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GEOMETRIES INHERENT TO N=1 SUPERGRAVITIES

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1. Introduction

At present it is becoming clear that the number N of gravitinos does not specify the kind of extended supergravity completely. Even in the simplest case, N=1, we are aware of, at least, three supergravities. Two N=2 versions are already known. For higher N one may expect even greater diversity. The versions differ by the content of auxiliary fields. Correspondingly, differences occur in the interactions with matter fields, in the mechanism of spontaneous symmetry breaking (when auxiliary fields get nonzero vacuum expectations); also, in some versions important additional local symmetries appear, etc. In view of all that it seems instructive to study the simplest case, N=1, in detail. In the first part of the present talk we shall discuss N=1 supergravity in the linearized limit, the structure of currents - sources in it and the free equations of motion. These quite elementary arguments are very useful in a preliminary sort out of the various possible sets of auxiliary fields.

In the second, main part of the talk the intrinsic geometries of the different N=1 theories and their action principle in superspace will be discussed. We shall show that each version has its own, inherent complex geometry in which the basic postulates and equations of the theory become natural and clear.

A special attention will be paid to the new version of N=1 supergravity with local $U(1)$ symmetry /1,2/. It reveals some unique geometric properties and poses new questions.

The content of the second part of the talk is as follows. First, a framework*) for the description of the various N=1 models is introduced. A complex superspace $\mathbb{C}^{4,4}$ **) is considered with

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*) It has already been used for both minimal /3,4/ and non-minimal /5,6/ N=1 supergravities.

**) $\mathbb{C}^{n,k}$ means a complex superspace with n vector and k spinor coordinates.

coordinate transformations leaving invariant the chiral $\mathbb{C}^{4,2}$ subspace. The physical real superspace $\mathbb{R}^{4,4}$ is embedded in $\mathbb{C}^{4,4}$ as a hypersurface specified by an axial $(\mathcal{H}^m)$ and a spinor $(\mathcal{H}^\mu, \bar{\mathcal{H}}^{\dot\mu})$ superfields. The Einstein supergravity is described by a one-parameter (n) family of supergroups, preserving a certain relation between the Berezinians (superdeterminants) of the $\mathbb{C}^{4,4}$ and $\mathbb{C}^{4,2}$ coordinate transformations. This relation becomes particularly simple for two values of n. For $n=-1/3$ the $\mathbb{C}^{4,2}$ supervolume is preserved and this is the case of minimal supergravity. For $n=0$ the $\mathbb{C}^{4,4}$ supervolume is preserved. This case exhibits a number of new features. First, in the Wess-Zumino gauge there is a local $U(1)$ invariance. Second, a peculiar geometric invariant emerges. It is the Berezinian of the change of variables from left to right-handed parametrization of $\mathbb{R}^{4,4}$ which in this and only this case transforms as a (dimensionless) scalar superfield. It corresponds to an invariant subset of 8+8 fields. The latter can, and moreover, have to be constrained in order to write down an action. Third, unlike all other cases of N=1 supergravity here the action is not the invariant volume $\mathbb{R}^{4,4}$ (the latter just vanishes (cf. /6b, 7/) when the whole 8+8 subset is eliminated). The action is now given by a new type of invariant/7/ involving the $U(1)$ part of the vielbeins. The constraint reducing the number of fields from 20+20 to 12+12 can be solved explicitly in terms of fields in the WZ gauge. The resulting theory is exactly the one of Ref. /1/. We can easily solve this constraint in terms of superfields at the linearized level reproducing the result of Ref. /8/. However, finding the full nonlinear superfield solution is still an open problem with possible implications for extended supergravity. Note also that another, weaker constraint leads to a theory with 16+16 fields ($U(1)$ supergravity interacting with 4+4 matter fields in a specific way). This version is at present under investigation and will be discussed only briefly in this talk.

An analysis of the $U(1)$ supergravity was already made in Ref. /7/ in the framework of the real $R^{4,4}$ geometry supplemented by appropriate algebraic constraints. When translated into this language our results are consistent with those of Ref. /7/.

2. Linearized Supermultiplets of Fields and Currents

It is well known that Einstein gravity can be considered as the theory of a symmetric tensor field h^{mn} generated by the symmetric energy-momentum tensor θ_{mn} of all fields including the gravitational one:[9]

$$\varepsilon_{mkls}\,\varepsilon_{nqr}{}^{s}\,\partial^{k}\partial^{q}h^{rs} = \varkappa\,\theta_{mn}. \qquad (1)$$

The operator in the l.h.s. of the equation is degenerate (owing to gauge invariance). For consistency of the theory the energy-momentum tensor must be conserved, $\partial_m \theta^{mn} = 0$. This means that as a Poincaré group representation θ_{mn} contains spins 2 and 0 (the latter corresponds to the trace $\theta^m{}_m$), i.e, just the spins of the interacting graviton[9]. In the case of conformal gravity (pure spin 2) the source of h^{mn} is the conserved tensor θ_{mn} with vanishing trace $\theta^m{}_m = 0$.

