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HIGHER ORDER CORRECTIONS FOR THE RUNNING COUPLING CONSTANT OF QCD AND AN EFFECTIVE POTENTIAL OF THE ONE-GLUON EXCHANGE IN t- AND s-CHANNELS

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1. INTRODUCTION

In the last years formulae of quantum chromodynamics (QCD) for the one-gluon exchange amplitude are widely used for calculations of different processes at short distances. So, for example, in ref.[1] an effective potential of the quarkonium is calculated as the Fourier transform of this amplitude. However, in this case considerable technical difficulties appear when the expression for the effective ("running") coupling constant $\alpha_s(Q^2)$ is used in the one-loop approximation. The use of formulae in the two-loop and three-loop approximations and, especially, the formulae which take into account the quark masses would lead to greater difficulties.

Moreover, the formulae for the effective constant obtained in QCD are singular at $Q^2 \to 0$. As a result, in the calculation of the effective potential as the Fourier transform of the QCD Born amplitude it is necessary to resort to the regularization of the expressions for $\alpha_s(Q^2)$ at small Q^2.

However, we can approach to the mentioned range of problems from a different point of view. Namely, we may pose the task of finding in the momentum space a simple enough expression regular at $Q^2 \to 0$ for the effective potential, that behaves at high Q^2 as the QCD one-gluon exchange amplitude.

This task in the one-loop approximations has been solved in ref.[2] where it is established that a Fourier transform of the Coulomb-like potential

$$V_0(r) = -g^2/r, \quad g^2 = \text{const} \tag{1}$$

in the relativistic configurational representation (its coordinates $\vec{r}$ are conjugate to rapidities but not to momenta)[3] has the form

$$V_0(Q^2) = -\frac{4\pi g^2}{m^2 y \sinh y} \Big|_{Q^2 \to \infty} \approx -\frac{8\pi g^2}{Q^2 \cdot \ln Q^2/m^2}. \tag{2}$$

Here $y = \cosh^{-1}(1 + Q^2/2m^2)$ is the rapidity, m is the effective quark mass. From (2) it follows that the behaviour of the potential at large Q^2 coincides in an asymptotic region with the Q^2-behaviour of the QCD one-gluon exchange amplitude

$$T(Q^2) \simeq \frac{\alpha_s(Q^2)}{Q^2} \sim \frac{1}{Q^2 \cdot \ln Q^2/\Lambda^2}. \qquad (3)$$

The aim of the present paper is to construct such a simple regular at $Q^2 \to 0$ expression for the effective coupling constant $\tilde{\alpha}_s(Q^2)$ that the amplitude

$$T(Q^2) = \frac{\tilde{\alpha}_s(Q^2)}{Q^2} \qquad (4)$$

coincides at $Q^2 \to \infty$ with eq. (3) and differs from eq. (3) by no more than 10% at moderate Q^2. For $\alpha_s(Q^2)$ in eq. (3) we will use two- and three-loop formulae and other presently known expressions and its generalizations. The denominators of formulae (3), (4) coincide with each other, therefore, we will compare the expressions for $\alpha_s(Q^2)$ and $\tilde{\alpha}_s(Q^2)$ with each other. In other words we want to find the function $\tilde{\alpha}_s(Q^2)$ that approximates all known formulae for $\alpha_s(Q^2)$ listed in the first section. In the second section the obtained expression for the effective amplitude is continued into the cross-channel and is compared with presently known continuations.

2. THE FORMULAE FOR THE QCD EFFECTIVE COUPLING CONSTANT

By now a number of formulae for the effective coupling constant $\alpha_s(Q^2) \equiv \bar{g}^2(Q^2)/16\pi^2$ is obtained. So, in the first papers on QCD /4/ the widely known expression for α_s in the one-loop approximation

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \beta_0 \alpha_s(\mu^2) \cdot \ln Q^2/\mu^2} \equiv \frac{1}{\beta_0 \cdot \ln Q^2/\Lambda^2} \qquad (5)$$

has been found. There $\beta_0 = 11 - 2/3 \cdot N_f$, N_f is the number of quark flavors, μ^2 is a subtraction point, Λ is a scale parameter of QCD.

The constant $\alpha_s(Q^2)$ has been calculated in ref. /5/ in the two-loop approximation and in ref. /6/ in the three-loop approximation (MS scheme):

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \cdot L} - \frac{\beta_1 \cdot \ln L}{\beta_0^3 \cdot L^2} + \frac{1}{\beta_0^5 \cdot L^3}(\beta_1^2 \cdot \ln L - \ln L + \beta_2 \beta_0 - \beta_1^2), \qquad (6)$$

$$L = \ln Q^2/\Lambda^2, \; \beta_1 = 102 - \frac{38}{3} N_f, \; \beta_2 = \frac{2857}{2} - \frac{5033}{18} N_f + \frac{524}{54} N_f^2 .$$

The first term corresponds to the contribution of one-loop diagrams; the second term, to the contribution of two-loop diagrams; and the third term, to the contribution of three-loop diagrams. A detailed analysis of the formulae (5), (6) is performed in ref. [7].

