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# RENORMALIZATION AND SHORT-DISTANCE PROPERTIES OF GAUGE INVARIANT GLUONIUM AND HADRON OPERATORS

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## 1. INTRODUCTION

Composite operators have always been the main tool for field theoretic description of bound states. In the framework of Bethe-Salpeter amplitudes these operators are used as non-local objects. In case of Quantum Chromodynamics gauge invariance poses special problems. The description of colour singlet bound states should be performed by means of gauge invariant composite operators. Especially if one wants to exploit short distance properties of composite operators one has to restrict consideration to gauge invariant ones, otherwise short distance properties depending on gauge fixing parameters would lack physical relevance.

It is well-known that gauge invariant composite operators have to be found with the help of phase factors [1]. Renormalization and operator product expansion of meson operator

$$M(x_1,x_2) = \bar{\psi}_\alpha(x_2)U_{\alpha\beta}(x_2,x_1)\psi_\beta(x_1), \qquad (1.1)$$
$$U(x_2,x_1) = P\exp ig\int_{x_1}^{x_2} dx_\mu A_\mu(x)$$

has already been studied [2]. For baryons the operator

$$B(x_1,x_2,x_3) = \psi_{\alpha_1}(x_1)\psi_{\alpha_2}(x_2)\psi_{\alpha_3}(x_3)\times \qquad (1.2)$$
$$\times U_{\beta_1\alpha_1}(x_0,x_1)U_{\beta_2\alpha_2}(x_0,x_2)U_{\beta_3\alpha_3}(x_0,x_3)\epsilon_{\beta_1\beta_2\beta_3}$$

has been proposed [3].

It seems worthwile to extend considerations to gauge invariant gluonium operators, too, which could be useful for the study of colour singlet states from gluons. In analogy to (1.1) one may define

$$G^{adj}_{\mu\nu\rho\sigma}(x_1,x_2) = F^a_{\mu\nu}(x_2)U_{ab}(x_2,x_1)F^b_{\rho\sigma}(x_1) \qquad (1.3)$$

with

$$U(x_2,x_1) = P\exp ig\int_{x_1}^{x_2} dx_\mu \tilde{A}_\mu(x) \qquad (1.4)$$
$$(\tilde{A}_\mu)_{ac} = A^b_\mu(x)(i f_{abc}).$$

Here in contrast to (1.1) the phase factor appears in the adjoint representation. The contour joining the points $x_1$ and $x_2$ should be smooth and without double points in order to avoid specific ultra violet divergences connected with such contours [4,5].

There is another possibility of building gluonium operators, which uses a closed contour passing through $x_1$ and $x_2$ together with phase factors in the fundamental representation:

$$G^{fund}_{\mu\nu\rho\sigma}(x_1,x_2)=\mathrm{tr}\{F_{\mu\nu}(x_2)U(x_2,x_1)F_{\rho\sigma}(x_1)U(x_1,x_2)\}. \qquad (1.5)$$

Such a construction is known from investigations of Wilson loop functional equations [4,6,7,8]. The basic tool used for the study of all of such gauge invariant operators is the formalism of auxiliary fields [6] which will be reviewed shortly in section 2. It allows a reduction of the nonlocal objects to products of local composite ones. As a first example we calculate Z factors and anomalous dimensions of the composite operators needed for the construction of (1.3). Then in section 3 renormalization and short distance behaviour of $G^{adj}$ itself is obtained. It turns out that both operators $G^{adj}$ and $G^{fund}$ show the same short-distance behaviour. In this sense the group theoretic representation has no influence on physical properties. In section 4 we discuss alternative constructions of hadronic operators. Again for the meson-operator (4.2) renormalization and short-distance properties are not changed. On the other hand, the new expression allows one to determine the renormalization properties of baryon operators (compare eq. (4.4)) in a convenient manner. In the last section we discuss modified hadron and gluonium operators with trivial Z factors, i.e., $Z = 1+O(g^4)$, which eventually could be useful in nonperturbative calculations.

## 2. GLUONIUM OPERATORS IN THE ADJOINT REPRESENTATION

In considerations of operators with phase factors the formalism of auxiliary field [6] has proved to be useful. Usually one introduces an anticommuting field z defined on the contour $x=x(\eta)$, $0\leq\eta\leq 1$, which transforms according to the fundamental representation of gauge group SU(N). This allows one to represent phase factors $U(x_1,x_2)$ from the fundamental representation by means of

$$U_{\alpha\beta}(x_2,x_1)=z_\alpha(\eta_2)\bar{z}_\beta(\eta_1), \qquad (2.1)$$

where $x_1=x(\eta_1)$, $x_2=x(\eta_2)$. The effective action for the enlarged system (QCD+auxiliary field) reads

