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DESCRIPTION OF THE MOMENTS OF NUCLEON STRUCTURE FUNCTION DATA AT $Q^2 \geq 30$ GeV^2 IN THE COMPOSITE MODEL

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Experiments on deep inelastic scattering of electrons and muon on nucleons carried out at "moderate" values of $Q^2 \leq 30 GeV^2$ showed that the prediction of Bjorken about the independence of structure functions on squared transfer momentum, scailing, is fulfilled only on the average. The 10-20% declination from observed at these values Q^2 was at first considered as an experimental confirmation of the prediction of asymptotically free non-Abelian gauge theory, Quantum Chromodynamics (QCD), related with a logarithmic decrease in the behaviour of the coupling constant as a function of Q^2 [1]. But lately the part of higher twist effects [2] has been realized. In the present paper it is shown that new data on the muon-carbon deep-inelastic scattering obtained by the BCDMS collaboration in the interval of Q^2=30-110 GeV^2 are well described by the quark wave functions in hadrons. We use as our experimental data the Nachtman momenta obtained by the BCDMS collaboration data on the structure functions for μ-N inclusive deep inelastic scattering.

The n-th moments of the second structure functions of hadrons $A (A = \pi, N, \dots)$

$$M_2^A(n, Q^2) = \int_0^1 dx\, x^{n-2} \nu W_2^A(x, Q^2) \tag{1}$$

in QCD are bound by formula [4]

$$\frac{M_N(n, Q_0^2)}{M_N(n, Q^2)} = \frac{M_\pi(n, Q_0^2)}{M_\pi(n, Q^2)}. \tag{2}$$

This formula, which has been used in paper [4] to formula, the behaviour of pion moments by that of nucleon moments is used by us for quite an opposite purpose.

For the calculation of the pion structure functions the single-time, two-particle wave functions [5] are used; the latter are the solutions of the quasipotential equation [6], with the interaction kernel as one-gluon exchange amplitude.

As is known, the hadron tensor entering the amplitude of deep-inelastic lepton-hadron scattering, is

$$W_{\mu\nu}(P, q) = \frac{1}{4\pi} \sum_N (2\pi)^4 \delta(P + q - P_N) \langle P | J_\mu^+(0) | N \rangle \langle N | J_\nu(0) | P \rangle ,$$

where the summation is over all final states, $J_\mu(0)$ being hadron current.

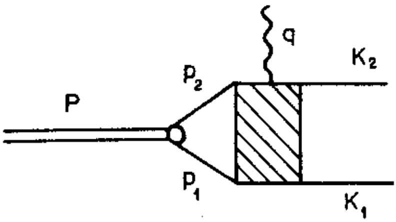

Fig. 1.

For the bound state of quark and antiquark, tensor in the impulse representation is

$$W_{\mu\nu}(P,q)=\frac{1}{2(2\pi)^3}\int\frac{d\vec{k}}{2k^0}\delta(\omega^2-2Pk_1-2qk_1)\times$$
$$\times\langle k_1k_2|J_\mu(0)|P\rangle^*\langle k_1k_2|J_\nu(0)|P\rangle \,, \quad (3)$$

where $\omega^2=(p+q)^2$. Graphically the matrix element $\langle k_1k_2|J_\mu(0)|P\rangle$ can be depicted as it is shown in fig. 1. By changing $k_1\to\Delta_k = L_p^{-1}k_1$ in the integrand, transform (3) to the following form /8/:

$$W'_{\mu\nu}(P,q')=\frac{1}{2(2\pi)^3}\int\frac{d\vec{\Delta}_k}{2\Delta_k^0}(\omega^2-2\omega\Delta_k^0-2q'\Delta_k)\times$$
$$\times\langle\Delta_k,\tilde{\Delta}_k|J_\mu(0)|M\rangle^*\langle\Delta_k\tilde{\Delta}_k|J_\nu(0)|M\rangle \,, \quad (4)$$

where $\tilde{\Delta}_k\equiv L_p^{-1}k_2$, $q'\equiv L_p^{-1}q$ and $W'_{\mu\nu}(M,q')=[L_p^{-1}W(P,q)]_{\mu\nu}$.
It is not difficult to see that the Lorentz-transforms give

$$q'^0=\frac{P\cdot q}{M}\equiv\nu \qquad \text{and} \qquad |\vec{q}|=\sqrt{(q'^0)^2-q^2}\,. \quad (5)$$