In the derivation of the formulae (5), (6) the quarks are considered as massless ones. However, in the attainable energy range it is necessary to take into account quark masses. This was carried out in ref. [8] in the one-loop approximation. The formula for α_s is complicated but it can be approximately represented in the form

$$\alpha_s(Q^2) \approx \frac{\alpha_s(\mu^2)}{1+\alpha_s(\mu^2)\cdot\left[11\cdot\ln\frac{Q^2}{\mu^2}-\frac{2}{3}\cdot\sum_{i=1}^{N_f}\ln\frac{Q^2+5m_i^2}{\mu^2+5m_i^2}\right]}, \qquad (7)$$

where $i = u, d, s, c, \ldots$, m_i is the mass of a quark with the i-th flavor.

In ref. [9] the masses of heavy quarks are taken into account in the two-loop approximation. The author obtained the following approximate formula:

$$\frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} = 1 + \alpha_s(\mu^2)\cdot[9\cdot\ln x - \frac{2}{3}(J'(x) - J'(1))] + \frac{64}{9}\alpha_s(\mu^2)\times \qquad (8)$$

$$\times \ln\{1+\alpha_s(\mu^2)\cdot[9\cdot\ln x - \frac{57}{32}(J'(x) - J'(1))]\},$$

where

$$J'(t) = \sum_{i=4}^{5} I_1(t/y_i), \qquad y_i = m_i^2/\mu^2, \quad x = Q^2/\mu^2, \qquad (9)$$

$$I_1(t) = 6\cdot\int_0^1 dx\cdot(1-x)x\cdot\ln[1+x(1-x)t]. \qquad (10)$$

However, the comparison of QCD predictions with the recent experimental data [10] shows that the scale parameter Λ_{MS} is of an order of 50 MeV [11]. In this connection it may be expedient to take into account the contribution of the light quarks also. If we will follow reasonings of ref. [9] and will take into account the masses of all the quarks, then, as it is easy to show, we will arrive at the new formula

$$\frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} = 1 + \alpha_s(\mu^2)\cdot[11\cdot\ln x - \frac{2}{3}(J(x) - J(1))] + \frac{102}{11}\alpha_s(\mu^2)\times$$
$$\times\ln\{1+\alpha_s(\mu^2)\cdot[11\cdot\ln x - \frac{209}{153}(J(x) - J(1))]\},\quad J(t) = \sum_{i=1}^{N_f} I_1(t/y_i). \qquad (11)$$

The basic difference of the formula (11) from (8) consists in that $J(t)$ in (11) is the sum over all quark flavours of the functions $I_1(t/y_i)$, where $y_i = m_i^2/\mu^2$, while $J'(t)$ in (8) is the sum over the b and c quarks only. Therefore the formulae differ from each other by numerical factors also.

3. THE EFFECTIVE POTENTIAL OF THE ONE-GLUON EXCHANGE AND A SIMPLE REGULAR AT $Q^2\to 0$ PARAMETRIZATION OF THE RUNNING COUPLING CONSTANT

Above listed formulae for the running coupling constant $\alpha_s(Q^2)$ (5)-(11) are correct for such values of Q^2 when the condition of weak coupling $4\pi\cdot\alpha_s(Q^2)\ll 1$ is hold. As a result, an extension of the formulae (5)-(11) on a range of small Q^2 is illegal. It cannot be performed because of technical reasons also: the formulae (5)-(11) are singular at $Q^2\to 0$. To use, nevertheless, the formulae for $\alpha_s(Q^2)$ at small Q^2 a regularization of the formulae is necessary. For this purpose in the problem of quarkonium the substitution $\ln Q^2/\Lambda^2 \to \ln(\zeta + Q^2/\Lambda^2)$, $\zeta > 1$ is used because it leads to a linear confining potential at large r [1].

On the other hand, the simple Coulomb-like potential (1) gives the expression (2) in the momentum space that represents at large Q^2 the Q^2-behaviour of the QCD one-gluon exchange amplitude (3).

The difference between the asymptotics of the expressions (2) and (3) is that the other unknown parameter, the quark mass m, enters into eq.(9) instead of the scale parameter Λ.

As is known [12] the magnitude of the scale parameter Λ is insignificant, if the one-loop approximation (5) is chosen for the "running" coupling constant $\alpha_s(Q^2)$ and calculations are restricted to the first order in $(\ln Q^2/\Lambda^2)^{-1}$.

Namely, any change in the scale parameter Λ in describing structure function moments, as is known [12], is equivalent in a nonleading order to a shift of coefficients of higher corrections proportional to the one-loop anomalous dimensions. Thus attempts to extract the value of Λ from experimental data with only the leading order formulae in QCD have no meaning.