The theories of supergravity can be treated analogously. There the energy-momentum tensor θ_{mn} and the spin-vector current of supersymmetry $j_{m\alpha}$ enter the same supermultiplet[10, 11]. The latter is the source of the supergravity multiplet[12]. It is very important that this current multiplet is not unique. Its different versions lead to different N=1 supergravities. As we shall see, the reason is the reducibility of the current multiplet with respect to the supersymmetry group. We recall that instead of spin in supersymmetry one considers superspin taking integer and half-integer values too. An irreducible representation with superspin Y contains spins $Y+{}^1/_2, Y, Y, Y-{}^1/_2$ and a superfield with an external

Lorentz index corresponding to spin j contains superspins $j+1/2$, $j, j, j-1/2$ /13,14/. Therefore, the simplest representation including spin 2 has superspin $^3/_2$. The superspin $^3/_2$ current multiplet includes the conserved γ_5-current j^m_5 besides θ_{mn} and $j_{m\alpha}$:

Spin	Current	Conservation Law	
2	$\theta_{mn}=\theta_{nm}$	$\partial_m \theta^{mn}=0$	$\theta^m{}_m=0$
$(^3/_2)^2$	$j_{m\alpha}$	$\partial_m j^m_\alpha=0$	$\bar\sigma_m{}^{\dot\alpha\beta} j^m_\beta=0$
1	j^m_5	$\partial_m j^m_5=0$	

Such a current multiplet generates the multiplet of fields of conformal supergravity containing the vierbein $e_a{}^m$, the gravitino ψ^m_α and the gauge vector A^m /15/. This field multiplet describes superspin $^3/_2$ in the interaction. The multiplets of fields and currents can be placed in a real axial superfield $h^m(x,\theta,\bar\theta)$ and an axial supercurrent $V_m(x,\theta,\bar\theta)$, respectively/11,12/. The latter obeys the conservation law

$$\bar\sigma_m{}^{\dot\alpha\beta} \mathcal{D}_\beta V^m = 0 \qquad (2)$$

which singles out superspin $^3/_2$.

In conformal supergravity the order of the equations of motion is too high. We are rather interested in Einstein supergravity with the usual order of the equations (second for bosons, first for fermions). There the dimension of the coupling constant $\varkappa$ is cm^1 $(\hbar=c=1)$ and the superconformal invariance is broken, so, in particular, $\theta^m{}_m \neq 0$. There are various ways to break down the symmetry. In ordinary minimal supergravity it is done as follows. The supercurrent $V_m(x,\theta,\bar\theta)$ has external spin 1 and 0, and, correspondingly, superspins Y = $^3/_2$, 1, 1, $^1/_2$, $^1/_2$, 0, 0. Consider the reducible current submultiplet with superspins Y = $^3/_2$, 0, 0. The superspins Y = 0,0 contain spins $(^1/_2)^2$, $(0)^4$ which can be carried by $\bar\sigma_m{}^{\dot\alpha\beta} j^m_\beta$, $\theta^m{}_m$, $\partial_m j^m_5$ (now the latter don't vanish). The two remaining spin 0 currents generate the auxiliary fields S and P . The axial field A^m ceases to be a gauge one: $\partial_m j^m_5 \neq 0$. Finally, the multiplet of minimal N=1 supergravity consists of the gauge fields $e_a{}^m, \psi^m_\alpha$ and the auxiliary fields A^m, S, P . The equation of motion is/12/

$$(q_m q_n + 3 q_n q_m) h^n = \varkappa V_m \tag{3}$$

$$q_m \equiv \frac{1}{4} \bar{\sigma}_m^{\dot{\alpha}\beta} [\mathcal{D}_\beta, \bar{\mathcal{D}}_{\dot{\alpha}}] \tag{3'}$$

The operator in the l.h.s. of Eq.(3) is proportional to the square root of the projectors for superspins $^3/_2$, 0, 0/12/. It is degenerate, so the r.h.s. must be conserved:

$$\left[\mathcal{D}_\alpha \bar{\mathcal{D}}^2 \eta_{mn} - 4 p_m (\sigma_n \bar{\mathcal{D}})_\alpha \right] V^n = 0 \tag{4}$$

Eq.(4) means that V^m contains superspins Y = $^3/_2$,0,0. The general algorithm for finding supercurrents obeying Eq. (4) is given in Ref./16/ and also Ref./17/.

This scenario is not unique. Instead of superspins 0,0 one can add superspin $^1/_2$ to the superspin $^3/_2$ of conformal supergravity. Consider first the superspin $^1/_2$ which is contained in the superfield V^m with $\partial_m V^m = 0$ (this is just the case of the new minimal version of supergravity). With the help of the projection operators/14/ one can find out that the spins 1, $(^1/_2)^2$, 0 in the superspin $^1/_2$ are distributed as follows: $\theta^m{}_m$ is spin 0, $\bar{\sigma}_m^{\dot{\alpha}\beta} j^m_\beta$ is $(^1/_2)^2$; spin 1 is carried by a conserved antisymmetric tensor generating a gauge antisymmetric auxiliary field a_{mn} ("notoph"/18/). Notice that all spins 0 are already used, so the axial current has to be conserved, $\partial_m j^m_5 = 0$. The multiplet of fields now consists of the physical gauge fields $e_a{}^m, \psi^m_\alpha$ and the auxiliary gauge fields A^m, a_{mn} . The linearized equation of motion is /8/

$$[q_m, q_n] h^n = \varkappa V_m . \tag{5}$$

The operator in the l.h.s. of Eq.(5) is proportional to the square root of the sum of projectors for the superspin $^3/_2$ and one of the superspins $^1/_2$. The conservation laws for the r.h.s. now are

$$\mathcal{D}^2 V_m = \bar{\mathcal{D}}^2 V_m = \partial_m V = 0. \tag{6}$$

It is important to realize that Eq.(5) is just the linearized equation of motion. The full nonlinear superfield theory involves both an axial and a spinor superfields (see below).

