ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E2-81-766

S.N.Nikolaev, A.V.Radyushkin

METHOD FOR COMPUTING HIGHER GLUONIC POWER CORRECTIONS TO QCD CHARMONIUM SUM RULES

Submitted to "Physics Letters"

1981

1. Introduction. The QCD sum rule approach to hadronic physics proposed by the ITEP group [1] is one of the most exciting developments of the last years. The basic idea of the approach is that at large momenta one can rely on perturbation theory (PT), whereas the deviations from PT at moderate momenta can be described and/or parametrized by nonvanishing vacuum matrix elements of certain local operators, such as $\langle g^2 G^a_{\mu\nu} G^a_{\mu\nu}\rangle$, $\langle\bar\psi\psi\rangle$, $\langle f_{abc} G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu}\rangle$, etc. In the absence of a complete theory of the QCD vacuum these matrix elements play the role of fundamental constants characterizing the quark-gluon interactions at large distances. The magnitude of the quark condensate term $\langle\bar\psi\psi\rangle$, as is well-known, can be obtained from the hadronic spectrum by the current algebra analysis (see ref. [1] and references therein) $\langle\bar u u\rangle = \langle\bar d d\rangle \simeq \langle\bar s s\rangle \simeq -(0.24\ \mathrm{GeV})^3$.

Starting from the QCD charmonium sum rules

$$M_n = \frac{1}{12\pi^2 e_c^2}\int_0^\infty \frac{R_c(s)\,ds}{s^{n+1}} =$$
$$= \frac{3}{4\pi^2}\,\frac{2^n(n+1)(n-1)!}{(2n+3)!!(4m_c^2)^n}\left\{1 + \alpha_s a_n - \frac{(n+3)!}{(n-1)!\,(2n+5)}\,\frac{\langle g^2 G^a_{\mu\nu} G^a_{\mu\nu}\rangle}{9(4m_c^2)^2} + \dots\right\} \qquad (1)$$

(where a_n are known coefficients of the lowest-order PT correction, m_c is the mass of the charmed quark, $e_c = 2/3$ is its electric charge and $R_c \equiv \sigma(e^+e^- \to \mathrm{charm})/\sigma(e^+e^- \to \mu^+\mu^-)$), Shifman, Vanishtein and Zakharov (SVZ) have estimated the gluon condensate term $\langle g^2 G^2\rangle$ (see ref. [1]).

$$\langle g^2 G^2\rangle \equiv \langle g^2 G^a_{\mu\nu} G^a_{\mu\nu}\rangle = (0.83\ \mathrm{GeV})^4. \qquad (2)$$

Calculating higher power terms in the r.h.s. of eq. (1) and comparing the results obtained with the experimental curve one can estimate the magnitude of the vacuum matrix elements of higher dimension operators (g^3G^3, g^4G^4, etc.), i.e., to extract more detailed information about the QCD vacuum structure. In the present paper we describe a new method that considerably simplifies the calculation of the gluonic power corrections in QCD. As an example, we present our results of computation of the $O(m_c^{-6})$- corrections to the sum rule (1). The method works also in other cases (higher twist effects in deep inelastic scattering, etc.).

2. <u>Straightforward Approach.</u> The simplest way is to treat vacuum gluons as an external field and to use the standard Feynman rules for diagrams like those shown in fig. 1. For instance, the contribution of fig.1c for vector current is

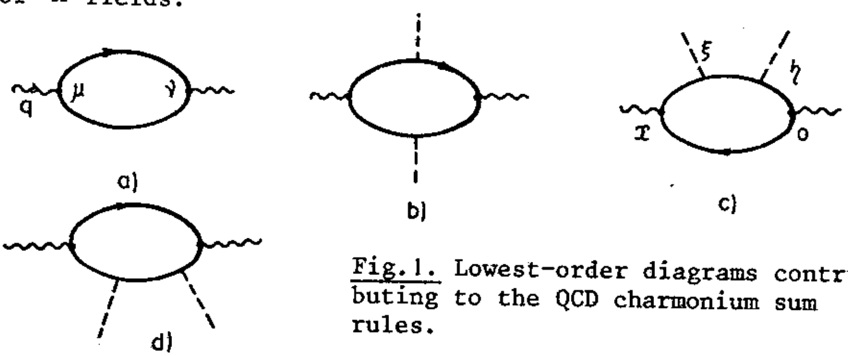
$$\Pi^{(1c)}_{\mu\nu}(q,A) = \int e^{iqx} d^4x \, Sp\{\gamma^\mu S^c(x)\gamma^\nu S^c(-\eta)\gamma^{\alpha_2}\tau_{a_2} S^c(\eta-\xi)\times$$
$$\times\gamma^{\alpha_1}\tau_{a_1} S^c(\xi-x)\}\langle A^{a_1}_{\alpha_1}(\xi) A^{a_2}_{\alpha_2}(\eta)\rangle d^4\xi \, d^4\eta, \qquad (3)$$