Analogously, in the first approximation in $(\ln Q^2)^{-1}$ the formula (2) coincides with the formula (3) with $\alpha_s(Q^2)$ taken in any form among (5)-(7), (11) and correction terms of order $(\ln Q^2)^{-n}$, $n>1$ give the smaller contribution in the coupling constant. Indeed,

$$\frac{1}{\ln Q^2/\Lambda^2} = \frac{1}{\ln Q^2/m^2 + \ln m^2/\Lambda^2} =$$

$$= \frac{1}{\ln Q^2/m^2} \cdot \frac{1}{1+\dfrac{\ln m^2/\Lambda^2}{\ln Q^2/m^2}} = \frac{1}{\ln Q^2/m^2}\left(1 - \frac{\ln m^2/\Lambda^2}{\ln Q^2/m^2} + \dots\right). \tag{12}$$

The coincidence of formulae (2), (3) at asymptotic values of Q^2 allows us to introduce the following approximation of the effective coupling constant

$$\alpha_s(Q^2) \approx \tilde{\alpha}_s(Q^2) \equiv \frac{g^2}{y \cdot \sinh y} \cdot \frac{Q^2}{2m^2}. \tag{13}$$

At $Q^2 \to \infty$

$$\frac{1}{\sinh y} \cdot \frac{Q^2}{2m^2} \to 1, \quad y \to \ln\frac{Q^2}{m^2} \approx \ln\frac{Q^2}{\Lambda^2},$$

so that $\tilde{\alpha}_s(Q^2) \to g^2/(\ln Q^2/\Lambda^2)$. On the other hand at $Q^2 \to 0$

$$\tilde{\alpha}_s(Q^2) = g^2 \frac{y}{\sinh y} \cdot \frac{Q^2}{y^2 \cdot 2m^2} \to g^2 \cdot 1 \cdot 1 = g^2,$$

i.e., the expression (13) is regular at $Q^2 \to 0$.

To compare $\tilde{\alpha}_s(Q^2)$ (13) with the formula (5), we will proceed as follows: Let us fix the boundaries Q_1^2, Q_2^2 of the interval of changing Q^2: $Q_1^2 \le Q^2 \le Q_2^2$; then divide the interval $[Q_1^2, Q_2^2]$ into 10 equal parts (in the log scale). Let us denote the obtained points by Q_j^2, $j = 1,2,\dots,11$. Further, let us fix the value of the scale parameter Λ and find by the formula (5) values of $\alpha_s(Q^2)$ at the points Q_j^2. Let us denote $\alpha_j \equiv \alpha_s(Q_j^2)$.

Now we can use the program "FUMILI" /13/ and find such values of the parameters m and g^2 in (13) that the $\tilde{\alpha}_s(Q^2)$ passes through the points α_j the most accurate way. It turns out that a good agreement between the formulae (5) and (13) can be achieved if one parameter g^2 will be considered as a free one and the quark mass m will be fixed in the interval 100-500 MeV. As a measure of the deviation of $\tilde{\alpha}_s(Q^2)$ from $\alpha_s(Q^2)$ we use the quantity

$$\epsilon = \max_{Q_1^2 \leq Q^2 \leq Q_2^2} \left| \frac{\alpha_s(Q^2) - \tilde{\alpha}_s(Q^2)}{\alpha_s(Q^2) + \tilde{\alpha}_s(Q^2)} \right|. \tag{14}$$

In table 1 the values of ϵ and g^2 are represented for Λ =50, 100, 150 and 200 MeV and m =100, 200, 300, 400 and 500 MeV at $N_f = 3$ and $N_f = 4$. The following interval of changing Q^2: $Q_1^2 = 5$ GeV^2, $Q_2^2 = 1000$ GeV^2 has been choose. The results of comparison of $\tilde{\alpha}_s(Q^2)$ with the formulae for $\alpha_s(Q^2)$ in the two- and three-loop approximations (see (6)) are represented in tables 2,3 (figs.1-3).

Comparing $\tilde{\alpha}_s(Q^2)$ (13) with the formula (7) we used values of the parameters such as in ref.[8] : $\mu^2 = 9$ GeV , $\alpha_s(\mu^2) = \frac{0.5}{16\pi^2}$, $m_e = 1.5$ MeV, $m_s = 0.4$ GeV, $m_u = m_d = m_s/20$. The results are represented in table 4 (fig.4). To compare $\tilde{\alpha}_s(Q^2)$ with the formula (11) we put as in ref.[9] $\mu^2 = 10$ GeV^2 and the following values: $100 \cdot \alpha_s(\mu^2) \equiv a = 1.2$, 1.5, 1.8, 2.0, 2.8 are subsequently taken. Values of the quark masses are taken such as in comparing with the formula (7). In both the cases we have chosen $Q_1^2 = 20$ GeV^2. The results of analysis are represented in tables 5,6 (fig.5).

From tables 1-6 it follows that the formula (13) for $\tilde{\alpha}_s(Q^2)$ well approximates the expressions for $\alpha_s(Q^2)$ obtained in the massless quarks case in the one-, two-, and three-loop approximations and in the case of massive quarks in the one- and two-loop approximations.

It is interesting that in the one-loop approximation for $\alpha_s(Q^2)$ the asymptotics of the wave function of a spherical symmetric state of a two-particle system does not depend on the scale parameter Λ. Really, in the single-time formulation of two-particle problem[14] an equation for the wave function (see, for example, ref.[15]) after integrating over angles (see ref.[16]) takes the form

$$2m\cosh\chi_p \cdot (M - 2m\cosh\chi_p) \cdot \sinh\chi_p \cdot \phi(\chi_p) = \tag{15}$$

$$= (2\pi)^{-2} \int d\chi_k \cdot \tilde{V}(\chi_p, \chi_k) \cdot \sinh\chi_k \cdot \phi(\chi_k),$$

where χ_p and χ_k are constituent rapidities. If as an effective potential we shall take the QCD one-gluon exchange amplitude in the one-loop approximation (10) then the kernel of eq.(15) will acquire the form:

$$\tilde{V}(\chi_p,\chi_k) = \ln \left| \frac{\ln\left|\frac{2m}{\Lambda}\sinh\frac{\chi_p-\chi_k}{2}\right|}{\ln\left|\frac{2m}{\Lambda}\sinh\frac{\chi_p+\chi_k}{2}\right|} \right|_{Q^2\to\infty} \approx \frac{\chi_k}{\chi_p},$$

i.e., it does not depend on Λ. As a result, the solution of eq.(15) does not depend on Λ. In the two- and three-loop approximations the kernels of eq. (15) contain the Λ-dependences.