If the superspin $^3/_2$ is combined with the superspin $^1/_2$ from $\partial_m V^m$, the spin 0 will be carried by $\partial_m j^m_5$; the spins $(^1/_2)^2$, by $\bar{\sigma}_m^{\dot{\alpha}\beta} j^m_\beta$ and the trace of energy-momentum tensor must vanish, $\theta^m{}_m = 0$. It is not hard to write down the linearized equations of motion for this case:

$$\left(\eta_{mn} \mathcal{D}^\alpha \bar{\mathcal{D}}^2 \mathcal{D}_\alpha - 2[q_m, q_n]\right) h^n = \varkappa V_m \tag{7}$$

and to find out the field content. The local γ_5 -invariance of the previous case is now replaced by local dilatations.This formulation does not include the unrestricted Einstein group of coordinate transformations and, apparently, it cannot be generalized for the interacting case.

Finally, there exists nonminimal N=1 supergravity/5/. We shall only notice that the current multiplet contains superspins $^3/_2$, $(^1/_2)^2$, $(0)^4$.

Here we end the brief description of the linearized limit of N=1 supergravity. The most natural way out of this approximation is to use superspace and its geometry.

3. Complex Superspace

Let us first recall the geometric framework for nonminimal supergravity/5/ applied in Ref. /6/ in the spirit of Ref./3/. Consider a complex superspace

$$\mathbb{C}^{4,4} = \{ z_L \} = \{ x_L^m, \theta_L^\mu, \bar{\varphi}_L^{\dot{\mu}} \}, \tag{8}$$

where x_L^m are 4 complex vector coordinates and θ_L^μ, $\bar{\varphi}_L^{\dot{\mu}}$ are 4 complex spinor ones. The conjugated coordinates will carry an index R :

$$\{Z_R\} = \{ x_R^m = (x_L^m)^\dagger,\ \bar\theta_R^{\dot\mu} = (\theta_L^\mu)^\dagger,\ \varphi_R^\mu = (\bar\varphi_L^{\dot\mu})^\dagger \}. \qquad (9)$$

To distinguish these two parametrizations of $\mathbb{C}^{4,4}$ we call them left and right-handed.

Now we introduce a gauge group in $\mathbb{C}^{4,4}$. We choose it to be the group of analytic transformations of the coordinates which leave the chiral subspace

$$\mathbb{C}^{4,2} = \{ \zeta_L \} = \{ x_L^m,\ \theta_L^\mu \} \qquad (10)$$

invariant. In other words, the group has a "triangular" structure

$$\begin{aligned} \delta x_L^m &= \lambda^m(x_L, \theta_L) \\ \delta \theta_L^\mu &= \lambda^\mu(x_L, \theta_L) \\ \delta \bar\varphi_L^{\dot\mu} &= \bar\rho^{\dot\mu}(x_L, \theta_L, \bar\varphi_L), \end{aligned} \qquad (11)$$

where λ^m and λ^μ are chiral superfunctions - parameters and $\bar\rho^{\dot\mu}$ is a general one.

The next step is to introduce the real superspace

$$\mathbb{R}^{4,4} = \{ Z \} = \{ x^m,\ \theta^\mu,\ \bar\theta^{\dot\mu} \} \qquad (12)$$

as a hypersurface in $\mathbb{C}^{4,4}$, e.g,

$$\begin{aligned} & x^m = \mathrm{Re}\, x_L^m,\quad \theta^\mu = \theta_L^\mu,\quad \bar\theta^{\dot\mu} = \bar\theta_R^{\dot\mu} \\ & \mathcal{H}^m(x,\theta,\bar\theta) = \mathrm{Im}\, x_L^m \\ & \mathcal{H}^\mu(x,\theta,\bar\theta) = \varphi_R^\mu - \theta_L^\mu,\quad \bar{\mathcal{H}}^{\dot\mu}(x,\theta,\bar\theta) = \bar\varphi_L^{\dot\mu} - \bar\theta_R^{\dot\mu}. \end{aligned} \qquad (13)$$

Here the coordinates of $\mathbb{C}^{4,4}/\mathbb{R}^{4,4}$ are made arbitrary functions of the coordinates of $\mathbb{R}^{4,4}$. The superfunctions $\mathcal{H}^m, \mathcal{H}^\mu, \bar{\mathcal{H}}^{\dot\mu}$ define the hypersurface and simultaneously determine the (curved) geometry of $\mathbb{R}^{4,4}$. The group (4) induces the following transformations

$$\begin{aligned} x'^m &= x^m + \tfrac{1}{2}[\lambda^m(x_L,\theta_L) + \bar\lambda^m(x_R,\bar\theta_R)] \\ \theta'^\mu &= \theta^\mu + \lambda^\mu(x_L,\theta_L) \\ \bar\theta'^{\dot\mu} &= \bar\theta^{\dot\mu} + \bar\lambda^{\dot\mu}(x_R,\bar\theta_R) \end{aligned} \qquad (14.a)$$

$$\begin{aligned} \delta\mathcal{H}^m &= \mathcal{H}'^m(x',\theta',\bar\theta') - \mathcal{H}^m(x,\theta,\bar\theta) = \tfrac{1}{2i}[\lambda^m(x_L,\theta_L) - \bar\lambda^m(x_R,\bar\theta_R)] \\ \delta\mathcal{H}^\mu &= \mathcal{H}'^\mu(x',\theta',\bar\theta') - \mathcal{H}^\mu(x,\theta,\bar\theta) = \rho^\mu(x_R,\bar\theta_R,\varphi_R) - \lambda^\mu(x_L,\theta_L) \end{aligned} \qquad (14.b)$$

with
$$x_L^m = x^m + i\mathcal{H}^m(x,\theta,\bar\theta)\ ,\ \theta_L^\mu = \theta^\mu$$
$$\bar\varphi_L^{\dot\mu} = \bar\theta^{\dot\mu} + \bar{\mathcal{H}}^{\dot\mu}(x,\theta,\bar\theta) \tag{14.c}$$
(and their conjugates) being now functions of $x, \theta, \bar\theta$ rather than independent coordinates. In what follows we shall refer to Z_L (Z_R) of Eq.(14.c) as left (or right)-handed parametrizations of $R^{4/4}$.