Following the paper (8), in the impulse approximation we have

$$\langle\Delta_k,\tilde{\Delta}_k|J_\mu(0)|M\rangle=\tilde{j}_\mu(\tilde{\Delta}_k,\tilde{\Delta}_p)\,\psi_{BM}(\vec{\Delta}_k)\,, \quad (6)$$

where $\tilde{j}_\mu(\tilde{\Delta}_k;\tilde{\Delta}_p)$ is antiquark current and $\tilde{\Delta}_p=(\Delta_k^0;-\vec{\Delta}_k)$.
Having combined formulas (4) and (6), one can write

$$W'_{\mu\nu}(M,q')=\frac{1}{(2\pi)^3}\int\frac{d\Delta_k}{2\Delta_k^0}\delta[\omega^2-2(\nu+M)\Delta_k^0+2\sqrt{\nu^2+Q^2}\,\Delta_k]\times$$
$$\times\tilde{j}^*_\mu(\tilde{\Delta}_k,\tilde{\Delta}_p)\,j_\nu(\tilde{\Delta}_k,\tilde{\Delta}_p)|\psi_{BM}(\vec{\Delta}_k)|^2\,. \quad (7)$$

Considering quarks in a pion as scalars, we put in expression (7) that

$$\tilde{j}_\nu(\tilde{\Delta}_k,\tilde{\Delta}_p)=-(\tilde{\Delta}_k+\Delta_p)_\nu\,, \quad \tilde{j}^*_\mu(\tilde{\Delta}_k,\tilde{\Delta}_p)=(\tilde{\Delta}_p+\tilde{\Delta}_k)_\mu\,. \quad (8)$$

We define the functions

$$V_1(Q^2,\nu)=g^{\mu\nu}W_{\mu\nu}(P,q)=g^{\mu\nu}W'_{\mu\nu}(M,q')\,,$$
$$V_2(Q^2,\nu)=p^\mu p^\nu W_{\mu\nu}(P,q)/M^2=W'_{00}(M,q')\,, \quad (9)$$

and take advantage of tensor decaying $W_{\mu\nu}$ over the structure functions $W_1(Q^2,\nu)$ and $W_2(Q^2,\nu)$:

$$2MW_{\mu\nu}(P,q) = 4m^2\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)W_1(Q^2,\nu) + 4\left(P_\mu - q_\mu \frac{P\cdot q}{q^2}\right)\left(P_\nu - q_\nu \frac{P\cdot q}{q^2}\right)W_2(Q^2,\nu). \tag{10}$$

From (9) and (10) it is easy to find a coupling between the functions V_1, V_2 and W_1, W_2

$$4MW_1(Q^2,\nu) = -V_1(Q^2,\nu) + \frac{Q^2}{\nu^2+Q^2}V_2(Q^2,\nu), \tag{11}$$

$$\frac{4M(\nu^2+Q^2)}{Q^2}W_2(Q^2,\nu) = -V_1(Q^2,\nu) + \frac{3Q^2}{\nu^2+Q^2}V_2(Q^2,\nu).$$

For the second structure function we have

$$W_2(x,Q^2) = \frac{\nu}{2(1+\nu^2/Q)\sqrt{\nu^2+Q^2}}\left\{\int_{\eta_-}^{\eta_+}\left[\frac{M+\nu}{m} - \frac{\omega^2}{4m^2} - \eta^2 - \frac{1}{2}\right]|\phi(\eta)|^2 d\eta + \right.$$

$$\left. + \frac{3}{1+\nu^2/Q^2}\int_{\eta_-}^{\eta_+}\frac{\omega^2}{8m^2}|\phi(\eta)|^2 d\eta\right\}, \tag{12}$$

where $\nu = \dfrac{Q^2}{2M_\pi x}$, $\omega^2 = M_\pi^2 + 2M_\pi\nu - Q^2$ are the usual kinetic variables of deep-inelastic scattering, and $\eta = \Delta_k^0/m$ is the dimensionless quantity. The limits of integration in (12) are given by the expressions

$$\eta_\pm = \frac{1}{2m}\left[M+\nu \pm \sqrt{(\nu^2+Q^2)\left(1-\frac{4m^2}{\omega^2}\right)}\right]. \tag{13}$$