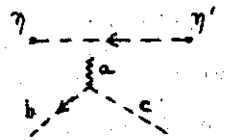
$$S = S^{\mathrm{eff}}_{QCD} + \int_0^1 d\eta\,(i\bar{z}\partial_\eta z + g\bar{z}A_\mu \dot{x}_\mu z), \tag{2.2}$$

with $A_\mu = A^a_\mu t^a$ ($t^a$ generators of SU(N) in fundamental representation). In order to get phase factors in the adjoint representation, we need auxiliary fields $z, \bar{z}$ transforming with respect to the adjoint representation so that the action takes the form

$$S = S^{\mathrm{eff}}_{QCD} + \int_0^1 d\eta\,\{i\bar{z}\partial_\eta z + ig\bar{z}^a A^b_\mu \dot{x}_\mu z^c f_{abc}\}. \tag{2.3}$$

The corresponding Feynman rules are

z propagator $\overbrace{z^a(\eta)\bar{z}^b}(\eta') = \theta(\eta-\eta')\delta^{ab}$




$\bar{z}^b z^c A^a$ vertex $-g\dot{x}_\mu f_{bac}$

We should emphasize that in spite of the modification of action (2.2) or (2.3) this theory does not really differ from QCD. There are no contributions of the auxiliary field neither to the usual Green functions of QCD nor to the $\beta$ function or the anomalous dimensions of quark, gluon or ghost fields. The reason is that because of the structure of the z propagator all closed z loop diagrams vanish. To avoid unnecessary complications with renormalization [4,5] we restrict us to smooth contours without double points which are strictly time-like or space-like. In the treatment of light-like contours special renormalization problems appear [9].

In turning to the study of gluonium operators we start with their definition in the framework of our auxiliary field formalism

$$F^a_{\mu\nu}(x_2)\Big(P\exp ig\int_{x_1}^{x_2} dx_\mu \tilde{A}_\mu(x)\Big)_{ab} F^b_{\rho\sigma}(x_1) = (F^a_{\mu\nu} z^a)(\eta_2)(\bar{z}^b F^b_{\rho\sigma})(\eta_1). \tag{2.4}$$

This is a consequence of eq. (2.1) of course also valid for the adjoint representation. Thereby the study of the nonlocal gluonium operator is reduced to that of the product of two local composite operators $F_{\mu\nu}z$ and $\bar{z}F_{\rho\sigma}$. With respect to the short-distance properties to be studied it is sufficient to determine the renormalization properties (i.e., Z factor and anomalous dimension) of these operators. We apply the technique outlined in [6,8,2]: all the calculations are done in x-space and we use the gluon and massless quark propagators

$$D_{\mu\nu}(x) = \frac{g_{\mu\nu}}{4\pi^2}\,\frac{\Gamma(\frac{n}{2}-1)}{(x^2-i0)^{n/2-1}}, \qquad \text{(Feynman gauge)}$$

$$S_F(x) = \frac{i\Gamma(\frac{n}{2})}{2\pi^2}\,\frac{\gamma x}{(x^2-i0)^{n/2}}$$

for the calculation of Z factors in dimensional renormalization scheme. Whereas the calculations of Z factor and anomalous dimensions are done in Feynman gauge later on renormalization group equations are used in Landau gauge. However general experience tells that anomalous dimensions of gauge invariant operators should not depend on gauge parameters used. An important tool are the following formulas

$$\int d^n z\, D^2(x-z) f(z) = \frac{i f(x)}{8\pi^2(n-4)} + \text{reg. terms},$$

$$\int d^n z\, D(x-z) f(z) \partial_\mu D(x-z) = \frac{i\partial_\mu f(x)}{16\pi^2(n-4)} + \text{r.t.},$$

$$\int d^n z\, D(x-z) f(z) D(y-z) = \tag{2.5}$$

$$= -i\Gamma(n/2-2)(16\pi^2)^{-1}\,[(x-y)^2]^{2-n/2}\{f(x) + \frac{1}{2}(y-x)_\mu \partial_\mu f(x) + \dots\}$$

(compare Appendix of ref. [8] ). At first one has to determine the Z-factor of z field:

$$-g^2 i f_{abc}\; i f_{cbd} \int_0^1 d\eta_1 \int_0^1 d\eta_2\, \bar z^a(\eta_1) z^d(\eta_2) \theta(\eta_1-\eta_2) D_{\mu\nu}(1-2)\dot x_\mu(\eta_1)\dot x_\nu(\eta_2)$$

$$= -\frac{C_A g^2}{4\pi^2}\,\frac{1}{4-n}\int_0^1 d\eta_1\, \bar z^a(\eta_1)\partial_\eta z^a(\eta_1) + \dots$$

which results in

$$Z_{3z} = 1 + \frac{C_A g^2}{4\pi^2}\,\frac{1}{4-n}. \tag{2.6}$$

For the composite operator $F_{\rho\sigma} z$ we have to calculate the following diagrams (the other operator $\bar z F_{\mu\nu}$, of course, yields identical results)

$$\frac{g^2}{4\pi^2} C_A \frac{(-1)}{4-n}\{z^a (A^a \dot x)(\ddot x_\rho \dot x_\sigma - \ddot x_\sigma \dot x_\rho)\} + \text{reg.}$$

$$\frac{g^2}{4\pi^2} C_A \frac{1}{4-n} \{ \partial_\eta z^a (A^a_\sigma \dot{x}_\rho - A^a_\rho \dot{x}_\sigma) + z^a (A^a_\sigma \ddot{x}_\rho - A^a_\rho \ddot{x}_\sigma) \} + \text{reg}.$$