The potential chosen in the form of the amplitude (3) with the coupling constant in any approximation can be represented according to (12) in the form

$$V(Q^2) \equiv T(Q^2) = V_0(Q^2)\cdot[1-(\ln m^2/\Lambda^2)/y + \dots], \qquad (16)$$

where the potential V_0 is given by the formula (2). For eq. (15) with the potential (2) an exact solution is known and solutions of eq.(15) with the potential (16) can be obtained within perturbation theory.

The solution of eq. (15) with the potential (2) was found in ref.[17] by the transition from the momentum space to the relativistic configurational representation[3]

$$\phi_{s\sigma}(\vec{p}) = \int d\vec{r}\cdot\xi(\vec{p},\vec{r})\cdot\phi_{s\sigma}(\vec{r}),$$

where the functions $\xi(\vec{p},\vec{r}) = (p^{\mu}n_{\mu}/m)^{-1-irm}$, n_{μ} $(1,\vec{r}/|\vec{r}|)$ realize unitary infinite-dimensional representations of the Lorentz group[18]. The solution has the form

$$\phi_{s\sigma}(\vec{r}) = C\cdot e^{-rm\chi_0},$$

where C is an i-periodical constant and the parameter χ_0 is introduced by the parametrization $M = 2m\cos\chi_0$.

4. THE CROSSING TRANSFORMATION FOR THE EFFECTIVE COUPLING CONSTANT

The approximation of the effective coupling constant by the expression (13) allows us to approach to the problem of the chromodynamic description of processes in the time-like transfer momentum range in a new way.

As an expansion parameter in perturbative calculations the quantities $\alpha_s(|Q^2|)$, $|\alpha_s(Q^2)|$, $\mathrm{Re}\,\alpha_s(Q^2)$ and the expression

$$\frac{1}{\pi\beta_0}\tan^{-1}\left(\frac{\pi}{\ln|Q^2|/\Lambda^2}\right) \tag{17}$$

(in the one-loop approximation) and other combinations were suggested (see ref.[19] and references therein). The formula for $\tilde{\alpha}_s(Q^2)$ (13) can be also continued into the cross-channel. At the crossing transformation we have: $y\to y+i\pi$, $\sinh y\to -\sinh y$[20] therefore

$$\tilde{\alpha}_s(Q^2) \longrightarrow \frac{-g^2}{(y+i\pi)\cdot\sinh y}\cdot\frac{Q^2}{2m^2}. \tag{18}$$

As a result, as an expansion parameters the quantities

$$|\tilde{\alpha}_s(Q^2)| = \frac{g^2}{\sqrt{y^2+\pi^2}\cdot\sinh y}\cdot\frac{|Q^2|}{2m^2}, \tag{19}$$

$$\mathrm{Re}\,\tilde{\alpha}_s(Q^2) = \frac{g^2 y}{(y^2+\pi^2)\cdot\sinh y}\cdot\frac{|Q^2|}{2m^2}, \tag{20}$$

and also $\tilde{\alpha}_s(|Q^2|)$ can be used. The behaviour of $\tilde{\alpha}_s(|Q^2|)$, $|\tilde{\alpha}_s(Q^2)|$ and $\mathrm{Re}\,\tilde{\alpha}_s(Q^2)$ as functions of $|Q^2|$ is depicted in fig.6 (curves 1,2,3, respectively). The quark mass value m was taken to be 200 MeV, the value of the constant $g^2 \simeq 0.928$ was obtained by comparing the formula (13) with the formula (5) at Λ=100 MeV. In this figure the behaviour of $\alpha_s(|Q^2|)$, $|\alpha_s(Q^2)|$, $\mathrm{Re}\,\alpha_s(Q^2)$ in the one-loop approximation (see eq. (5)) and the expression (17) is also depicted (the curves 4,5,6,7). In all the cases the value of the parameter Λ was taken to be equal to 100 MeV.

The best parameter of the perturbative expansion is the modulo one. From fig.6 it follows that it is the parameter (20).