The wave functions $\phi(\eta)$, which are combined with Bethe-Salpeter's wave functions by the following correlation:

$$\phi_M(t,\vec{x}_1,\vec{x}_2) = \langle 0|\psi(t_1,\vec{x}_1)\psi(t_2,\vec{x}_2)|M\rangle|_{t_1=t_2} = \tag{14}$$

$$= \langle 0|\psi(t,\vec{x}_1)\psi(t,\vec{x}_2)|M\rangle$$

are the solution of covariant single-time, two-particle equation [6]

$$2\Delta^0(2\Delta^0 - M)\phi_M = \int V\phi_M \frac{d^3\Delta_k}{2\Delta_k^0},$$

$$\phi_M(\vec{\Delta}_p) = 2\Delta_p^0\,\psi_M(\Delta_p). \tag{15}$$

where $\lambda_p = P/M$ and $\Delta_k = L_P^{-1}k$ and

$$\psi_M(\Delta_{p,m\lambda_p}) = 2\Delta_k^0 \int dx\, e^{i(P_1-P_2)x}\,\delta(\lambda_p\cdot x)\langle 0|T\psi(x)\bar{\psi}(0)|M\rangle. \tag{16}$$

By means of changing [9]

$$\phi_M(\vec{\Delta}_k) = \int d\vec{r}\,\xi(\vec{\Delta}_k,\vec{r})\,\Phi_M(\vec{r}) \tag{17}$$

in which the role of flat waves is played by the functions [10]

$$\xi(\vec{\Delta}, \vec{r}) = [\frac{\Delta_r n^r}{m}]^{-1-irm} \tag{18}$$

$$\vec{r} = r \cdot \vec{n}, \quad \vec{n}^2 = 1$$

we turn to the relativistic configuration representation (RCR).

With due regard to the spherical symmetry, that is $\phi_M(\vec{\Delta}_k) = \phi_M(|\vec{\Delta}_k|)$, from (17) we get

$$\phi_M(|\vec{\Delta}_k|) = \frac{4\pi}{m\,\mathrm{sh}y} \int dr\, r\, \Phi_M(r) \sin mry\,, \tag{19}$$

where $y = \ln(\Delta_k^0 + |\vec{\Delta}_k|/m$ is the quark rapidity. Equation (15) in RCP will have the form

$$\hat{H}_0(M - \hat{H}_0)\Phi_M(r) = 2m V(r)\Phi_M(r)\,, \tag{20}$$

$$\hat{H}_0 = 2m\,\mathrm{ch}\frac{i}{m}\frac{\partial}{\partial r} + \frac{2i}{r}\,\mathrm{sh}\frac{i}{m}\frac{\partial}{\partial r}.$$

It is shown in paper [7] that the amplitude of one-gluon changing in QCD in the one-loop approximation for $\alpha_s(Q^2)$

$$T(Q^2) = \frac{-4\pi\alpha_s(Q^2)}{Q^2} = -\frac{4\pi}{Q^2 \ln Q^2/\Lambda^2} \tag{21}$$

and the form of quasipotential of the "Coulomb" type in RCP

$$V_{QCD}(r) = -g^2/r \tag{22}$$

in the impulse space at large Q^2 have the same asymptotics. Therefore, the role of the scale parameter is assigned to the quark mass m.