$$\frac{g^2}{4\pi^2} C_A \frac{(-1)}{4-n} \frac{3}{4} z^a F^a_{\rho\sigma} + \text{reg}.$$

$$\frac{g^2}{4\pi^2} C_A \frac{1}{4-n} \{ \frac{1}{4} z^a F^a_{\rho\sigma} + z^a (A^a_\rho \ddot{x}_\sigma - A^a_\sigma \ddot{x}_\rho) + \frac{1}{2} z^a (F^a_{\rho\alpha} \dot{x}_\alpha \dot{x}_\sigma - F^a_{\sigma\alpha} \dot{x}_\alpha \dot{x}_\rho) + z^a (A^a \dot{x})(\ddot{x}_\rho \dot{x}_\sigma - \ddot{x}_\sigma \dot{x}_\rho) + \frac{1}{2} \partial_\eta z^a (A^a_\rho \dot{x}_\sigma - A^a_\sigma \dot{x}_\rho) \} + \text{reg}.$$

(The parametrization of the contour is normalized so that $\dot{x}^2 \equiv 1$). As usual counter terms of new structure are demanded. The following operators are allowed by the BRS transformation (compare ref./10,11/)

$$\Omega^1_{\rho\sigma} = F^a_{\rho\sigma} z^a$$
$$\Omega^2_{\rho\sigma} = \dot{x}_\alpha (F^a_{\rho\alpha} \dot{x}_\sigma - F^a_{\sigma\alpha} \dot{x}_\rho) z^a \tag{2.7}$$
$$\Omega^3_{\rho\sigma} = (\dot{x}_\rho A^a_\sigma - \dot{x}_\sigma A^a_\rho)(Dz)^a$$
$$(Dz)^a = \partial\eta\, z^a + g f_{abc} A^b z^c .$$

Collecting our results we get

$$<\Omega^1 A \bar{z}> = \frac{g^2}{4\pi^2} C_A \frac{1}{4-n} \times <\{ -\frac{1}{2}\Omega^1 + \frac{1}{2}\Omega^2 + \frac{1}{2}\Omega^3 \} A\bar{z}>^{(g^0)} + \text{reg.} + O(g^4). \tag{2.8}$$

As usual the radiative corrections of mixing partner are also to be determined.

$$<\Omega^2 A \bar{z}> = \frac{g^2}{4\pi^2} C_A \frac{1}{4-n} \frac{1}{2} <\Omega^3 A \bar{z}>^{(g^0)} + \text{reg.} + O(g^4). \tag{2.9}$$

This could be obtained immediately from (2.8) by taking into account definition (2.7).

For $\Omega^3_{\rho\sigma}$ one obtains

$$\frac{g^2}{4\pi^2} C_A \frac{1}{4-n} \frac{3}{4} z^a \{ (\dot{x}\partial)(A^a_\sigma \dot{x}_\rho - A^a_\rho \dot{x}_\sigma) + (\dot{x}_\sigma \partial_\rho - \dot{x}_\rho \partial_\sigma)(A^a \dot{x}) \} + \text{reg.}$$

$$\frac{g^2}{4\pi^2} C_A \frac{1}{4-n} \{ \partial_\eta z^a (A^a_\sigma \dot{x}_\rho - A^a_\rho \dot{x}_\sigma) + z^a (A^a \dot{x})(\ddot{x}_\sigma \dot{x}_\rho - \ddot{x}_\rho \dot{x}_\sigma) \} + \text{reg.}$$

$$\frac{g^2}{4\pi^2} C_A \frac{1}{4-n} z^a (A^a \dot{x})(\ddot{x}_\rho \dot{x}_\sigma - \ddot{x}_\sigma \dot{x}_\rho) + \text{reg.}$$

$$\frac{g^2}{4\pi^2} C_A \frac{1}{4-n} \frac{3z^a}{4} \{ (\dot{x}_\rho \partial_\sigma - \dot{x}_\sigma \partial_\rho)(A^a \dot{x}) + (\dot{x}\partial)(A^a_\rho \dot{x}_\sigma - A^a_\sigma \dot{x}_\rho) \} + \text{reg.}$$

so that

$$\langle \Omega^3 A \bar{z} \rangle = \frac{g^2}{4\pi^2} C_A \frac{(-1)}{4-n} \langle \Omega^3 A \bar{z} \rangle^{(g^0)} + \text{reg.} + O(g^4). \qquad (2.10)$$

Comparing eqs. (2.8), (2.9), (2.10) with the definition of the $Z$ matrix for operator renormalization

$$\langle \Omega^i A \bar{z} \rangle = Z_{ij} Z_3^{-1/2} Z_{3z}^{-1/2} \langle \Omega^j A \bar{z} \rangle_{\text{reg.}} \qquad (2.11)$$

and using (2.6) and

$$Z_3 = 1 + \frac{g^2}{4\pi^2} \frac{1}{4-n} \{ \frac{10}{12} C_A - \frac{n_f}{3} \}$$