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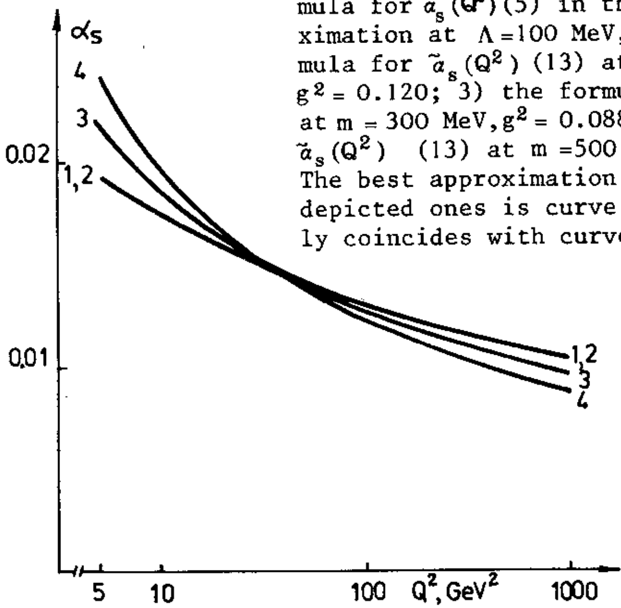


Fig.1. The dependence of the effective coupling constant on Q^2: 1) the QCD formula for $\alpha_s(Q^2)$ (5) in the one-loop approximation at Λ = 100 MeV, $N_f = 4$; 2) the formula for $\tilde{\alpha}_s(Q^2)$ (13) at m = 100 MeV, $g^2 = 0.120$; 3) the formula for $\tilde{\alpha}_s(Q^2)$ (13) at m = 300 MeV, $g^2 = 0.088$; 4) the formula $\tilde{\alpha}_s(Q^2)$ (13) at m = 500 MeV, $g^2 = 0.072$. The best approximation of $\alpha_s(Q^2)$ among depicted ones is curve 2 (that practically coincides with curve 1).

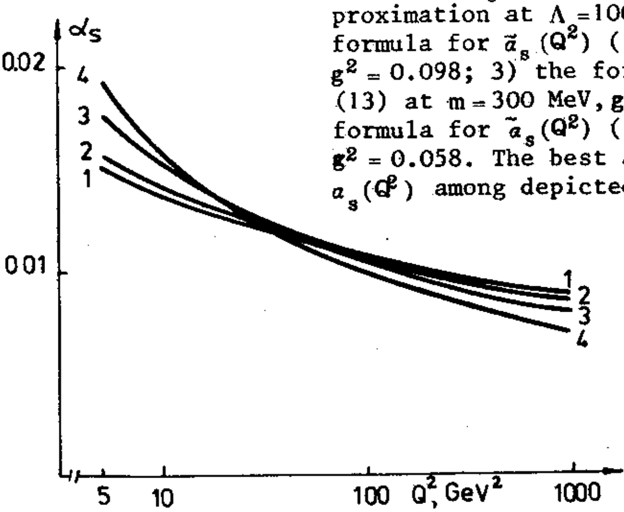
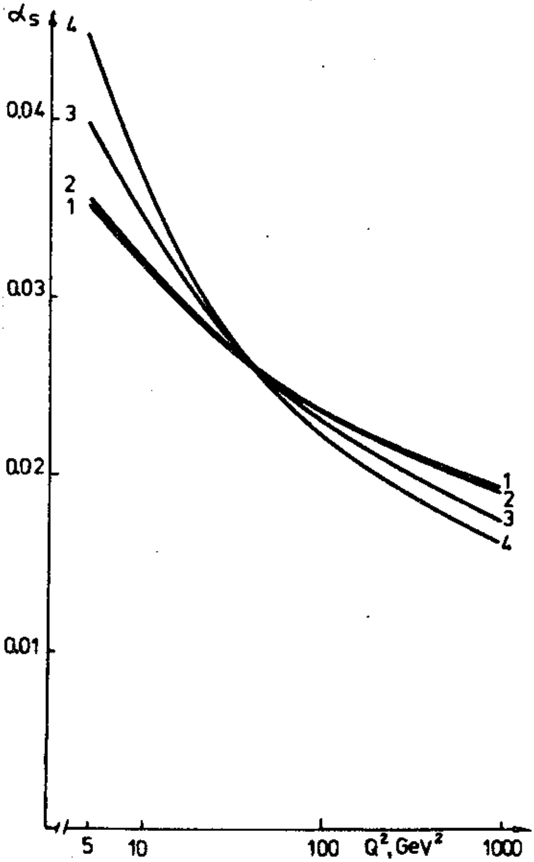


Fig.2. The behaviour of the effective coupling constant on Q^2: 1) the QCD formula for $\alpha_s(Q^2)$ (6) in the two-loop approximation at Λ = 100 MeV, $N_f = 4$; 2) the formula for $\tilde{\alpha}_s(Q^2)$ (13) at m = 100 MeV, $g^2 = 0.098$; 3) the formula for $\tilde{\alpha}_s(Q^2)$ (13) at m = 300 MeV, $g^2 = 0.072$; 4) the formula for $\tilde{\alpha}_s(Q^2)$ (13) at m = 500 MeV, $g^2 = 0.058$. The best approximation of $\alpha_s(Q^2)$ among depicted ones is curve 2.

Fig.3. The dependence of the effective coupling constant on Q^2: 1) the QCD formula for $\alpha_s(Q^2)$ (6) in the three-loop approximation at $\Lambda = 100$ MeV, $N_f = 4$; 2) the formula for $\tilde{\alpha}_s(Q^2)$ (13) at m=100 MeV, $g^2 = 0.222$; 3) the formula for $\tilde{\alpha}_s(Q^2)$ (13) at m=300 MeV, $g^2 = 0.163$; 4) the formula for $\tilde{\alpha}_s(Q^2)$ (13) at m=500 MeV, $g^2 = 0.134$. The best approximation of $\alpha_s(Q^2)$ among depicted ones is curve 2.



