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STOCHASTIC PSEUDOFIELDS AND THE QUANTUM FIELD THEORY

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1. Introduction

In this paper we shall develop further the idea of stochastic pseudofields [1,2] regular and generalized ones, and indicate their possible use in quantum field theory. We will show that a mathematical scheme of the probabilistic type dealing with pseudofields (ρ- fields for shortness) incorporates the main results of the functional methods. The word "pseudo" indicates that we are using a complex-valued pseudomeasure in defining average operation instead of a genuine measure as in theory of probabilities [3,4] . This differs our approach from that by Nelson and others [5,6] developing the so-called stochastic quantization program which although rigorously deals with the quantities of not direct physical interest. Our scheme based on the use of pseudoprocesses and pseudofields, seems to be very natural in the framework of quantum mechanics but is formulated on the physical level, i.e., it is so far formal. We believe, however, that it may be made rigorous following the ideas which recently appeared in the literature [7,8] .

2. Basic definitions

Let as is usual in theory of probabilities [9] Ω mean a set of elementary events $\omega \in \Omega$ and $\mathcal{B}(\Omega)$ some σ- algebra of its subsets. Let further ξ be a real random variable, i.e., a mapping

$$\xi : \Omega \to \mathbb{R}$$

$$\xi^{-1}(A) \in \mathcal{B}(\Omega) \tag{2.1}$$

for all Borel subsets A of $\mathbb{R}$.

We assume that a complex valued average operation Q, (p- average), is defined on some random variables

$$Q : \xi \to Q\{\xi\} \in \mathbb{C}$$

such that

1° $Q\{a\xi + b\eta\} = aQ\{\xi\} + bQ\{\eta\}$

for any complex numbers a, b and

2° $Q\{1\} = 1$.

The probability amplitude of an event A is given by

$$M\{A\} = Q\{\chi_A\} , \tag{2.2}$$

where χ_A is the indicator of this event.

We shall write formally the p-average operation as an integral over the amplitude and its distribution corresponding to given ξ

$$Q\{f\circ\xi\} = \int_{\Omega} f\circ\xi(\omega) M(d\omega) = \int_{\mathbb{R}} f(a)\mu_{\xi}(da) = \int_{\mathbb{R}} f(a)\,dm_{\xi}(a) , \tag{2.3}$$

where $f\circ\xi$ is a composite function, and

$$\mu_{\xi}(da) = M\{\xi \in [a, da]\} = dm_{\xi}(a) \tag{2.4}$$

$$m_{\xi}(a) = M\{\xi < a\} .$$

Similarly, when we have a function of several variables

$$\eta = f(\xi_1, \dots, \xi_m) \tag{2.5}$$

then we may write

$$Q\{\eta\} = \int_{\mathbb{R}} a\, dm_{\eta}(a) = \int_{\mathbb{R}^n} f(a_1, \dots, a_n)\mu_{\xi_1,\dots,\xi_m}(da_1, \dots, da_n) , \tag{2.6}$$

where

$$\mu_{\xi_1,\dots,\xi_m}(da_1, \dots, da_n) = M\{\xi_1 \in [a_1, da_1], \dots, \xi_m \in [a_n, da_n]\} . \tag{2.7}$$

By the characteristic function of a random variable we mean the p-average

$$Q\{e^{i\lambda\xi}\} = \int_{\mathbb{R}} e^{i\lambda a} dm_{\xi}(a) . \tag{2.8}$$

We call the two random variables ξ, η equal when they are equivalent under the Q - operation sign, i.e.,

$$Q\{f\circ\xi\} = Q\{f\circ\eta\} \tag{2.9}$$

for any bounded continuous function f.

We will say that the random pseudoprocess (p-process) is given if to each $t \in T$ a random variable $\xi(t)$ is given. Similarly, we will talk about the p-fields over some space M (of more than one dimension) when for each $x \in M$ the random variable $\phi(x)$ is given .

If for some test functions $\varphi_1(x), \dots, \varphi_m(x) \in K$ the random variables $\phi(\varphi_1), \dots, \phi(\varphi_m)$ are given such that

$$1^\circ \ \phi(\alpha\varphi_1 + \beta\varphi_2) = \alpha\phi(\varphi_1) + \beta\phi(\varphi_2)$$

$$2^\circ \ \lim_{k\to\infty} Q\{f\circ\phi(\varphi_k)\} = Q\{f\circ\phi(\varphi)\}$$

when $\varphi_k \to \varphi$ in the space K and f is an arbitrary bounded continuous function, then we say that the generalized random p-field is given.