( $n_f$ =number of quark flavours) we finally get

$$Z = 1 + \begin{pmatrix} \frac{5}{12} C_A - \frac{1}{6} n_f & 1/2\, C_A & 1/2\, C_A \\ 0 & \frac{11}{12} C_A - \frac{n_f}{6} & 1/2\, C_A \\ 0 & 0 & \frac{1}{12} - \frac{n_f}{6} \end{pmatrix} \frac{g^2}{4\pi^2} \frac{1}{4-n} . \qquad (2.12)$$

From this the matrix of anomalous dimensions can be extracted according to $Z = 1 - \frac{\gamma}{4-n} + O(g^4)$.

$$\gamma^{adj} = -\frac{g^2}{4\pi^2}\begin{pmatrix} \frac{5C_A}{12}-\frac{n_f}{6} & 1/2C_A & 1/2C_A \\ 0 & \frac{11C_A}{12}-\frac{n_f}{6} & 1/2C_A \\ 0 & 0 & -\frac{C_A}{12}-\frac{n_f}{6} \end{pmatrix} \tag{2.13}$$

It has the following eigenvectors and eigenvalues

$$\tilde{\Omega}_1 = \Omega^1 - \Omega^3 \quad \to \gamma_1 = \frac{g^2}{4\pi^2}\left(-\frac{5C_1}{12}+\frac{n_f}{6}\right)$$

$$\tilde{\Omega}_2 = \Omega^2 + \frac{1}{2}\Omega^3 \quad \to \gamma_2 = \frac{g^2}{4\pi^2}\left(-\frac{11}{12}C_A+\frac{n_f}{6}\right)$$

$$\tilde{\Omega}_3 = \Omega^3 \quad \to \gamma_3 = \frac{g^2}{4\pi^2}\left(\frac{1}{12}C_A+\frac{n_f}{6}\right).$$

The physical consequences are discussed in the next section.

## 3. SHORT-DISTANCE BEHAVIOUR OF GLUONIUM OPERATORS

Turning back to our original problem the short distance behaviour of gluonium operators $G^{adj}$ can be determined in the following manner

$$G^{adj}_{\mu\nu\rho\sigma}(x_1,x_2) = F^a_{\mu\nu}(x_2)\left(P \exp ig \int_{x_1}^{x_2} dx_\mu \tilde{A}_\mu\right)_{ab} F^b_{\rho\sigma}(x_1)$$

$$= \Omega^1_{\mu\nu}(\eta_2)\Omega^1_{\rho\sigma}(\eta_1)$$

$$= (\tilde{\Omega}_1+\tilde{\Omega}_2-\frac{1}{2}\tilde{\Omega}_3)_{\mu\nu}(\eta_2)(\tilde{\Omega}_1+\tilde{\Omega}_2-\frac{1}{2}\tilde{\Omega}_3)_{\rho\sigma}(\eta_1).$$

Here $\Omega^1$ has been expressed in eigenvector combinations with respect to renormalization, so that we can take advantage of the anomalous dimensions (2.13). Furthermore noticing that $\tilde{\Omega}_3$ is a second class operator contributing contact terms to Green's functions only, it will be neglected in the following.

$$G^{adj}_{\mu\nu\rho\sigma}(x_1,x_2) \,\hat{=}\, \tilde{\Omega}^1_{\mu\nu}(\eta_2)\tilde{\Omega}^1_{\rho\sigma}(\eta_1) + \tilde{\Omega}^1\tilde{\Omega}^2 + \tilde{\Omega}^2\tilde{\Omega}^1 + \tilde{\Omega}^2\tilde{\Omega}^2 . \tag{3.1}$$

The operator products $\tilde{\Omega}_i\tilde{\Omega}_j$ can now be represented by standard methods in form of an operator product expansion

$$\tilde{\Omega}_i(\eta_2)\tilde{\Omega}_j(\eta_1) = \sum_n f_n^{ij}(x_1-x_2)O_n\left(\frac{x_1+x_2}{2}\right), \tag{3.2}$$

where $O_n(x)$ are local operators, among which also the operators occur known from the operator product expansion of currents in the case of forward scattering [12]. The asymptotic behaviour of coefficient functions $f_n^{ij}(x_1-x_2)$ is obtained by means of standard methods [14] (supposed that operators $O_n$ have already been diagonalized and $\gamma_n = g^2/4\pi^2 c_n$ are their anomalous dimension [13]):

$$f_n^{ij}\left(\frac{x_1-x_2}{\lambda}\right) \underset{(\lambda\to\infty)}{=} \lambda^{4-d_n}\left[\frac{g^2}{\bar{g}^2(\lambda)}\right]^{\frac{a_i+a_j-c_n}{2b}} f_n^{ij}\left(x_1-x_2, \bar{g}(\lambda), \frac{m}{\lambda}\right), \tag{3.3}$$

where $b = \frac{11C_A}{12} - \frac{n_f}{6}$ is the coefficient which appears in the $\beta$ function, $\bar{g}^2(\lambda) = g^2\left[1 + \frac{g^2 b}{4\pi^2}\log\lambda^2\right]^{-1}$ and $a_i, a_j$ are defined by

$$\begin{aligned} a_1 &= \frac{-5C_A}{12} - \frac{n_f}{6}, \quad a_2 = -\frac{11C_A}{12} + \frac{n_f}{6} \\ a_3 &= 1/12\,C_A + n_f/6 . \end{aligned} \tag{3.4}$$