The solution of equation (15) with the quasipotential (21) is given in the following form:

$$\phi_M^{(n)}(\Delta_p, m\lambda_p) = \frac{C_0\chi_p}{\mathrm{ch}\chi_p\,\mathrm{sh}\chi_p[2\mathrm{ch}\chi_p - M/m](\chi_p^2+\chi_0^2)}, \tag{23}$$

where $\chi_0 = \mathrm{Arch}\frac{M}{2m}$.

At large

$$\phi_M^{(n)}(\Delta_p, m\lambda_p) = \frac{1}{\eta\sqrt{\eta^2-1}(\eta - M/2m)\ \ln(\eta + \sqrt{\eta^2-1})}, \tag{24}$$

$$\chi_p = \mathrm{Arcg}\,\eta.$$

In paper [4] formula (2) has been derived for a nonsinglet combination of structure functions. As far as the carbon target used in BCDMS experiment is an isoscalar, the measured structure functions will be pure singlets. However, for the moments with $n \geq 4$ the nonsinglet formulas nevertheless are a good approximation.

Using formulas (1), (2), (12) and (23) we get the moments of the nucleon structure functions through an explicit form of pion wave functions.

Fit has been carried out for the fourth and sixth moments simultaneously in the interval $Q^2 = 30 \div 110$ GeV^2. The values of the nucleon moment $M_N(n, Q_0^2)$ have been taken from the experiment at the point $Q_0^2 = 30$ GeV^2 for each moment. The pion moment $M_\pi(n, Q_0^2)$ has been calculated at the same point by formulae (12) and (23). The results are shown in fig.2. The best description has been reached at the value of the only model parameter quark mass $m = 126$ MeV with $\chi^2_{d.f.} = 1.1$.

So, it is shown that the wave function at given values of quark mass provides the behaviour of calculated moments of the structure functions which reflects the behaviour of the values of moments measured in the experiment. In other words it provides necessary disappearing of the scaling part of moment. In our model the scale parameter of QCD is equal to the value of free parameter, the mass of valent quark in π-meson. It is interesting to admit, that the value of Λ found from our analysis coincide in the limits of errors with the value of Λ found out in /3/ in the QCD analysis with the use of the nonsinglet approximation in the first order in $\alpha_s(Q^2)$.

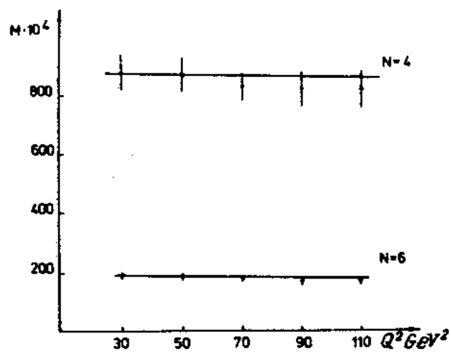

Fig.2.

REFERENCES

1. Gross D.J., Welchek F. Phys.Rev.Letters, 1973, 30, p. 1343.
   Politzer H.D. Phys.Rev.Letters, 1973, 30, p. 1346.

2. Abbott L.F., Barnett R.M. Ann. of Phys., 1980, 125, p. 276.
Abbott L.F., Atwood W.B., Barnett R.M. Phys.Rev., 1980, D22, p. 582.
3. Bollini D. et al. CERN 81-58, Subm. to Phys.Lett.B.
4. De Rujula A.De, Martin F. Preprint MIT-CTP-851, 1980.
5. Logunov A.A., Tavkhelidze A.N. Nuovo Cim., 1963, 29, p.380.
6. Kadyshevsky V.G. Sov. J. JETP, 1964, 19, p. 443, 597.
7. Savrin V.I., Skachkov N.B. CERN, preprint TH. 2822, 1980; Letters Nuovo Cim., 1980, 29, p. 363.
8. Kapshay V.N. et al. CERN, preprint TH. 3022, 1981.
9. Kadyshevsky V.G., Mir-Kasimov R.M., Skachkov N.B. Nuovo Cim., 1968, 55A.
10. Shapiro I.S. Sov. J.Doklady, 1956, 1, p. 91.
11. Buras A.J. Rev.Mod.Phys., 1980, 52, p. 1.

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