As is known from the theory of distributions [10] we may always perform the following operations on the generalized p-fields :

(i) Addition and multiplications by numbers

$$(\alpha\phi_1 + \beta\phi_2, \varphi) = \alpha\phi_1(\varphi) + \beta\phi_2(\varphi) \ ,$$

(ii)Multiplication by a function

$$(f\phi, \varphi) = \phi(f\cdot\varphi) \qquad \text{when } f\cdot\varphi \in K \ ,$$

(iii) Defferentiation

$$(\partial_\mu\phi, \varphi) = -\phi(\partial_\mu\varphi)$$

(iv) Shifting

$$\phi_h(\varphi) = \phi(\varphi_h)$$

$$\varphi_h(x) = \varphi(x-h)$$

Any generalized p-field generates functionals on K by

means of taking its moments

$$Q\{\phi(\varphi)\} = \int_{\mathbb{R}} a \, dm_{\phi(\varphi)}(a) = \Pi(\varphi) \ . \tag{2.10}$$

According to the linearity of generalized p-field and Q -operation we have the same property for $\Pi(\varphi)$,

$$\Pi(\alpha\varphi_1 + \beta\varphi_2) = \alpha\Pi(\varphi_1) + \beta\Pi(\varphi_2) . \tag{2.11}$$

For any generalized p-field we may write the decomposition

$$\phi(\varphi) = \phi_0(\varphi) + \phi_1(\varphi) , \tag{2.12}$$

where

$$\phi_0(\varphi) = \Pi(\varphi)$$

is not a random linear functional on K and $\phi_1(\varphi)$ is a random generalized p-field with vanishing average value.

The second moment of the generalized p-field is called the correlation functional

$$K(\varphi, \psi) = Q\{\phi(\varphi)\cdot\phi(\psi)\} . \tag{2.13}$$

Clearly, it is a bilinear functional on K . In the same way we may introduce higher moments as well.

3. Gaussian generalized p-fields

The generalized p-field $\phi(\varphi)$ is called Gaussian one when for any linearly independent functions $\varphi_1, \dots, \varphi_n$ we have

$$Q\{f[\phi(\varphi_1), \dots, \phi(\varphi_n)]\} = \left[(2\pi\lambda)^n \det B_n\right]^{-1/2} \int_{\mathbb{R}^n} f(a_1, \dots, a_n) \exp\{-\tfrac{1}{2\lambda}(B_n^{-1}a, a)\} d^n a, \tag{3.1}$$

where B_n^{-1} is a positive-definite symmetric matrix containing small imaginary part $i\epsilon a^2$ for regularization and λ is an imaginary number; $\lambda = i\gamma$. We classify all the Gaussian p-fields into two classes according to the sign of γ; $\gamma > 0$ the first class, $\gamma < 0$ – the second one.

If we put for $f(a_1, \dots, a_n) = a_j a_k$ we will have from (3.1) for the correlation functional

$$Q\{\phi(\varphi_j)\phi(\varphi_k)\} = \left[(2\pi\lambda)^n \det B_n\right]^{-1/2} \cdot \int_{\mathbb{R}^n} a_j a_k \exp\{-\tfrac{1}{2\lambda}(B_n^{-1}a, a)\} d^n a =$$

(3.2)

$$= (-i)^2 \partial^j \partial^k \exp\{-\tfrac{\lambda}{2}(p, B_n p)\}\Big|_{p=0} = \lambda \cdot (B_n)_{jk}.$$

From this we have obviously

$$\lambda B_n = \| K(\varphi_j, \varphi_k) \|_1^n. \tag{3.3}$$

For the characteristic functional of the Gaussian generalized p-field we shall have from (3.1) and (3.3) the formula

$$L\{\varphi\} = Q\{\exp i\phi(\varphi)\} = \left[2\pi\lambda \det B_1\right]^{-1/2} \cdot \int_{\mathbb{R}} \exp\{ia - \tfrac{a^2}{2\lambda B_1}\} da = \tag{3.4}$$

$$= \exp\{-\tfrac{\lambda}{2} B_1\} = \exp\{-\tfrac{1}{2} K(\varphi, \varphi)\}.$$