where τ_a are matrices of the gauge group $SU(3)_c$ in the quark representation. Then, performing the Taylor expansion of the A^a_α fields at some spatial point (say, at zero) one gets the vacuum matrix elements of local operators $\langle A^{a_1}_{\alpha_1}(0) A^{a_2}_{\alpha_2}(0)\rangle$, $\langle(\partial_\mu A^{a_1}_{\alpha_1}(0)) A^{a_2}_{\alpha_2}(0)\rangle$, etc.

In charmonium calculations (as well as in all cases when quark masses cannot be neglected) it is convenient to proceed further using momentum representation, e.g., the contribution of fig.1c reads

$$\Pi^{(1c)}_{\mu\nu}(q,A) = \sum_{n=0}^{\infty}\frac{1}{n!}\sum_{\ell=0}^{\infty}\frac{1}{\ell!}\int\frac{d^4k}{(2\pi)^4} Sp\{\gamma^\mu\frac{\hat{k}+\hat{q}+m}{(k+q)^2-m^2}\gamma^\nu\frac{\hat{k}+m}{k^2-m^2}\times$$
$$\times\gamma^{\alpha_2}\tau_{a_2}\frac{\partial}{\partial k_{\mu_1}}\cdots\frac{\partial}{\partial k_{\mu_n}}[\frac{\hat{k}+m}{k^2-m^2}\gamma^{\alpha_1}\tau_{a_1}\frac{\partial}{\partial k_{\nu_1}}\cdots\frac{\partial}{\partial k_{\nu_\ell}}(\frac{\hat{k}+m}{k^2-m^2})]\}\times \qquad (4)$$
$$\langle(\partial_{\mu_1}\cdots\partial_{\mu_n} A^{a_1}_{\alpha_1}(0))(\partial_{\nu_1}\cdots\partial_{\nu_\ell} A^{a_2}_{\alpha_2}(0))\rangle.$$

In the final result, any matrix element $\langle O_i\rangle$ will be accompanied by factor $m_c^{-d_i}$, where d_i is the mass dimension of O_i. Thus, one should calculate first the contribution of the lowest dimension operators, then the next power correction, next-to-next, etc. To get operators of higher dimensions, one has to increase either the number of derivatives or the number of A-fields.



<u>Fig. 1.</u> Lowest-order diagrams contributing to the QCD charmonium sum rules.

If the number of derivatives and/or A-fields is large, then it is difficult to calculate the integrand of eq. (4) by hand, but the corresponding manipulations can be easily performed by a computer with the help of, say, the SCHOONSHIP program. The resulting 1-loop integrals are standard and, if necessary, this step can be also performed by a computer.

At the next step one observes, however, that due to gauge invariance there appear numerous cancellations between contributions of different diagrams, and the final result must be expressed in terms of the operators containing only the gluon field strength $G^a_{\mu\nu} = \partial_\nu A^a_\mu - \partial_\mu A^a_\nu - g f_{abc} A^b_\mu A^c_\nu$ and its covariant derivatives. The reexpansion of the operators $(\partial)^{k_1} A \ldots (\partial)^{k_n} A$ over the operators $(D)^{\ell_1} G \ldots (D)^{\ell_m} G$ is very tedious, especially in a non-Abelian theory where, say, the operator $G_{\mu_1\nu_1} G_{\mu_2\nu_2}$ comes from diagrams with 2,3 and 4 external gluon lines. What is still worse, it is rather difficult to computerize this step.