Collecting (3.1), (3.2), (3.3) the operator product expansion of $G^{adj}_{\mu\nu\rho\sigma}$ is determined. In the following we will discuss another definition of gauge invariant gluonium operator suggested by the consideration of Wilson functional equations [4,6,7,8,11,15]. Taking a closed smooth contour we define in the fundamental representation

$$\begin{aligned} g^2 G^{fund}_{\mu\nu\rho\sigma} &= \mathrm{tr}\{g F_{\mu\nu}(x_2) U(x_2, x_1) g F_{\rho\sigma}(x_1) U(x_1, x_2)\} = \\ &= z_\alpha(1)(g\bar{z}F_{\mu\nu}z)(\eta_2)(g\bar{z}F_{\rho\sigma}z)(\eta_1)\bar{z}_\alpha(0). \end{aligned} \tag{3.5}$$

Here the usual auxiliary field transforming according to the fundamental representation appears. Let us review the results of [8,11]. The set of operators mixing under renormalization is

$$\begin{aligned} \omega^1_{\mu\nu} &= g\bar{z}F_{\mu\nu}z \\ \omega^2_{\mu\nu} &= g\bar{z}(\dot{x}_\nu F_{\mu\alpha} - \dot{x}_\mu F_{\nu\alpha})z\dot{x}_\alpha \\ \omega^3_{\mu\nu} &= g\{\bar{z}(\dot{x}_\mu A_\nu - \dot{x}_\nu A_\mu)Dz - \bar{z}\overleftarrow{D}(\dot{x}_\mu A_\nu - \dot{x}_\nu A_\mu)z\} \\ \omega^4_{\mu\nu} &= i(\dot{x}_\mu\ddot{x}_\nu - \dot{x}_\nu\ddot{x}_\mu)(\bar{z}Dz + \bar{z}\overleftarrow{D}z) \end{aligned} \tag{3.6}$$

with the matrix of anomalous dimensions

$$\gamma^{fund} = \begin{pmatrix} 1/2C_A & -1/2C_A & -C_A/4 & C_F \\ 0 & 0 & -C_A/4 & C_F \\ 0 & 0 & C_A/4 & -C_F \\ 0 & 0 & 0 & 0 \end{pmatrix} \frac{g^2}{4\pi^2} \qquad (3.7)$$

and $Z^{fund} = 1 - \frac{\gamma^{fund}}{4-n} + O(g^4)$.

Eigenvectors and eigenvalues are

$$\bar\omega^1 = \omega^1 - \omega^2 \quad \to \quad \gamma^1 = \frac{g^2}{4\pi^2}\frac{1}{2}C_A$$

$$\bar\omega^2 = \omega^2 + \omega^3 \quad \to \quad \gamma^2 = 0$$

$$\bar\omega^3 = -\omega^3 + \frac{4C_F}{C_A}\omega^4 \to \gamma^3 = \frac{g^2}{4\pi^2}\frac{1}{4}C_A$$

$$\bar\omega^4 = \omega^4 \quad \to \quad \gamma^4 = 0.$$

There are two second class operators, namely $\omega^3$ and $\omega^4$ which can be neglected in the short distance analysis. In analogy to eqs. (3.1)-(3.4) one obtains the leading short distance behaviour (restricting to $O_n = 1$ with $d_n = c_n = 0$)

$$\begin{aligned} g^2 G^{fund}\left(\frac{x_1}{\lambda},\frac{x_2}{\lambda}\right) \underset{(\lambda\to\infty)}{=} & \lambda^4 \left[\frac{g^2}{\bar g^2(\lambda)}\right]^{C_A/2b} \langle z(1)\bar\omega^1(\eta_2)\bar\omega^1(\eta_1)\bar z(0)\rangle \Big|_{\bar g(\lambda)} \\ & + \lambda^4 [\ldots]^{C_A/4b} \langle z(1)(\bar\omega^1\bar\omega^2 + \bar\omega^2\bar\omega^1)\bar z(0)\rangle \Big|_{\bar g(\lambda)} \\ & + \lambda^4 \langle z(1)\bar\omega^2(\eta_2)\,\bar\omega^2(\eta_2) z(0)\rangle \Big|_{\bar g(\lambda)} . \end{aligned} \qquad (3.8)$$

Because of the explicit coefficient g appearing in the operators (3.6) there is an additional factor $(\log\lambda)^{-1}$ originating from the matrix elements. To compare this with our earlier results we have to multiply the operators $\Omega^i$ (eq.(2.7)) by g thereby changing the Z and $\gamma$ matrices

$$Z_{adj} \to Z_g Z = \left(1 - \frac{b}{4\pi^2} g^2 \frac{1}{4-n}\right) Z$$

and

$$\gamma^{adj} \to \frac{g^2}{4\pi^2}\begin{pmatrix} 1/2C_A & -1/2C_A & -1/2C_A \\ 0 & 0 & -1/2C_A \\ 0 & 0 & C_A \end{pmatrix}$$

with eigenvalues $1/2 C_A \frac{g^2}{4\pi^2}$, $C_A \frac{g^2}{4\pi^2}$. Insertion into the analogue of eq.(3.3) and restricting to the first term $n=0$ yields a result identical to (3.8).

