m_1^2}{\eta} - \frac{\check{\vec{p}}_\perp^2 + m_2^2}{1-\eta}]} \tag{3.31}$$

and, correspondingly, the quasipotential equation will be

$$[P^2 - \frac{\check{\vec{p}}_\perp^{\,2} + m_1^2}{\eta} - \frac{\check{\vec{p}}_\perp^{\,2} + m_2^2}{1-\eta}]\ \psi_P(\eta, \check{\vec{p}}_\perp) = \tag{3.32}$$

$$= \frac{1}{\eta(1-\eta)} \int_0^1 d\eta' \int d\check{\vec{p}}_\perp{}' \, V(P;\eta,\check{\vec{p}}_\perp;\eta',\check{\vec{p}}_\perp{}')\,\psi_P(\eta',\check{\vec{p}}_\perp{}').$$

The form factors of composite particles can be considered in a similar manner.

Note, that in a number of recent papers (see, e.g.,/10,13,14/) composite particles are described on the basis of the old-fashioned three-dimensional perturbation theory in the infinite momentum frame, which has been applied to the relativistic theory some years ago by Weinberg /11/. A close analogy was indicated between the equation of ref./11/ and the equations of the quasipotential type /1,2,15/ . Equation, which has been obtained in the present section, reproduces in the lowest approximation the equation of ref. /11/ . At the same time such an approach provides a regular method of constructing the quasipotential in higher approximations, possessing correct projection properties.

4. Conclusion

It should be noted that the propagation function for two free particles the coordinates of which are related by the condition of belonging to the "light front" hyperplane

$$x_0 + x_3 = 0$$

has the form

$$\tilde{G} \sim \frac{1}{P^2 - \frac{(\vec{p}_\perp + (1/2-x)\vec{P}_\perp)^2 + m_1^2}{x} - \frac{(\vec{p}_\perp + (1/2-x)\vec{P}_\perp)^2 + m_2^2}{1-x}} \tag{4.1}$$

which coincide with the propagation function usually assumed in the free partons model.

In the general case of interacting partons the wave function should be sought as the solution of the quasipotential equations (I.5) and (3.17) with normalization condition (I.6). This fact should be taken into account, e.g., when analyzing the density distribution of partons in a hadron.

The authors are deeply grateful to N.N.Bogolubov, A.A.Logunov, R.M.Muradyan for stimulating discussions and valuable remarks. Thanks are due to D.I.Blokhintsev, V.G.Kadyshevsky, S.P.Kuleshov, M.D.Mateev, R.M.Mir-Kasimov, D.V.Shirkov, A.N.Sissakian and L.A.Slepchenko for useful dicussions.

References:

1. A.A.Logunov, A.N.Tavkhelidze. Nuovo Cim., 29, 380 (1962). V.G.Kadyshevsky, A.N.Tavkhelidze. in "Problems of Theoretical Physics", dedicated to N.N.Bogolubov in the ocassion of his 60-th birthday, Nauka, M., 1969.
2. V.A.Matveev, R.M.Muradyan, A.N.Tavkhelidze. JINR preprints, E2-3498 (1967); P2-3900 (1968), Dubna.
3. A.N.Tavkhelidze. Talk at the Solvey Congress. 1967.
4. N.N.Bogolubov, V.A.Matveev, A.N.Tavkhelidze. Proceedings of the Varna Seminar on Elementary Particle Theory, 1968.
5. V.A.Matveev. JINR preprint, P2-3847, Dubna (1968).
6. R.N.Faustov. TMF, 3, 240 (1970); R.N.Faustov. Ann. Phys., 78, 176 (1973).
7. P.N.Bogolubov. JINR preprints, P2-5684, P2-6181, Dubna, 1971.
8. N.N.Bogolubov, D.V.Shirkov. Introduction to the Theory of Quantized Fields, Nauka, M., 1973.
9. S.Mandelstam. Proc. Roy. Soc., A233, 248 (1955).
10. J.F.Gunion, S.J.Brodsky, R.Blankenbecler. Phys.Rev., D8, 287 (1973).
11. S.Weinberg. Phys.Rev., 150, 1313 (1966).
12. P.A.M.Dirac. Rev. Mod. Phys., 21, 392 (1949).
13. L.Susskind. Phys.Rev., 165, 1535 (1968).
14. A.Atanasov. JINR preprint, E2-6202, Dubna, 1973.
15. R.Blankenbecker, R.Sugar. Phys.Rev., 142, 1051 (1966).

C.Fronsdal, L.E.Lundberg. Phys.Rev., D1, 3247 (1970).
I.Todorov. Phys. Rev., D3, 2351 (1971).

16. S.P.Kuleshov, A.N.Kvinikhidze, V.A.Matveev, A.N.Sissakian, L.A.Slepchenko. JINR preprint, E2-8128, Dubna (1974).

Received by Publishing Department
on July 19, 1974.