The transformations (14) correspond to conformal supergravity. Restricting them appropriately one can obtain the transformation group of Einstein supergravity. Owing to the triangular structure of the group (4) the Berezinians of both the $C^{4,4}$ and $C^{4,2}$ transformations have multiplicative property. So we can single out subgroups by imposing a natural restriction
$$\left[Ber\left(\frac{\partial Z_L'}{\partial Z_L}\right)\right]^{3n+1} = \left[Ber\left(\frac{\partial \zeta_L'}{\partial \zeta_L}\right)\right]^{2n} \tag{15}$$
or, infinitesimally,
$$(3n+1)\frac{\partial}{\partial\bar\varphi_L^{\dot\mu}}\bar\rho^{\dot\mu} = (n+1)\left(\frac{\partial\lambda^m}{\partial x_L^m} - \frac{\partial\lambda^\mu}{\partial\theta_L^\mu}\right). \tag{16}$$
Each value of n corresponds to a nonminimal formulation of supergravity with 20+20 fields[5]. There are only two exceptions.

At $n = -1/3$ Eq.(15) takes the form
$$Ber\left(\frac{\partial \zeta_L'}{\partial \zeta_L}\right) = 1, \tag{17}$$
i.e, the transformations preserve the supervolume of $C^{4,2}$. In this case the parameters $\rho^\mu, \bar\rho^{\dot\mu}$ are not restricted and with their help the spinor superfields $\mathcal{H}^\mu, \bar{\mathcal{H}}^{\dot\mu}$ can be gauged away (just as in conformal supergravity). Thus one recovers the minimal formulation with 12+12 fields. It has been described in detail earlier[3,4] and we are not going to discuss it here.

The second exceptional value, $n = 0$, corresponds to the preservation of the total supervolume of $C^{4,4}$. At $n = 0$ Eq.(15) reduces to
$$Ber\left(\frac{\partial Z_L'}{\partial Z_L}\right) = 1. \tag{18}$$
Respectively, the supervolume element $d^4x_L\, d^2\theta_L\, d^2\bar\varphi_L$ is invariant.

This value of n is connected with the new minimal version of supergravity as will be explained below.

4. Field Content and Transformations

The field content of each of the above-described formulations and the meaning of the field transformations are revealed in the Wess-Zumino gauge. We shall do it here with the intention to show how the local $U(1)$ group emerges in the case $n=0$.

The parameters $\lambda^m, \lambda^\mu, \bar{\rho}^{\dot{\mu}}$ have the following decomposition consistent with Eq.(16)

$$
\begin{aligned}
\lambda^m(x_L,\theta_L) &= a^m + ib^m + \theta_L^\mu \chi^m_\mu + \theta_L\theta_L (c^m + id^m) \\
\lambda^\mu(x_L,\theta_L) &= \varepsilon^\mu + \theta_L^\nu \left[\delta_\nu^\mu (a+ib) + \omega_{(\nu}{}^{\mu)} \right] + \theta_L\theta_L \eta^\mu \\
\bar{\rho}^{\dot{\mu}}(x_L,\theta_L,\bar{\varphi}_L) &= \bar{E}^{\dot{\mu}} + \bar{\varphi}_L^{\dot{\mu}} \frac{n+1}{3n+1}\left(-a-ib+\tfrac{1}{2}\partial_m a^m + \tfrac{i}{2}\partial_m b^m\right) + \\
&+ \bar{\varphi}_L^{\dot{\nu}} \bar{\Omega}_{(\dot{\nu}}{}^{\dot{\mu})} + \theta_L^\nu C_\nu{}^{\dot{\mu}} + \theta_L\theta_L \bar{D}^{\dot{\mu}} + \theta_L^\nu \bar{\varphi}_L^{\dot{\mu}} \frac{n+1}{3n+1}\left(-\eta_\nu + \tfrac{1}{2}\partial_m \chi^m_\nu\right) + \qquad (19) \\
&+ \theta_L^\nu \bar{\varphi}_L^{\dot{\nu}} \sigma_{\nu(\dot{\nu}}{}^{\dot{\mu})} + \theta_L\theta_L \bar{\varphi}_L^{\dot{\mu}} \frac{n+1}{2\cdot(3n+1)} \partial_m (c^m + id^m) + \theta_L\theta_L \bar{\varphi}_L^{\dot{\nu}} \rho_{(\dot{\nu}}{}^{\dot{\mu})}
\end{aligned}
$$

All parameters in the r.h.s. of Eq.(19) are functions of x_L .

From Eqs.(14),(19) one finds that $\mathcal{H}^m$ can be gauged into

$$
\mathcal{H}^m(x,\theta,\bar{\theta}) = \theta^\mu \bar{\theta}^{\dot{\mu}} e^m_{\mu\dot{\mu}} + \bar{\theta}^2 \theta^\mu \psi^m_\mu + \theta^2 \bar{\theta}_{\dot{\mu}} \bar{\psi}^{m\dot{\mu}} + \theta^2 \bar{\theta}^2 A^m \qquad (20)
$$

by means of fixing the parameters $b^m, \chi^m_\mu, c^m, d^m$ in Eq.(19). Note that a^m in (19) remains unrestricted and it serves as the parameter of general coordinate transformations. Further, $\mathcal{H}^\mu$ transforms as follows