We therefore conclude that both constructions of gauge invariant gluonium operators are equaivalent-with respect to the short distance properties. It is an amusing fact that the identity

$$U^{adj}_{ab}(C) = \mathrm{tr}\{t^a U(c) t^b U^{-1}(C)\}^{\mathrm{fund}}$$

or

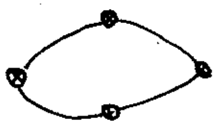
$$z^a(\eta_2)\bar{z}^b(\eta_1)_{adj} = z(1)(\bar{z}t^a z)(\eta_2)(\bar{z}t^a z)(\eta_1)\bar{z}(0)_{\mathrm{fund}}$$

valid for classical fields has some reflection in terms of Z factors. The l.h.s. has the Z-factor $Z_{adj} = 1 + \frac{g^2}{4\pi^2} C_A \frac{1}{4-n}$ (see eq. (2.6)) whereas for the r.h.s. one obtaines easily

$$Z^2_{(\bar{z}t\,z)} Z_{z(1)\bar{z}(0)} = (1 + \frac{g^2}{8\pi^2} C_A \frac{1}{4-n})^2\, 1 = Z_{adj}$$

(up to order $g^2$). Of course the contour used in the fundamental representation is degenerated. However, as is well known /4/, cusps of angle 180° do not produce singularities and moreover they are without influence on anomalous dimensions.

A final remark concerns more complex gauge invariant operators which can be constructed most conveniently with the help of phase factors in the fundamental representation: namely by insertion of a number of field-strength tensor into the Wilson loop. In the framework of auxiliary fields this is equivalent to insertions of an appropriate number of operators $g\bar{z}F_{\mu\nu}z$ in analogy to eq. (3.5). Of course the previously calculated anomalous dimensions determine the short-distance properties of these objects, too.



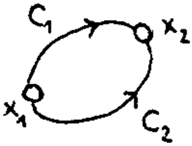
## 4. ALTERNATIVE CONSTRUCTIONS OF MESON AND BARYON OPERATORS

The two possibilities discussed in the last section pose the question, whether there are different constructions of gauge invariant meson or baryon operators, too. The most popular construction

$$M(x_1,x_2) = \bar{\psi}_\alpha(x_2)\Gamma U_{\alpha\beta}(x_2,x_1)\psi_\beta(x_1), \quad \Gamma = 1, \gamma_5, \gamma_\mu, \dots \tag{4.1}$$

$$= (\bar{\psi}_\alpha z_\alpha)(\eta_2)\Gamma(\bar{z}_\beta \psi_\beta)(\eta_1)$$

has already been analysed[2]. In analogy to the gluonium operators it is possible to define a meson operator which relies on a closed contour passing through $x_1$ and $x_2$. In case of colour group SU(3) we build




$$M'(x_1,x_2)=\bar{\psi}_{\alpha_1}(x_1)U_{\beta_2\beta_1}(x_2,x_1,C_1)\epsilon^{\alpha_1\beta_1\gamma_1}\times$$
$$\times\Gamma U_{\gamma_2\gamma_1}(x_2,x_1,C_2)\psi_{\alpha_2}(x_2)\epsilon^{\gamma_2\beta_2\alpha_2} \tag{4.2}$$

In order to rewrite this with the help of auxiliary fields, one now has to introduce two such fields $z^{(1)}$ and $z^{(2)}$ defined on the contours $C^1$ and $C^2$, respectively. Each field contributes $\int d\eta_i\, \bar{z}^{(i)} D z^{(i)}(\eta_i)$ to the action; there are no contractions between $z^{(1)}$ and $z^{(2)}$. Note that in contrast to the construction of the gluonium operators the contour now consists of two parts, which both are directed from $x_1$ to $x_2$. Then eq. (4.2) can be rewritten as

$$M'(x_1,x_2)=(\bar{\psi}_{\alpha_1}\bar{z}^{(1)}_{\beta_1}\bar{z}^{(2)}_{\gamma_1})(1)\epsilon^{\alpha_1\beta_1\gamma_1}\Gamma(z^{(2)}_{\gamma_2}z^{(1)}_{\beta_2}\psi_{\alpha_2})(2)\epsilon^{\gamma_2\beta_2\alpha_2}. \tag{4.3}$$