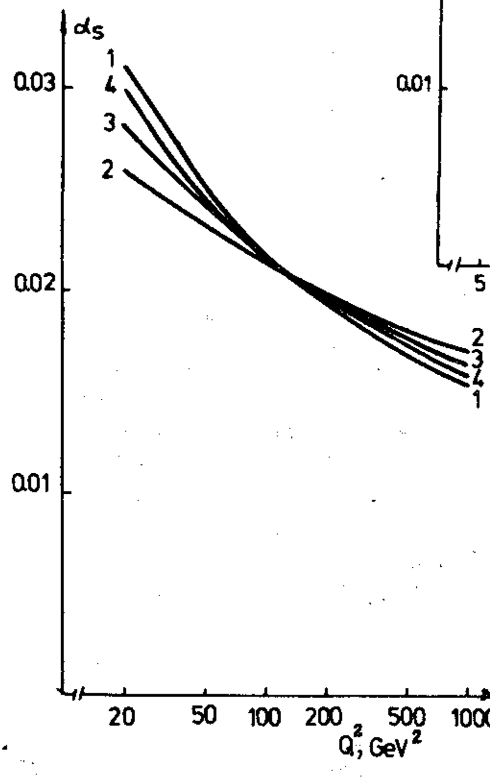


Fig.4. The dependence of the effective coupling constant on Q^2: 1) the QCD formula for $\alpha_s(Q^2)$ (7); 2) the formula for $\tilde{\alpha}_s(Q^2)$ (13) at m=100 MeV, $g^2 = 0.197$; 3) the formula for $\tilde{\alpha}_s(Q^2)$ (13) at m = 300 MeV, $g^2 = 0.152$; 4) the formula for $\tilde{\alpha}_s(Q^2)$ (13) at m = 500 MeV, $g^2 = 0.131$. The best approximation of $\alpha_s(Q^2)$ among depicted ones is curve 4.

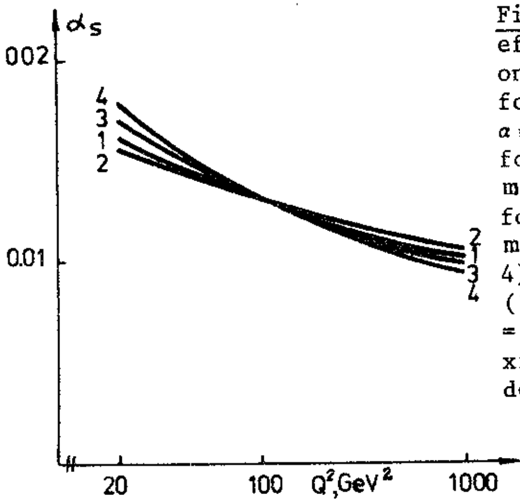


Fig.5. The dependence of the effective coupling constant on Q^2: 1) the QCD formula for $\alpha_s(Q^2)$ (11) at $\alpha \equiv \alpha_s(\mu^2) \cdot 100 = 1.8$; 2) the formula for $\tilde{\alpha}_s(Q^2)$ (13) at $m = 100$ MeV, $g^2 = 0.120$; 3) the formula for $\tilde{\alpha}_s(Q^2)$ (13) at $m = 300$ MeV, $g^2 = 0.092$; 4) the formula for $\tilde{\alpha}_s(Q^2)$ (13) at $m = 500$ MeV, $g^2 = 0.078$. The best approximation of $\alpha_s(Q^2)$ among depicted ones is curve 2.

Fig.6. The behaviour of the effective constant on $|Q^2|$ in the time-like transfer-momentum region: 1) the formula for $\tilde{\alpha}_s(|Q^2|)$, where $\tilde{\alpha}_s(|Q^2|)$ is given by the expression (13); 2) the formula for $|\tilde{\alpha}_s(Q^2)|$ (19); 3) the formula for $\mathrm{Re}\,\tilde{\alpha}_s(Q^2)$ (20); 4) the QCD formula for $\alpha_s(|Q^2|)$, where $\alpha_s(Q^2)$ is given by the expression (5); 5) the QCD formula for $|\alpha_s(Q^2)|$; 6) the QCD formula for $\mathrm{Re}\,\alpha_s(Q^2)$; 7) the QCD formula (17).

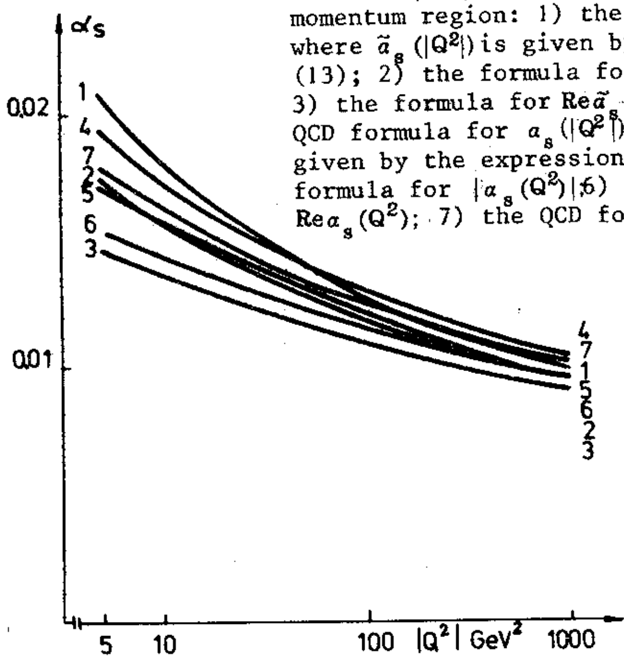


Table 1

The results of comparison of the formulae (13) and (5)