$$
\begin{aligned}
\delta\mathcal{H}^\mu = E^\mu - \varepsilon^\mu + \theta^\mu &\left[-\frac{4n+2}{3n+1}\cdot a - \frac{2n}{3n+1}\cdot ib + \frac{n+1}{2(3n+1)}\partial_m a^m \right] + \\
&+ \theta^\nu \left[\Omega_{(\nu}{}^{\mu)} - \omega_{(\nu}{}^{\mu)} \right] + \bar{\theta}_{\dot{\nu}} C^{\dot{\nu}\mu} - \theta\theta\eta^\mu + \bar{\theta}\bar{\theta} D^\mu + \\
&+ \theta^\mu \bar{\theta}_{\dot{\nu}} \frac{n+1}{3n+1}\left(-\bar{\eta}^{\dot{\nu}} - i\partial_m (\bar{\sigma}^m \varepsilon)^{\dot{\nu}}\right) + \theta^\nu \bar{\theta}_{\dot{\nu}} \bar{\sigma}^{\dot{\nu}}{}_{(\nu}{}^{\mu)} + \\
&+ \bar{\theta}^2 \theta_\nu \bar{P}^{(\nu\mu)} + \dots \qquad (21)
\end{aligned}
$$

where the dots denote field-dependent terms. Now one sees that for $n \neq 0, -1/2, -1/3$ one can gauge $\mathcal{H}^{\mu}$ into[5]

$$\mathcal{H}^{\mu}(x,\theta,\bar\theta) = \theta^{\mu}\bar\theta_{\dot\mu}\bar\xi^{\dot\mu} + \bar\theta^{2}\theta^{\mu}B + \theta^{2}\bar\theta_{\dot\mu}(v+iw)^{\dot\mu\mu} + \theta^{2}\bar\theta^{2}\beta^{\mu} \tag{22}$$

by means of fixing all parameters except $\varepsilon^{\mu}(x)$ (local supersymmetry) and $\omega^{(\nu\mu)}(x)$ (local Lorentz). The components in Eq.(20),(22) correspond to the nonminimal set of fields.

5. Peculiarities of the $n=0$ Case: $U(1)$ Local Group and Existence of an Invariant

It is remarkable that for $n=0$ the parameter $ib(x)$ (of local γ_5, or $U(1)$ transformations) drops out of Eq.(21) so it cannot be fixed and $\mathcal{H}^{\mu}$ becomes

$$\mathcal{H}^{\mu}(x,\theta,\bar\theta) = \theta^{\mu}iA + \theta^{\mu}\bar\theta_{\dot\mu}\bar\xi^{\dot\mu} + \bar\theta^{2}\theta^{\mu}B + \theta^{2}\bar\theta_{\dot\mu}(v+iw)^{\dot\mu\mu} + \theta^{2}\bar\theta^{2}\beta^{\mu}. \tag{23}$$

In comparison with Eq.(22) an additional real pseudoscalar field $A(x)$ appears. At the same time $V^{\dot\mu\mu}$ undergoes now gradient transformations, so the total number of components is again 20+20*).

So, in the family of nonminimal sets of fields there is one and only one allowing for local $U(1)$ transformations. This is not yet the set for the new minimal version of N=1 supergravity as we still have 20+20 fields instead of 12+12. However, it turns out that 8+8 fields of this set form a subset closed under supersymmetry transformations. This can be shown by the following clear geometrical reasoning. As was stressed above, for $n=0$ the $C^{4,4}$ supervolume is preserved. Consequently, both $d^{8}z_{L}$ and $d^{8}z_{R} = (d^{8}z_{L})^{\dagger}$ are invariant. On the real hypersurface (13) $d^{8}z_{L}$ and $d^{8}z_{R}$

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*) Note that for $n=-1/2$ the parameter $a(x)$ drops out but $\partial_m a^m(x)$ remains and the gauge can still be fixed as in Eq.(22) although thus restricting the general coordinate transformations[5].

are connected by the change of variables(see Eq.(14.c)) $Z_L \to Z \to Z_R$:

$$d^8 Z_L = Ber\left(\frac{\partial Z_L}{\partial Z}\right) d^8 Z = Ber\left(\frac{\partial Z_L}{\partial Z}\right) \cdot Ber\left(\frac{\partial Z}{\partial Z_R}\right) \cdot d^8 Z_R \,. \tag{24}$$

Therefore the quantity

$$U(x,\theta,\bar\theta) = Ber\left(\frac{\partial Z_L}{\partial Z_R}\right) = Ber\left(\frac{\partial Z_L}{\partial Z}\right) \cdot Ber^{-1}\left(\frac{\partial Z_R}{\partial Z}\right) \tag{25}$$

is invariant under the transformations (11),(15) for and only for $n = 0$. The explicit form of $U(x,\theta,\bar\theta)$ can be easily calculated

$$Ber\left(\frac{\partial Z_L}{\partial Z}\right) = Ber\begin{pmatrix} \delta_n{}^m + i\partial_n \mathcal{H}^m & 0 & \partial_n \bar{\mathcal{H}}^{\dot\mu} \\ i\partial_\nu \mathcal{H}^m & \delta_\nu{}^\mu & \partial_\nu \bar{\mathcal{H}}^{\dot\mu} \\ i\bar\partial_{\dot\nu} \mathcal{H}^m & 0 & \delta_{\dot\nu}{}^{\dot\mu} + \bar\partial_{\dot\nu} \bar{\mathcal{H}}^{\dot\mu} \end{pmatrix} =$$

$$= \frac{\det\left(\delta_n{}^m + i\partial_n \mathcal{H}^m\right)}{\det\left(\delta_{\dot\nu}{}^{\dot\mu} + \bar\Delta^{\dot\mu} \bar{\mathcal{H}}_{\dot\nu}\right)} \,. \tag{26}$$