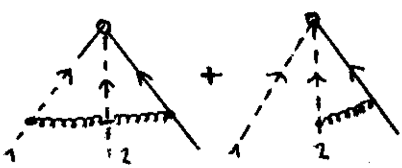
The new composite operator $\bar{\psi}_\alpha\bar{z}^{(1)}_\beta\bar{z}^{(2)}_\gamma\epsilon^{\alpha\beta\gamma}$ is group theoretically equivalent to $\bar{\psi}_\alpha z_\alpha$ used in eq. (4.1). Both operators together can be applied to form a gauge invariant baryon operator without introducing a fourth auxiliary point $x_0$ in eq.(1.2)

$$B'(x_1,x_2,x_3)=\psi_{\alpha_1}(1)U_{\beta_1\alpha_1}(3,1)\psi_{\alpha_2}(2)U_{\beta_2\alpha_2}(3,2)\psi_{\alpha_3}(3)\epsilon^{\beta_2\beta_1\alpha_3}$$
$$=(\psi_{\alpha_1}\bar{z}^{(1)}_{\alpha_1})(1)(\psi_{\alpha_2}\bar{z}^{(2)}_{\alpha_2})(2)(z^{(2)}_{\beta_2}z^{(1)}_{\beta_1}\psi_{\alpha_3}\epsilon^{\beta_2\beta_1\alpha_3})(3). \tag{4.4}$$

In order to obtain renormalization and short-distance properties of both M' and B' one has to determine the Z factors of $\psi_\alpha\bar{z}^{(i)}_\alpha$ and $z^{(2)}_\alpha z^{(1)}_\beta\psi_\gamma\epsilon^{\alpha\beta\gamma}$. From[2] one gets

$$Z_{\psi_\alpha\bar{z}_\alpha}=1+\frac{3C_F}{4}\frac{g^2}{4\pi^2}\frac{1}{4-n}$$
$$\gamma_{\psi_\alpha\bar{z}_\alpha}=-\frac{3C_F}{4}\frac{g^2}{4\pi^2}. \tag{4.5}$$

For the other operators the following diagrams have to be calculated




$$\frac{g^2}{4\pi^2}\cdot\frac{2}{3}\cdot\frac{1}{4-n}$$

$$-\frac{g^2}{4\pi^2}\,\frac{2}{3}\,\frac{1}{4-n}$$

Taking into account the Z factors of the fields $\psi$ and $z^{(i)}$ (compare eq. (2.11))

$$Z_{3z}^{\text{fund}} = 1 + C_F\,\frac{g^2}{4\pi^2}\,\frac{1}{4-n} \qquad \text{and} \qquad Z_2 = 1 - \frac{C_F}{2}\,\frac{g^2}{4\pi^2}\,\frac{1}{4-n}$$

this leads to

$$Z_{(zz\psi)} = 1 + \frac{g^2}{4\pi^2}\,\frac{1}{4-n}, \qquad \gamma_{(zz\psi)} = -\frac{g^2}{4\pi^2}. \tag{4.6}$$

Taking into account $C_F\,(N=3) = 4/3$, we obtain that Z factors and anomalous dimensions of both operators $\psi_\alpha \bar z^{(i)}_\alpha$ and $z^{(2)}_\alpha z^{(1)}_\beta \psi_\gamma \epsilon^{\alpha\beta\gamma}$ coincide. This has the consequence that renormalization and short distance properties of M and M′ coincide, too. It is now straight forward to discuss the baryon operator $B'(x_1,x_2,x_3)$. Its Z factor looks like

$$Z_{B'} = (Z_{\psi\bar z})^2\, Z_{(zz\psi)} = 1 + 3\,\frac{g^2}{4\pi^2}\,\frac{1}{4-n},$$

$$\gamma_{B'} = -3\,\frac{g^2}{4\pi^2} = C_{B'}\,\frac{g^2}{4\pi^2}. \tag{4.7}$$

Here a remark concerning the Dirac matrices $\Gamma$ is in order. As is well known, in case of local composite operators the $\gamma$ structure is of essential influence on renormalization properties (compare $\bar\psi\psi$ and $\bar\psi\gamma_\mu\psi$). The nonlocal operators M′ are reduced by the auxiliary field formalism to products of local operators at different points in between $\Gamma$ can be inserted without changing the ultra violet divergencies and Z factors.

If it is necessary to study the short distance behaviour of B′ one could apply the operator product expansion

$$B'(x_1,x_2,x_3) = \sum_n f_n(x_1,x_2,x_3)\,O_n\!\left(\frac{x_1+x_2+x_3}{3}\right), \tag{4.8}$$

where the coefficients behave as

$$f_n\!\left(\frac{x_1}{\lambda},\frac{x_2}{\lambda},\frac{x_3}{\lambda}\right) \underset{(\lambda\to\infty)}{=} \lambda^{9/2-d_n}\left[\frac{g^2}{\bar g^2(\lambda)}\right]^{\frac{c_{B'}-c_n}{2b}} f_n\!\left(x_1,x_2,x_3;\bar g(\lambda),\frac{\bar m(\lambda)}{\lambda}\right) \tag{4.9}$$

asymptotically.