3. Schwinger Gauge. Fortunately, there exists a class of gauges [2] (see also [3-6])

$$(x^\mu - z_0^\mu) A_\mu(x) = 0 \tag{5}$$

in which $A_\mu(x)$ can be expressed just in terms of $G_{\mu\nu}$ and its covariant derivatives*

$$\begin{aligned} A^a_\mu(x) &= (x^\nu - z_0^\nu) \int_0^1 t\, dt\, G_{\mu\nu}(z_0 + t(x - z_0)) = \\ &= \sum_{n=0}^{\infty} (x^\nu - z_0^\nu) \frac{(x^{\mu_1} - z_0^{\mu_1}) \ldots (x^{\mu_n} - z_0^{\mu_n})}{(n+2)\, n!} G^a_{\mu\nu;\mu_1 \ldots \mu_n}(z_0) . \end{aligned} \tag{6}$$

Here z_0 is an arbitrary spatial point serving as a gauge parameter. Of course, z_0 should disappear in the final result.

In the momentum representation, the expansion (6) generates Feynman rules for vertices where the quark interacts with the $G^a_{\mu\nu;\mu_1 \ldots \mu_n}(z_0)$ gluon field. It is convenient to use the usual graphical notation for these vertices indicating in some way the number of covariant derivatives.

Diagrams having only one external gluon line vanish owing to the color conservation, so one has to consider only diagrams with two or more (generalized) gluon vertices. The lowest-dimension operator $G^a_{\mu_1\nu_1}(z_0) G^b_{\mu_2\nu_2}(z_0)$ corresponds to diagrams with two external gluon lines (figs.1b-d). Our expli-

---

*Note that due to eq. (5) one can treat the derivatives in eq. (6) as covariant ones (see refs. [5,6] and eq. (16) below).

cit calculations show that the coefficient of $\langle G^a_{\mu_1\nu_1}(z_0) G^b_{\mu_2\nu_2}(z_0)\rangle$ is z_0-independent. Furthermore, the matrix element itself also does not depend on z_0 because of translation invariance. Hence, the z_0-dependence has disappeared, as expected. Finally, using the fact that

$$\langle G^a_{\mu_1\nu_1} G^a_{\mu_2\nu_2}\rangle = \frac{1}{12}(g_{\mu_1\mu_2} g_{\nu_1\nu_2} - g_{\mu_1\nu_2} g_{\mu_2\nu_1}) \quad \langle G^c_{\mu\nu} G^c_{\mu\nu}\rangle \tag{7}$$

(see, e.g., ref. /1/) we obtained the same result as that given in ref. /1/. In the next approximation one should calculate contributions of operators with dimension 6. These are $G^a_{\mu_1\nu_1} G^b_{\mu_2\nu_2} G^c_{\mu_3\nu_3}$, $G^a_{\mu_1\nu_1;\alpha\beta} G^b_{\mu_2\nu_2}$ and $G^a_{\mu_1\nu_1;\alpha} G^b_{\mu_2\nu_2;\beta}$. The total number of diagrams (constructed according to Feynman rules generated by eq.(6)) is 13=4+6+3. For matrix elements we used formulas that are straightforward generalizations of eq. (7). However, they are too lengthy to be presented here. Still, it is easy to realize that to get the set of operators with vacuum quantum numbers, one must contract in all possible ways the Lorentz and color indices of the original operators. Then one should use the equation of motion

$$G^a_{\mu\nu;\mu} = j^a_\nu = \bar\psi \gamma_\nu \tau^a \psi \tag{8}$$

for operators containing $G_{\mu\nu,\mu}$. In a similar way, $G_{\mu\nu,\alpha\alpha}$ can be expressed in terms of $G_{\mu\alpha} G_{\nu\alpha}$ and $j_{\mu;\nu}$. Finally, incorporating translation invariance reduces $\langle j^a_{\mu,\nu} G^a_{\mu\nu}\rangle$ to $\langle j^a_\mu j^a_\mu\rangle$. Thus, the dimension-6 contribution is expressed in terms of 2 operators *: $g^3 G^3 \equiv g^3 f_{abc} G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu}$ and $j^2 \equiv j^a_\mu j^a_\mu$.