In a regular case when $\phi(\delta_x) = \phi(x)$ makes sense as a random variable (δ_x – is the Dirac δ – function located at the point $x \in M$) and M is the one-dimensional space then we will say that a random p-process is given. As an example, we shall consider the Wiener p-process on a real line defined as follows

$$z(t) \in \mathbb{R}^1, \quad z(0) = 0$$

and for $0 \le t_1 \le \dots \le t_n$ we have

$$Q\{f[\phi(t_1), \dots, \phi(t_n)]\} = \left[(2\pi\lambda)^n t_1 (t_2 - t_1) \dots (t_n - t_{n-1})\right]^{-1/2} \cdot \tag{3.5}$$

$$\cdot \int_{\mathbb{R}^1} f(a_1, \dots, a_n) \exp\{-\tfrac{1}{2\lambda}\left[\tfrac{a_1^2}{t_1} + \tfrac{(a_2 - a_1)^2}{t_2 - t_1} + \dots + \tfrac{(a_n - a_{n-1})^2}{t_n - t_{n-1}}\right]\} d^n a.$$

Hence, for $s<t$ we will find from it

$$\lambda B_2 = \lambda \begin{Vmatrix} s, & s \\ s, & t \end{Vmatrix} \tag{3.6}$$

and, using the general relation (3.3), we infer for the correlation function

$$K(s,t) = Q\{\phi(s)\cdot\phi(t)\} = \lambda \min(s,t) \tag{3.7}$$

and for the correlation functional we will get after some simple calculations

$$\begin{aligned} K(\varphi,\psi) &= \lambda \int_0^\infty ds \int_0^\infty dt\, \varphi(s)\psi(t) \min(s,t) = \\ &= \lambda \int_0^\infty dt\, \psi(t) \int_0^t ds\, \varphi(s) s + \lambda \int_0^\infty ds\, \varphi(s) \int_0^s dt\, \psi(t) t = \\ &= \lambda \int_0^\infty [\hat\varphi(t) - \hat\varphi(\infty)][\hat\psi(t) - \hat\psi(\infty)]\, dt \ , \end{aligned} \tag{3.8}$$

where we denoted

$$\hat\varphi(s) = \int_0^s \varphi(\tau) d\tau \ . \tag{3.9}$$

There are of course two classes of the Wiener p-processes depending on the sign of γ . Both are needed for the construction of the relativistic p-processes which is useful in relativistic quantum field theory [11,12] . His role is analogous to that played by the relevant Wiener process in Euclidean quantum field theory [13,14] .

4. Connection between the Gaussian generalized p-fields and the quantum fields

Let $\{e_n(x)\}_1^\infty$ be an orthogonal base in the Hilbert space $L^2(M)$ of square integrable functions on M . Let be a generalized random scalar p-field then, according to the formula (3.1), we have

$$\begin{aligned} &Q\{f[\phi(e_1),\ldots,\phi(e_n)]\} = \\ &= \left[(2\pi\lambda)^n \det B_n\right]^{-1/2} \int_{\mathbb{R}^n} f(a_1,\ldots,a_n) \exp\{-\tfrac{1}{2\lambda}(B_n^{-1}a,a)\}\, d^n a \end{aligned} \tag{4.1}$$

and extending the matrix B_n beyond the n-th order we get

$$= \frac{\int f(a_1,\ldots,a_n) \exp\{-\frac{1}{2\lambda}\sum_{j,k=1}^\infty (B^{-1})_{jk} a_j a_k\} \prod_{k=1}^\infty da_k}{\int \exp\{-\frac{1}{2\lambda}\sum_{j,k=1}^\infty (B^{-1})_{jk} a_j a_k\} \prod_{k=1}^\infty da_k} \ , \tag{4.2}$$

where the integration is carried out, in fact, on the Hilbert space ℓ^2 since the presence of the regularizing term $-\epsilon\sum_{k=1}^\infty a_k^2$ which is hidden in B^{-1} . We may consider the variables of integration a_k as the Fourier coefficients of some function $q(x)$ with respect to the base $\{e_m\}_1^\infty$