## 5. HADRON OPERATORS WITHOUT Z FACTORS

The results of the foregoing sections can now be exploited to look for modified hadron operators showing a simpler behaviour with respect to renormalization. Returning to the basic composite operators $\bar{z}_\alpha \psi_\alpha$ or $z^{(2)}_\alpha z^{(1)}_\beta \psi_\gamma \epsilon^{\alpha\beta\gamma}$ with the identical Z factor $Z = 1 + 3g^2 C_F/(16\pi^2(4-n))$ (eq. 4.6) and comparing this with the well-known factor of multiplicative mass renormalization [14] (m denotes a quark mass)

$$Z_m = 1 - \frac{3g^2}{8\pi^2} C_F \frac{1}{4-n} \tag{5.1}$$

(which is also independent of the gauge parameter) we observe that the operators $\sqrt{m}\,\bar{z}\psi$ or $\sqrt{m}\, z^{(2)}_\alpha z^{(1)}_\beta \psi_\sigma \epsilon^{\alpha\beta\gamma}$ have trivial renormalization factors $Z = 1 + O(g^4)$. Therefore turning back to the hadron operators of eqs. (4.1), (4.3), (4.4) we conclude that the following modified meson and baryon operators

$$\begin{gathered} m\,\bar{\psi}(x_1)U(x_1,x_2)\Gamma\psi(x_2), \quad \Gamma = 1, \gamma_5, \gamma_\mu, \dots \\ m^{3/2}\psi_{\alpha_1}(1)U_{\beta_1\alpha_1}(3,1)\psi_{\alpha_2}(2)U_{\beta_2\alpha_2}(3,2)\psi_{\alpha_3}(3)\epsilon^{\beta_2\beta_1\alpha_3} \end{gathered} \tag{5.2}$$

have a trivial Z factor. We would like to emphasize that this statement is obtained in the framework of dimensional renormalization (minimal subtractions) which at the same time is the single scheme to deal with phase factors at present. The renormalization properties of operators (eq. (5.2)) should be compared with that of similar local operators: $m(\bar{\psi}\psi)(x)$ has a trivial Z factor [16] but not $m(\bar{\psi}\gamma_\mu\psi)(x)$ (because $(\bar{\psi}\gamma_\mu\psi)(x)$ has $Z=1$).

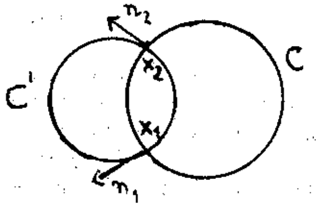
In the case of gluonium operators the problem is more involved. The operator $g^2 G^{\text{fund}}_{\mu\nu\rho\sigma}$ has nontrivial renormalization. Its basic composite operator $g\bar{z}F_{\mu\nu}z$ has, as a member of a set of mixing partners, a nontrivial Z matrix (compare eq. (3.7)). Surprisingly an essential simplification occurs if the field strength tensor is contracted with its tangent vector. If in addition the second class operators $\omega^3_{\mu\nu}$ and $\omega^4_{\mu\nu}$ (in eq. (3.6)) are neglected then because of $\omega^1_{\mu\nu}\dot{x}_\nu = \omega^2_{\mu\nu}\dot{x}_\nu$ one arrives at $(\omega^1_{\mu\nu}\dot{x}_\nu)^{\text{ren}} = (\omega^1_{\mu\nu}\dot{x}_\nu)^{\text{un}}$. Consequently the gluonium operator

$$g^2\dot{x}_\nu(2)\dot{x}_\sigma(1)G^{\text{fund}}_{\mu\nu\rho\sigma}(x_1,x_2) = \mathrm{tr}\{g(\dot{x}_\nu F_{\mu\nu})(2)U(2,1)g(\dot{x}_\sigma F_{\rho\sigma})(1)U(1,2)\} \tag{5.3}$$

has the trivial Z factor $Z=1$. The same of course is true for

$$g^2 \dot{x}_\nu(2) \dot{x}_\sigma(1) G^{adj}_{\mu\nu\rho\sigma}(x_1,x_2) = g(\dot{x}_\nu F^a_{\mu\nu})(2) U^{ab}(2,1) g(\dot{x} F_{\rho\sigma})(1). \qquad (5.4)$$

The result just obtained seems to be of a somewhat paradoxial character. Let us consider the gauge invariant quantities $g^2 G_{\mu\nu\rho\sigma}(x_1,x_2) n^1_\sigma n^2_\nu$ with arbitrary (normalized) vectors $n^1$ and $n^2$. The closed curve C connects the points $x_1$ and $x_2$. Astonishingly enough the renormalization of this operator depends drastically on the geometry of the curve C, e.g.,




C: $n^1, n^2$ are not tangent vectors: nontrivial renormalization

C': $n^1 = \dot{x}(x_1)$, $n^2 = \dot{x}(x_2)$: trivial renormalization.

Letting shrink both curves to one point $(x_\mu(\eta,\lambda) = \lambda x_\mu(\eta))$ one gets a different short distance behaviour of

$g^2 G_{\mu\nu\rho\sigma}(x_1,x_2;C) n^1_\sigma n^2_\nu$ and $g^2 G_{\mu\nu\rho\sigma}(x_1,x_2;C') n^1_\sigma n^2_\nu$. May be,

the distinct behaviour of the basic operators $g\bar{z} F_{\mu\nu} n_\nu z$ and $g\bar{z} F_{\mu\nu} \dot{x}_\nu z$ in both cases is of a common origin with the additional Z factors for Wilson functionals with corners/5/. Obviously the gluonium operators (5.3), (5.4) play a distinguished role having no avoidable Z factor and showing a definite shortdistance behaviour.

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