where [4]

$$\bar\Delta_{\dot\mu} = -\bar\partial_{\dot\mu} - \bar\partial_{\dot\mu} \mathcal{H}^m \cdot (1 - i\partial\mathcal{H})^{-1}{}_m{}^n\, \partial_n$$

so

$$U(x,\theta,\bar\theta) = \frac{\det\left(\delta_n{}^m + i\partial_n \mathcal{H}^m\right)}{\det\left(\delta_{\dot\nu}{}^{\dot\mu} + \bar\Delta^{\dot\mu} \bar{\mathcal{H}}_{\dot\nu}\right)} \cdot \frac{\det\left(\delta_\nu{}^\mu + \Delta_\nu \mathcal{H}^\mu\right)}{\det\left(\delta_n{}^m - i\partial_n \mathcal{H}^m\right)} \,. \tag{27}$$

Clearly, $U U^\dagger = 1$, therefore $U = \exp(iu)$. The real superfield $u(x,\theta,\bar\theta)$ is the carrier of the invariant 8+8 subset. It is a new quantity not yet encountered either in minimal or nonminimal supergravity. Its roots are essentially in the complex structure of $\mathbb{C}^{4/4}$ and it cannot be explained in the framework of real superspace geometry. It is neither a torsion nor a curvature component, nor anything else known in real supergeometry.

6. Constraints on the Prepotentials

Since U is an invariant object it can be used to write down constraints. In fact, one must do that if one wishes to construct an action. Indeed, as was mentioned above, the field $V_a = \frac{1}{2}(\sigma_a)_{\mu\dot\mu} V^{\dot\mu\mu}$ in Eq.(23) (as well as A^m in Eq.(20)) transforms as a gauge field for $U(1)$. However, its dimension is cm^{-2} *) so it cannot have a normal kinetic term of the type $F_{mn}F^{mn}$. The only way it can enter a Lagrangian is to be coupled to a divergenceless (i.e, constrained) axial vector field. This is, indeed, the case realized in the new minimal version of Ref./1,2/. The corresponding constraint is

$$V = 1. \tag{28}$$

The solution to it is easily found in terms of components in the WZ gauge (20),(23):

$$A = 0, \ \bar{\xi}^{\mu} = 0, \ B = 0, \ w_a = -\frac{1}{2}\partial_m e_a^{\ m}, \ \beta_{\mu} = i\partial_m \psi^m_{\ \mu} \tag{29.a}$$

$$\partial_m (A^m - e_a^{\ m} V^a) = 0. \tag{29.b}$$

Eq. (29.b) means that

$$A^m - e_a^{\ m} V^a = \varepsilon^{mnkl}\partial_n a_{kl} \quad , \ a_{kl} = -a_{lk} \tag{30}$$

so the "notoph"/18/ a_{kl} of Ref. /1,2/(together with its additional invariance $\delta a_{kl} = \partial_k b_l - \partial_l b_k$) appears as a solution to the constraint.

7. Invariant Integrals and Action Principle

The constraint (28) enables us to write down an action. To this end we first need an invariant integral for $R^{4/4}$. Let $R^{4/4}$ be parametrized by z_L^M (or their conjugates z_R^M) defined in Eq.(14.c) instead of z^M . Then, according to the geometric meaning of our gauge group (11),(18) the following integrals

$$\begin{aligned} I_L &= \int d^8 z_L \ \phi_L(z_L) = \int d^8 z \ \mathrm{Ber}\left(\frac{\partial z_L}{\partial z}\right) \phi(z) \\ I_R &= \int d^8 z_R \ \phi_R(z_R) = \int d^8 z \ \mathrm{Ber}\left(\frac{\partial z_R}{\partial z}\right) \phi(z) \end{aligned} \tag{31}$$

*) $[\mathcal{H}^\mu] = [\theta^\mu] = cm^{1/2}$ but all components of $\mathcal{H}^\mu$ have to include a factor $\varkappa$, $[\varkappa] = cm^1$ since they vanish in the flat limit.

are invariant. Here $\phi(z)$ is a real scalar superfield, and $\phi_L(z_L)=\phi_R(z_R)^\dagger=\phi(z)$. Further, as a consequence of the constraint (28)

$$\mathrm{Ber}\left(\frac{\partial z_L}{\partial z}\right)=\mathrm{Ber}\left(\frac{\partial z_R}{\partial z}\right)=$$
$$=\left[\frac{\det(\delta_n{}^m+i\partial_n\mathcal{H}^m)}{\det(\delta_\nu{}^\mu+\bar{\Delta}^\mu\bar{\mathcal{H}}_\nu)}\cdot\frac{\det(\delta_n{}^m-i\partial_n\mathcal{H}^m)}{\det(\delta_\nu{}^\mu+\Delta_\nu\mathcal{H}^\mu)}\right]^{1/2}\equiv E \tag{32}$$

therefore

$$I_L=I_R=\int d^8z\; E\cdot\phi(z)\,. \tag{33}$$

Note that the density E is in fact the Berezinian of vielbeins for the curved $R^{4,4}$ with local $U(1)$ in the tangent space (see Sect.8). If we choose $\phi(z)=1$ in Eq.(31) the integrals will vanish and so will the integral in Eq.(33), i.e., the invariant volume of $\mathbb{R}^{4,4}$ (the same phenomenon was observed in Ref./7/, see also Ref./6b/). So, the supervolume of $R^{4,4}$ is not an adequate action for $n=0$ unlike all cases with $n\neq 0$. If we had some nontrivial dimensionless scalar superfield ϕ constructed out of the prepotentials we could put it in Eq.(33) and try this as an action; however, the only such object is U (27) and it is 1 in our case.