$$a_k = (q \cdot e_k) \equiv q \cdot e_k = \int_M q(x) e_k(x) dx \quad (4.3)$$

Moreover, we may write for the matrix elements

$$(B^{-1})_{jk} = (e_j, B^{-1} e_k) \quad (4.4)$$

where the operator B^{-1} is defined by its action on the basic elements

$$B^{-1} e_k = \sum_{j=1}^{\infty} (B^{-1})_{jk} e_j \quad . \quad (4.5)$$

Thus, using the completeness relation of the base we will have the formulae

$$\sum_{j,k=1}^{\infty} (B^{-1})_{jk} a_j a_k = (q, B^{-1} q) \quad (4.6)$$

$$\int \exp\{-\tfrac{1}{2\lambda} \sum_{j,k=1}^{\infty} (B^{-1})_{jk} a_j a_k\} \prod_{m=1}^{\infty} da_m = \int_{L^2(M)} \exp\{-\tfrac{1}{2\lambda}(q, B^{-1} q)\} dq \quad (4.7)$$

$$\int f(a_1, \ldots, a_n) \exp\{-\tfrac{1}{2\lambda} \sum_{j,k=1}^{\infty} (B^{-1})_{jk} a_j a_k\} \prod_{m=1}^{\infty} da_m = \quad (4.8)$$

$$= \int_{L^2(M)} f[(q, e_1), \ldots, (q, e_n)] \exp\{-\tfrac{1}{2\lambda}(q, B^{-1} q)\} dq \quad .$$

Therefore we obtain finally for the p-average

$$Q\{f[\phi(e_1), \ldots, \phi(e_n)]\} = \frac{\int_{L^2(M)} f(q \cdot e_1, \ldots, q \cdot e_n) \exp\{-\tfrac{1}{2\lambda}(q, B^{-1} q)\} dq}{\int_{L^2(M)} \exp\{-\tfrac{1}{2\lambda}(q, B^{-1} q)\} dq} \quad (4.9)$$

$$= \int_{L^2(M)} f(q \cdot e_1, \ldots, q \cdot e_n) \mu_B(dq) \; , \quad (4.10)$$

where $\mu_B(dq)$ denotes the pseudomeasure

$$\mu_B(dq) = \frac{\exp\{-\tfrac{1}{2\lambda}(q, B^{-1} q)\} dq}{\int_{L^2(M)} \exp\{-\tfrac{1}{2\lambda}(q, B^{-1} q)\} dq} \quad . \quad (4.11)$$

If we put $-\frac{1}{2\lambda} B^{-1} = \frac{i}{2} K - \epsilon = \frac{i}{2}(\Box - m^2 + 2i\epsilon)$ then according to the well known connection between the functional and operator formulation of the quantum field theory, we may write

$$\frac{\int_{L^2(M)} f(q \cdot e_1, \ldots, q \cdot e_n) \exp(\tfrac{i}{2} q K q) dq}{\int_{L^2(M)} \exp(\tfrac{i}{2} q K q) dq} = \langle 0 | T^* f(a \cdot e_1, \ldots, a \cdot e_n) | 0 \rangle \quad (4.12)$$

where $a(x)$ is a free scalar neutral quantum field

$$\begin{aligned} &(\Box - m^2) a(x) = 0 \\ &[a(x), a(y)] = -i \Delta(x-y; m) \quad . \end{aligned} \quad (4.13)$$

In this case we shall call the corresponding p-filed $\phi(x)$ the free, neutral scalar random p-field. We have the following connection between various averages

$$Q\{f(\phi\cdot e_1,\dots,\phi\cdot e_n)\} = \langle 0|T^* f(a\cdot e_1,\dots,a\cdot e_n)|0\rangle = \tag{4.14}$$

$$= \frac{\int_{L^2(M)} f(q\cdot e_1,\dots,q\cdot e_n)\exp(\frac{i}{2}qKq)dq}{\int_{L^2(M)} \exp(\frac{i}{2}qKq)dq} .$$

In particular, we may write for the generating functional of the τ- functions

$$J[p] = \frac{\langle 0|T^* \exp(ia\cdot p)S[a]|0\rangle}{\langle 0|S[a]|0\rangle} = \frac{Q\{\exp(i\phi\cdot p)S[\phi]\}}{Q\{S[\phi]\}} = \tag{4.15}$$

$$= \frac{\int \exp\{\frac{i}{2}qKq + iL_{int}[q] + iqp\}dq}{\int \exp\{\frac{i}{2}qKq + iL_{int}[q]\}dq} ,$$

where the integration is carried out over the Hilbert space $L^2(M_4)$, M_4 is the Minkowski space, The S-matrix is replaced in the second part of the above formula by a functional

$$S[\phi] = \exp(iL_{int}[\phi]) , \tag{4.16}$$

i.e., the T^*-product operation sign is removed and the field operator a is replaced by the p-field ϕ in the integration functional $L_{int}[a]$.