	m, MeV \ Λ, MeV	50		100		150		200	
		g^2	ε	g^2	ε	g^2	ε	g^2	ε
$N_f=3$	100	0.096	2.51	0.111	0.0004	0.123	2.02	0.132	3.83
	200	0.080	5.95	0.093	3.61	0.103	1.72	0.111	0.006
	300	0.070	8.65	0.082	6.46	0.091	4.67	0.098	3.05
	400	0.063	11.05	0.074	9.00	0.082	7.31	0.0089	5.77
	500	0.057	13.32	0.067	11.40	0.075	9.81	0.081	8.36
$N_f=4$	100	0.103	2.51	0.120	0.0004	0.133	2.02	0.143	3.83
	200	0.086	5.95	0.100	3.61	0.111	1.72	0.120	0.006
	300	0.075	8.65	0.088	6.46	0.098	4.67	0.106	3.05
	400	0.068	11.05	0.079	9.00	0.088	7.31	0.096	5.77
	500	0.061	13.32	0.072	11.40	0.081	9.81	0.088	8.36

Table 2

The results of comparison of the formula (13) with the formula (6) in the two-loop approximation

	m, MeV \ Λ, MeV	50		100		150		200	
		g^2	ε	g^2	ε	g^2	ε	g^2	ε
$N_f=3$	100	0.078	4.22	0.089	2.20	0.097	0.54	0.104	0.97
	200	0.065	7.53	0.074	5.66	0.081	4.11	0.087	2.69
	300	0.057	10.13	0.065	8.37	0.071	6.92	0.077	5.57
	400	0.051	12.42	0.059	10.78	0.064	9.42	0.069	8.15
	500	0.046	14.59	0.053	13.06	0.058	11.79	0.063	10.59
$N_f=4$	100	0.085	4.09	0.098	2.03	0.107	0.50	0.114	1.20
	200	0.071	7.41	0.081	5.50	0.089	3.92	0.096	2.47
	300	0.062	10.01	0.072	8.22	0.075	6.74	0.084	5.37
	400	0.056	12.32	0.064	10.65	0.071	9.25	0.076	7.96
	500	0.051	14.49	0.058	12.94	0.064	11.63	0.069	10.42

Table 3

The results of comparison of the formulae (13) and (6)

	m, MeV \ Λ, MeV	50		100		150		200	
		g^2	ε	g^2	ε	g^2	ε	g^2	ε
$N_f=3$	100	0.177	2.93	0.205	0.48	0.226	1.52	0.243	3.33
	200	0.147	6.34	0.171	4.06	0.189	2.18	0.204	0.48
	300	0.129	9.02	0.150	6.88	0.167	5.11	0.180	3.49
	400	0.116	11.39	0.136	9.38	0.150	7.71	0.163	6.18
	500	0.105	13.63	0.123	11.76	0.140	10.19	0.149	8.75
$N_f=4$	100	0.192	2.93	0.222	0.50	0.245	1.49	0.264	3.28
	200	0.160	6.35	0.185	4.08	0.205	2.22	0.221	0.53
	300	0.140	9.02	0.163	6.89	0.181	5.14	0.195	3.54
	400	0.126	11.39	0.147	9.40	0.163	7.75	0.176	6.23
	500	0.114	13.63	0.134	11.77	0.148	10.22	0.161	8.79

Table 4

The results of comparison
of the formulae (13) and (7)

	$N_f=3$		$N_f=4$	
m MeV	g^2	ε	g^2	ε
100	0.192	10.11	0.197	9.14
200	0.164	7.77	0.169	6.82
300	0.148	5.97	0.152	5.05
400	0.137	4.43	0.140	3.52
500	0.127	3.03	0.131	2.14
600	0.120	1.72	0.123	0.85
700	0.113	0.46	0.116	0.40
800	0.108	0.77	0.111	0.16
900	0.103	1.98	0.105	2.80
1000	0.098	3.20	0.101	3.99

Table 5

The results of comparison of the formulae (13) and (11) at $N_f = 3$ $(a \equiv a_s(\mu^2) \cdot 100)$

m MeV \ a	1.2		1.5		1.8		2.0		2.4		2.8	
	g^2	ε	g^2	ε	g^2	ε	g^2	ε	g^2	ε	g^2	ε
100	0.087	1.77	0.103	0.12	0.117	1.96	0.125	3.14	0.139	5.40	0.152	7.51
200	0.074	3.73	0.088	1.91	0.100	0.15	0.107	0.99	0.119	3.18	0.130	5.24
300	0.067	5.21	0.079	3.45	0.089	1.74	0.096	0.64	0.107	1.49	0.117	3.50
400	0.061	6.48	0.072	4.77	0.082	3.11	0.088	2.03	0.099	0.15	0.108	2.01
500	0.057	7.62	0.067	5.96	0.076	4.34	0.082	3.29	0.092	1.26	0.100	0.66
600	0.053	8.68	0.063	7.07	0.072	5.50	0.077	4.47	0.086	2.49	0.094	0.65
700	0.050	9.69	0.059	8.13	0.068	6.60	0.073	5.60	0.081	3.70	0.089	1.82
800	0.047	10.67	0.056	9.16	0.064	7.66	0.069	6.69	0.077	4.80	0.085	2.99
900	0.045	11.64	0.053	10.16	0.061	8.72	0.065	7.77	0.074	5.92	0.080	4.16
1000	0.043	12.59	0.051	11.17	0.058	9.76	0.062	8.84	0.070	7.04	0.077	5.32