Fortunately, the unique properties of the superspace in this case provide another way of constructing an action. Suppose that ϕ in Eq.(33) is not a scalar but transforms as follows:

$$\delta\phi(x,\theta,\bar\theta)=L+R\,, \tag{34}$$

where

$$L=L(x_L,\theta_L)\;,\quad R=R(x_R,\bar\theta_R)\;;\quad R=L^\dagger$$

are some left and right-handed $\mathbb{C}^{4,2}$ (chiral) parameters. Then

$$\delta\int d^8z\,E\cdot\phi=\int d^4x_L d^2\theta_L\, d^2\bar\varphi_L\cdot L(x_L,\theta_L)+h.c.=0$$

because L (R) is independent of $\bar\varphi_L$ (φ_R). Such type of invariant was proposed in Ref./7/. In our approach the superfield ϕ can be constructed in terms of prepotentials:

$$\phi = \ln F$$

$$F = \det{}^{-1/4}\left(\tfrac{1}{4}[\Delta, \sigma_a \bar{\Delta}]\mathcal{H}^m\right) \times \det{}^{-1/8}\left(\delta_n{}^m + \partial_n \mathcal{H}^k \partial_k \mathcal{H}^m\right) \times \tag{35}$$
$$\times \left[\det\left(\delta_\nu{}^\mu + \Delta_\nu \mathcal{H}^\mu\right) \cdot \det\left(\delta_{\dot\nu}{}^{\dot\mu} + \bar{\Delta}^{\dot\mu} \bar{\mathcal{H}}_{\dot\nu}\right)\right]^{3/8}.$$

It transforms according to Eq.(34) with

$$L(x_L, \theta_L) = \frac{\partial \lambda^m}{\partial x_L{}^m} - \frac{\partial \lambda^\mu}{\partial \theta_L{}^\mu} \tag{34'}$$

being the variation of the $C^{4,2}$ volume element. In fact, F is a part of the vielbeins $E_\alpha{}^M$, $E_{\dot\alpha}{}^M$ (see below).

Now we are prepared to write down the action for the new minimal version. Putting Eqs.(32),(35) into the invariant integral one finds

$$S = \frac{1}{\varkappa^2} \int d^4x\, d^2\theta\, d^2\bar{\theta}\; E \cdot \ln F \tag{36}$$

which should be considered together with the constraint (28). Inserting the component field solution (29),(30) to this constraint into the action (36) one obtains exactly the action of Ref./1/.

A major question in the present formulation is how to solve the constraint (28) in terms of <u>superfields</u>. It is easily done in the linearized limit

$$\mathcal{H}^m = \theta \sigma^m \bar{\theta} + \varkappa h^m, \quad \mathcal{H}^\mu = \varkappa h^\mu, \quad \bar{\mathcal{H}}^{\dot\mu} = \varkappa \bar{h}^{\dot\mu}.$$

The solution is (up to gauge freedom)

$$h^\mu = \frac{1}{2} \bar{\sigma}_m{}^{\mu\dot\mu} \bar{\mathcal{D}}_{\dot\mu} h^m \tag{37}$$

and the linearized action (36) is in agreement with Ref./8/ (see also Eq.(5)). In the nonlinear case the answer is not yet known. It seems likely that the local Lorentz gauge (in X - space) has to be fixed (such a possibility in the theory of relativity is known/9/). After that the antisymmetric part of the vierbein $e_a{}^m$ will play the role of the tensor a_{mn}. Then a single vector prepotential will describe all the fields of the model and the unconstrained variation of the action (36) will produce the vector equation of motion the linear approximation to which is Eq.(15). In any case, it is important to investigate this point because it might help to solve the analogous problem in N=2 supergravity. There/6/ the superfields $\mathcal{H}^m, \mathcal{H}^\mu, \bar{\mathcal{H}}^{\dot\mu}$ are also constrained, the volume also vanishes as a consequence of the constraints. The linearized solution is known/19/ but it is not clear how to generalize it to the nonlinear case.

8. Differential Geometry in $\mathbb{R}^{4,4}$

In order to compare the results of the present approach with those of Ref. /7/ one has to develop the differential geometry formalism for $\mathbb{R}^{4,4}$. It is a straightforward procedure (see Ref./4/). Notice that it can be done before imposing the constraint (28) (the latter is needed only for the action).

The derivative

$$\nabla_\alpha \phi = (1+\Delta\mathcal{H})^{-1}{}_\alpha{}^\beta \Delta_\beta \phi \tag{38}$$

of a scalar superfield transforms covariantly under the group (11), (15) (infinitesimally):

$$\delta(\nabla_\alpha \phi) = -(\nabla_\alpha \rho^\beta)\nabla_\beta \phi = \tfrac{1}{2}(\nabla\rho)\nabla_\alpha\phi - (\nabla_{(\alpha}\rho^{\beta)})\nabla_\beta\phi \,. \tag{39}$$

The second term in Eq.(36) is an induced Lorentz transformation in the tangent superspace, while the first one is an induced Weyl one. In fact, the analysis of the component structure suggests that only the $U(1)$ part of the induced Weyl tangent group is essential. The dilatation part can be compensated for by introducing a factor F into the definition of the spinor covariant derivative of a scalar-weightless superfield

$$\mathcal{D}_\alpha \phi = F\nabla_\alpha \phi = E_\alpha{}^M \partial_M \phi \tag{40}$$

with the transformation law (see Eqs.(16),(34'))

$$\delta F = -\tfrac{1}{4}(\nabla\rho + \bar\nabla\bar\rho)F = \tfrac{1}{4}(L+R)F \,. \tag{41}$$

Introducing Lorentz and $U(1)$ connections (the latter is just $\nabla_\alpha \ln F$) and defining

$$\mathcal{D}_a = \frac{i}{4}\, \bar{\sigma}_a^{\dot{\alpha}\alpha} \{ \mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}} \} \equiv E_a{}^M \partial_M \tag{42}$$

one finds expressions for all vielbeins $E_A{}^M$. Further, changing variables from z^M to $z_L{}^M$ or $z_R{}^M$ one finds left or right-handed vielbeins $\ell_A{}^M (r_A{}^M)$. Their Berezinians, according to Eq.(18), transform as scalars, so they can be put equal to some function of the scalar U (27) thus obtaining equations for the factors $F, \bar{F}$ (40).