According to the well-known formulas for the functionals [15] we obtain for $J[p]$

$$J[p] = N^{-1}\exp(iL_{int}[-i\tfrac{\delta}{\delta p}])\exp(-\tfrac{i}{2}pK^{-1}p) = \tag{4.17}$$

$$= N^{-1}\exp(-\tfrac{i}{2}pK^{-1}p)\exp(\tfrac{i}{2}\tfrac{\delta}{\delta p}K\tfrac{\delta}{\delta p})\exp(iL_{int}[-K^{-1}p]),$$

$$-K^{-1} = \Delta^c$$

with N determined by the condition

$$J[0] = 1 \tag{4.18}$$

As a simple example, we shall calculate the two-point function in the case of vanishing interaction

$$\tau(x_1,x_2) = (-i)^2 \frac{\delta^2 J[p]}{\delta p(x_1)\delta p(x_2)}\Big|_{p=0} = \langle 0|T^* a(x_1)a(x_2)|0\rangle = \tag{4.19}$$

$$Q\{\phi(x_1)\phi(x_2)\} = -\frac{\delta^2}{\delta p(x_1)\delta p(x_2)}\exp(\tfrac{i}{2}p\Delta^c p)\Big|_{p=0} =$$

$$= -i\Delta^c(x_1-x_2) = -\frac{i}{(2\pi)^4}\int\frac{e^{ip(x_1-x_2)}}{\varkappa^2-p^2-i\epsilon}d^4p .$$

One sees from this that at the coinciding points this p-average does not make sense which indicates that $\phi(x)$ is a generalized random p-field for which a smearing with some smooth functions is necessary in order to make the multiplications of random variables possible.

The generalization to the case of several independent generalized random p-field $\phi_1, \dots, \phi_n$ is strightforward

$$J[p_1, \dots, p_n] = \frac{\langle 0| T^* \exp\left(i\sum_{\alpha=1}^{n} a_\alpha \cdot p_\alpha\right) S |0\rangle}{\langle 0|S|0\rangle} = \frac{Q\{\exp\left(i\sum_{\alpha=1}^{n} \phi_\alpha \cdot p_\alpha\right) S\}}{Q\{S\}} = \tag{4.20}$$

$$= N^{-1} \int \exp\left\{\frac{i}{2}\sum_{\alpha=1}^{n} q_\alpha K q_\alpha + i L_{int}[q_1, \dots, q_n] + i\sum_{\alpha=1}^{n} q_\alpha p_\alpha\right\} dq_1 \cdots dq_n =$$

$$= N^{-1} \cdot \exp\left(i L_{int}\left[-i\frac{\delta}{\delta p_1}, \dots, -i\frac{\delta}{\delta p_n}\right]\right) \exp\left(-\frac{i}{2}\sum_{\alpha=1}^{n} p_\alpha K^{-1} p_\alpha\right) =$$

$$= N^{-1} \cdot \exp\left(-\frac{i}{2}\sum_{\alpha=1}^{n} p_\alpha K^{-1} p_\alpha\right) \exp\left(\frac{i}{2}\sum_{\alpha=1}^{n} \frac{\delta}{\delta p_\alpha} K \frac{\delta}{\delta p_\alpha}\right) \exp\left(i L_{int}\left[-K^{-1} p_1, \dots, -K^{-1} p_n\right]\right).$$

The notion of independence of the random variables is understood in the sense of the theory of probabilities [16] while the independence of quantum fields means , as usually, their commutativity on space-like distances.

In the case of n-even we may introduce the complex fields as the combinations of the basic real ones. We shall demonstrate this construction on the simplest case of the two fields.