Table 6

The results of comparison of the formulae (13) and (11) at
$N_f = 4$ ($a \equiv a_s(\mu^2)\cdot 100$)

m, MeV \ α	1.2		1.5		1.8		2.0		2.4		2.8	
	g^2	ε	g^2	ε	g^2	ε	g^2	ε	g^2	ε	g^2	ε
100	0.089	2.34	0.105	0.56	0.120	1.15	0.128	2.26	0.144	4.40	0.160	6.42
200	0.076	4.27	0.090	2.58	0.102	0.92	0.110	0.15	0.123	2.22	0.134	4.18
300	0.068	5.74	0.080	4.10	0.092	2.50	0.098	1.46	0.110	0.56	0.121	2.47
400	0.062	6.98	0.074	5.39	0.084	3.84	0.090	2.83	0.102	0.86	0.111	1.00
500	0.57	8.11	0.068	6.57	0.078	5.06	0.084	4.07	0.095	2.16	0.104	0.33
600	0.054	9.15	0.064	7.66	0.073	6.19	0.079	5.23	0.089	3.36	0.097	1.57
700	0.051	10.15	0.060	8.70	0.069	7.27	0.074	6.33	0.084	4.51	0.092	2.77
800	0.048	11.12	0.057	9.71	0.065	8.32	0.070	7.41	0.079	5.63	0.087	3.92
900	0.045	12.07	0.054	10.70	0.062	9.35	0.067	8.47	0.076	6.73	0.083	5.07
1000	0.043	13.01	0.052	11.69	0.059	10.38	0.064	9.52	0.072	7.83	0.079	6.21

REFERENCES

1. Gelmaster W., Georgi H., Machachek M. Phys.Rev., 1978, D17, p.879; Gelmaster W., Henyey F. Phys.Rev., 1978, D18, p.1688; Richardson J.L. Phys.Lett., 1979, 82B, p.272; Rafelski J., Viollier R.D. CERN TH.2673, Geneva, 1979.
2. Savrin V.I., Skachkov N.B. CERN TH. 2822, Geneva, 1980; Lett.Nuovo Cim., 1980, 29, p.363.
3. Kadyshevsky V.G., Mir-Kasimov R.M., Skachkov N.B. Nuovo Cim., 1968, 55A, p.233.
4. Gross D.J., Wilczek F. Phys.Rev.Lett., 1973, 30, p.1343; Politzer H.D. Phys.Rev.Lett., 1973, 30, p.1346.
5. Jones D.R.T. Nucl.Phys., 1974, B75, p.531; Caswell W.E. Phys.Rev.Lett., 1974, 33, p.244.
6. Tarasov O.V., Vladimirov A.A. JINR, E2-80-483, Dubna, 1980; Tarasov O.V., Vladimirov A.A., Zharkov A.Yu. Phys. Lett., 1980, 93B, p.429.
7. Shirkov D.V. JINR, E2-80-609, Dubna, 1980.
8. Georgi H., Politzer H.D. Phys.Rev., 1976, D14, p.1829.
9. Shirkov D.V. JINR, E2-81-80, Dubna, 1981.
10. Bollini D. et al. Phys.Lett., 1981, 104B, p.403.
11. Fadeev N.G. et al. JINR, E1-81-585, Dubna, 1981.
12. Bace M. Phys.Lett., 1978, 78B, p.132; Bardeen W.A. et al. Phys.Rev., 1978, D18, p.3998; Buras A.J. Rev.Mod.Phys., 1980, 52, p.199.
13. Silin I. JINR, D-810, Dubna, 1961; CERN, P810, Geneva.
14. Logunov A.A., Tavkhelidze A.N. Nuovo Cim., 1963, 29, p.380.
15. Linkevich A.D., Savrin V.I., Skachkov N.B. JINR, E2-81-611, Dubna, 1981.
16. Kapshay V.N., Skachkov N.B. JINR, E2-12919, Dubna, 1979.
17. Skachkov N.B., Solovtsov I.L. Sov.J.Nucl.Phys., 1979, 30, p.1079.
18. Shapiro I.S. Sov.J.Dokl.Akad,Nauk SSSR, 1956, 106, p.642.
19. Pennington M.R., Ross G.G. Phys.Lett., 1981, 102B, p.167; Radyushkin A.V. JINR, E2-81-123, Dubna, 1981.
20. Smorodinsky Ya.A. Lecture in Collection "Voprosy Fiziki Elementarhych Chastits"(Problems of Elementary Particles Physics), Pub.Armenian Acad. of Science, Yerevan, 1963, vol.3, p.242.

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