The particular choice

$$\mathrm{Ber}\,(\ell_A{}^M) = U^{-1/2}$$

leads to the form of $F = \bar{F}$ given in Eq.(35). Further, $\mathrm{Ber}(E_A{}^M)$ calculated with the above value of F is indeed equal to E^{-1} from Eq.(32).

The last step is to calculate the invariant tensors (torsion components) using the covariant derivatives already defined. Our results agree with those of Ref./7/ but we ought to point out the following. The quantity U (27) is an invariant of the theory although there is no room for it among the torsion components. However, its covariant derivatives do appear as torsion components, e.g., $T_{\alpha b}{}^b$ is expressed in terms of $\mathcal{D}_\alpha U$, $T_{\alpha b}{}^{\dot{\gamma}}$ in terms of $\mathcal{D}\mathcal{D}U$,etc. So, the constraint (28) yields the vanishing of all those torsion components. In the framework of real superspace geometry U is not present. There, however, there is the constraint

$$T_{\alpha b}{}^b = T_{\dot{\alpha} b}{}^b = 0$$

which is equivalent to

$$\mathcal{D}_\alpha U = \bar{\mathcal{D}}_{\dot{\alpha}} U = 0 \tag{43}$$

in our language. Eq.(43) implies U = const which is essentially the same as Eq.(28). This explains the agreement between the two approaches.

9. A Weaker Constraint for the Case $n=0$

Here we would like to discuss briefly a weaker constraint on the superfield U. In this case we find a superanalogue of a "notoph"/18/ (superspin 0 on-shell and $^1/_2$ off-shell) which interacts with $U(1)$ supergravity.

Consider the integral

$$I_1 = \int d^8 z_L \ln F = \int d^8 z \, \mathrm{Ber}\left(\frac{\partial z_L}{\partial z}\right) \cdot \ln F$$

taken over $R^{4/4}$ in the left-handed parametrization. According to Eq.(34)

$$\delta I_1 = \int d^8 z_L \, (L+R) = \int d^8 z_L \cdot R$$

because L does not depend on $\bar{\varphi}_L$. Further, going to the right-handed parametrization, we find

$$\delta I_1 = \int d^8 z_R \cdot U \cdot R = \int d^4 x_R \, d^2 \bar{\theta}_R \left(\frac{\partial^2}{\partial \varphi_R^2} U\right) R$$

because now R does not depend on φ_R. So, I_1 will be invariant if

$$\frac{\partial^2}{\partial \varphi_R^2} U = 0 \tag{44}$$

which is covariant constraint (the l.h.s. of Eq.(43) transforms as a scalar with a chiral weight) <u>weaker</u> than Eq.(28). Notice that the quantity I_1 is not real since U is not 1 now. Furthermore, we can write down another nontrivial complex invariant

$$I_2 = \int d^8 z_L \, f(U),$$

where $f(U)$ is some function of the scalar U.

The constraint (44) has been solved in the WZ gauge and only linearly. The fields A, ξ_α (23) now remain unconstrained, $B=0$, $\tilde{W}_a$, $\tilde{V}_a$ become divergenceless*) and β_α is expressed in terms of $\psi_{m\alpha}$ again. In other words, under the weaker

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*) $\tilde{W}_a = W_a + \frac{1}{2}\partial_m e_a{}^m$, $\tilde{V}_a = V_a - A_a + \frac{1}{2}\partial_a A$

constraint (43) the superfield U describes a superspin $^1/_2$ multiplet ($A, \xi_\alpha, \widetilde{W}_a$)*). Inserting this linearized solution into the action $S_1 = \mathrm{Re}\, I_1$, one finds a sum of the action of Ref./1,2/ and an action for the superspin $^1/_2$ matter multiplet. The second action $S_2 = \mathrm{Re}\, I_2$ produces the superspin $^1/_2$ kinetic terms once again thus allowing to regulate their sign (or eliminate them completely). This alternative version is now under investigation. Details will be reported elsewhere.

10. Concluding Remarks

We have seen that the usage of the adequate complex geometry makes transparent the meaning of the basic facts in all N=1 supergravity models. Apparently, the results obtained can be generalized to the N=2 case and the existence of an N=2 version with local $U(1)$ symmetry is plausible. Is there also a version with local $U(2)$ symmetry? Will this remarkable mechanism of auxiliary fields appearing as gauge ones work for higher N, e.g.,N=8?

There exists an opinion /1,7,20/ that in the component field approach anomalies will break down the local $U(1)$ symmetry of the new version. However, then supersymmetry will be broken too. Couldn't the quantization be performed in a manifestly supersymmetric way thus avoiding this difficulty? Even with the most sceptical attitude towards these possibilities they are worth a very careful investigation.

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*) Remarkably, on-shell this multiplet of fields describes superspin 0. The divergenceless vector $\widetilde{W}_a$ is the field strength of a notoph /18/ (spin 0 on-shell, spin 1 off-shell).

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