Let ξ_i $i = 1, 2$ stand for one of the fields p_i, a_i, ϕ_i, q_i. Then we introduce the complex quantities as follows

$$\xi_1 = \frac{1}{\sqrt{2}}(\xi + \xi^*), \quad \xi_2 = \frac{-i}{\sqrt{2}}(\xi - \xi^*)$$

$$\xi = \frac{1}{\sqrt{2}}(\xi_1 + i\xi_2), \quad \xi^* = \frac{1}{\sqrt{2}}(\xi_1 - i\xi_2) \tag{4.21}$$

$$\frac{\delta}{\delta\xi_1} = \frac{1}{\sqrt{2}}\left(\frac{\delta}{\delta\xi} + \frac{\delta}{\delta\xi^*}\right), \quad \frac{\delta}{\delta\xi_2} = \frac{i}{\sqrt{2}}\left(\frac{\delta}{\delta\xi} - \frac{\delta}{\delta\xi^*}\right)$$

and the notation

$$F[\xi_1, \xi_2] = F\left[\frac{1}{\sqrt{2}}(\xi + \xi^*), \frac{-i}{\sqrt{2}}(\xi - \xi^*)\right] \to F[\xi, \xi^*] \tag{4.22}$$

$$F\left[-i\frac{\delta}{\delta\xi_1}, -i\frac{\delta}{\delta\xi_2}\right] = F\left[\frac{-i}{\sqrt{2}}\left(\frac{\delta}{\delta\xi} + \frac{\delta}{\delta\xi^*}\right), \frac{1}{\sqrt{2}}\left(\frac{\delta}{\delta\xi} - \frac{\delta}{\delta\xi^*}\right)\right] \to F\left[-i\frac{\delta}{\delta\xi^*}, -i\frac{\delta}{\delta\xi}\right]$$

$$d\xi_1 d\xi_2 = c\, d\xi d\xi^*.$$

We will get from the formula (4.20) at $n = 2$

$$J[p, p^*] = \frac{\langle 0| T^* \exp\left(i a \cdot p^* + i a^+ \cdot p\right) S[a, a^+] |0\rangle}{\langle 0| S[a, a^+] |0\rangle} =$$

$$= \frac{Q\{\exp\left(i\phi \cdot p^* + i\phi^* \cdot p\right) S[\phi, \phi^*]\}}{Q\{S[\phi, \phi^*]\}} = \tag{4.23}$$

$$= N^{-1} \int \exp\{i q K q^* + i L_{int}[q, q^*] + i q \cdot p^* + i q^* \cdot p\} dq\, dq^*$$

$$= N^{-1} \cdot \exp\left(i L_{int}\left[-i\frac{\delta}{\delta p^*}, -i\frac{\delta}{\delta p}\right]\right) \cdot \exp\left(-i p K^{-1} p^*\right) =$$

$$= N^{-1}\exp\left(-ip\bar{K}^{-1}p^*\right)\exp\left(i\frac{\delta}{\delta p}K\frac{\delta}{\delta p^*}\right)\exp\left(iL_{int}[-\bar{K}^{-1}p, -\bar{K}'^{-1}p^*]\right) .$$

A generalization to the case of several complex variables is strightforward. Thus we completed the considerations of fields with integer spins. For the sake of completeness we shall consider also a case of the Dirac spin 1/2 field. In order to fixe the notation we shall write formulae relevant for the free Dirac field. Namely we have for $\psi(x)$ and $\bar{\psi}(x)$ operators the conditions

$$\begin{aligned}
&(i\gamma^\mu\partial_\mu - m)\psi(x) = 0 \\
&\bar{\psi}(x)(i\gamma^\mu\overleftarrow{\partial}_\mu + m) = 0 \quad , \quad \bar{\psi}(x) = \psi^+(x)\gamma^0 \\
&\{\psi(x), \bar{\psi}(y)\} = -iS(x-y) \ ,
\end{aligned} \tag{4.24}$$

where the γ- matrices satisfy the conditions

$$\begin{aligned}
&\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu} \quad , \quad g^{00} = -g^{kk} = 1 \quad , \ k = 1,2,3 \\
&(\gamma^\mu)^+ = g^{\mu\mu}\gamma^\mu \ .
\end{aligned} \tag{4.25}$$

The simplest particular realization of this algebra is given by the formulae

$$\gamma^0 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} , \quad \gamma^1 = \begin{pmatrix} & & & 1 \\ & & 1 & \\ & -1 & & \\ -1 & & & \end{pmatrix} \tag{4.26}$$

$$\gamma^2 = \begin{pmatrix} & & & -i \\ & & i & \\ & i & & \\ -i & & & \end{pmatrix} , \quad \gamma^3 = \begin{pmatrix} & & 1 & \\ & & & -1 \\ -1 & & & \\ & 1 & & \end{pmatrix} .$$

A generating functional for the τ- functions of the Dirac field is given by one of the equivalent expressions

$$J[\eta,\bar\eta] = \frac{\int \exp\left(i\bar\psi D\psi + iL_{int}[\psi,\bar\psi] + i\bar\eta\psi + i\bar\psi\eta\right)d\psi\, d\bar\psi}{\int \exp\left(i\bar\psi D\psi + iL_{int}[\psi,\bar\psi]\right)d\psi\, d\bar\psi} = \tag{4.27}$$

$$= N^{-1}\cdot\exp\left(iL_{int}\left[-i\frac{\delta}{\delta\bar\eta}, i\frac{\delta}{\delta\eta}\right]\right)\exp\left(-i\bar\eta S^c\eta\right) \tag{4.28}$$

$$= N^{-1}\cdot\exp\left(-i\bar\eta S^c\eta\right)\exp\left(-i\frac{\delta}{\delta\eta}D\frac{\delta}{\delta\bar\eta}\right)\exp\left(iL_{int}[-S^c\eta, -\bar\eta S^c]\right) \tag{4.29}$$

$$= \frac{\langle 0_F|T^*\exp\left(i\bar\psi\eta + i\bar\eta\psi\right)S[\psi,\bar\psi]|0_F\rangle}{\langle 0_F|S[\psi,\bar\psi]|0_F\rangle} \tag{4.30}$$

($|0_F\rangle$ - the mathematical Fermi vacuum).

The notation used here is

$$D = i\gamma^\mu\partial_\mu - m \quad , \tag{4.31}$$

$$\begin{aligned}
&S(x) = (i\gamma^\mu\partial_\mu + m)\Delta(x) \quad , \\
&S^c(x) = (i\gamma^\mu\partial_\mu + m)\Delta^c(x) \ ,
\end{aligned} \tag{4.32}$$

$\eta, \bar\eta$ areanticommuting spinors , the derivatives over them are both left and satisfy the conditions

$$\left\{\frac{\delta}{\delta\eta(x)}, \eta(y)\right\} = \delta(x-y) \quad , \quad \left\{\frac{\delta}{\delta\bar\eta(x)}, \bar\eta(y)\right\} = \delta(x-y) ,$$
$$\left\{\frac{\delta}{\delta\eta(x)}, \bar\eta(y)\right\} = 0 \qquad , \quad \left\{\frac{\delta}{\delta\bar\eta(x)}, \eta(y)\right\} = 0 . \tag{4.33}$$

The factor N is determined by the normalization condition

$$J[0,0] = 1 . \tag{4.34}$$

We introduce now the spinor p-fields $\psi(x,\omega)$ and $\bar\psi(x,\omega)$ as random variables over the Minkowski space, $x \in M_4$, $\omega \in \Omega$. They separately anticommute, and $\psi(x)$ and $\bar\psi(y)$ become independent random variables when $x-y$ is space-like. Using these fields one may express the $J[\eta,\bar\eta]$ generating functional in the form of p-average over the pseudomeasure

$$\mu_D(d\psi, d\bar\psi) = \frac{\exp(i\bar\psi D\psi)\, d\psi\, d\bar\psi}{\int \exp(i\bar\psi D\psi)\, d\psi\, d\bar\psi} \tag{4.35}$$

Namely, we have the formula

$$J[\eta,\bar\eta] = \frac{Q_F\{\exp(i\bar\eta\psi + i\bar\psi\eta)\, S[\psi,\bar\psi]\}}{Q_F\{ S[\psi,\bar\psi]\}} \tag{4.36}$$

from which the τ - functions may be calculated.

We shall close this paper with a remark concerning the symmetry principles and their form in the stochastic framework. For instance, the relativistic invariance of a theory means, e.g., that for scalar p-fields , the random variables

$$\phi(Lx_1+a), \ldots, \phi(Lx_n+a) \quad \text{and} \quad \phi(x_1), \ldots, \phi(x_n) \tag{4.37}$$

have the same p-distributions for all $L \in \mathcal{P}_+^\uparrow(\mathbb{R})$ and all $a \in M_4$. It means that the variables $\phi(Lx+a)$ and $\phi(x)$ are stochastically equivalent. Similar, conclusions, with usual complications for multicomponent case, may be established for fields of higher spins.

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