Note that G^3-terms come not only from GGG-diagrams, but also from GG-diagrams with derivatives. It is worth mentioning also that, according to our calculations, both GGG- and GG-contributions are z_0-dependent (i.e., not gauge-invariant) and only summing them we have obtained the z_0-independent result:

$$\begin{aligned} M_n = M_n^{(0)}\{1 + \alpha_s a_n &- \frac{(n+3)!}{(n-1)!(2n+5)} \frac{\langle g^2 G^2\rangle}{9(4m_c^2)^2} - \\ &- \frac{2}{45} \frac{(n+4)!\,(3n^2+8n-5)}{(n-1)!\,(2n+5)(2n+7)} \frac{\langle g^3 G^3\rangle}{9(4m_c^2)^3} - \\ &- \frac{8}{135} \frac{(n+2)!(n+4)}{(n-1)!} \frac{(3n^3+47n^2+244n+405)}{(2n+5)(2n+7)} \frac{\langle g^2 j^2\rangle}{9(4m_c^2)^3}\}. \end{aligned} \tag{9}$$

*Absence of the $d_{abc} G^a G^b G^c$ -operators is a manifestation of the QCD Furry theorem.

We have performed calculations also for other currents ($\bar{c}c$, $\bar{c}\gamma^5 c$, $\bar{c}\gamma^5\gamma_\mu c$); the results will be published elsewhere.

The calculations may be simplified by a particular choice of z_0, say, $z_0=0$. In this case we have used various tricks that do not work if z_0 is arbitrary. Thus, our $z_0=0$ calculation was independent of the $z_0 \neq 0$ one (i.e., it was not just putting $z_0=0$ in the $z_0 \neq 0$ program), and we used the $z_0=0$ calculation as another check for the $z_0 \neq 0$ ones (the first check was the absence of the z_0-dependent terms).

4. Discussion of Results. Comparing predictions of eq. (9) with experimental data as described in ref. [1] one can estimate $\langle g^3 G^3 \rangle$. As emphasized in refs. [1,7], one should compare with data the ratio $r_n = M_n/M_{n-1}$ rather than the moments themselves. Note that according to dominance of the vacuum intermediate state, $\langle g^2 j^2 \rangle$ is not a free parameter:

$$\langle g^2 j^2 \rangle = -\frac{4}{3} g^2 \langle \bar{u} u \rangle^2 = -(0.38\ \mathrm{GeV})^6 \tag{10}$$

(for details see ref. [1]). However, this quantity is very small compared to $(4m_c^2)^3 = (2.5\ \mathrm{GeV})^6$. As a result, for $n \le 10$ the j^2-contribution is smaller than 1%, and we can safely neglect it. The magnitude of the G^3 matrix element can be estimated using the dilute instanton-gas approximation (DIGA) that gives [1]

$$\langle g^3 f_{abc} G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu} \rangle_{\mathrm{DIGA}} \simeq \frac{12}{5} \rho_c^{-2} \langle g^2 G^a_{\mu\nu} G^a_{\mu\nu} \rangle , \tag{11}$$

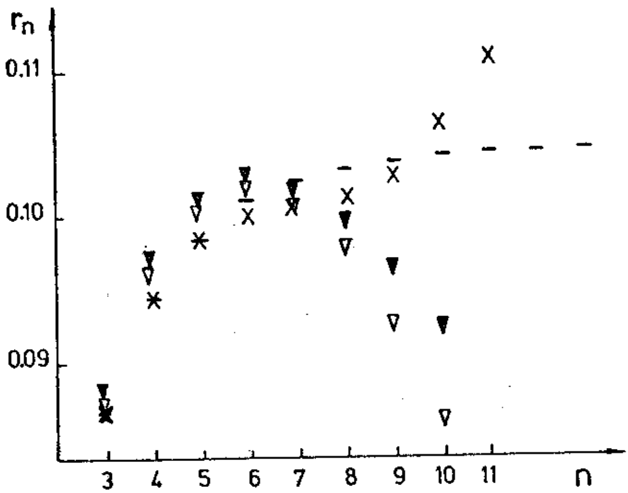


Fig.2. Ratio $r_n = M_n/M_{n-1}$, (—) experiment; (▼) SVZ fit: $\langle g^2 G^2 \rangle = (0.83\ \mathrm{GeV})^4$, $\langle g^3 G^3 \rangle = 0$; (∇) SVZ corrected by G^3-contribution estimated in DIGA: $\langle g^3 G^3 \rangle = (0.59\mathrm{GeV})^6$ (×) $\langle g^2 G^2 \rangle = (0.94\ \mathrm{GeV})^4$, $\langle g^3 G^3 \rangle = -(0.80\ \mathrm{GeV})^6$.

where $\rho_c = (200 \text{ MeV})^{-1}$. If we adhere to this estimate (namely, take $\langle g^3 G^3 \rangle = (0.59 \text{ GeV})^6$), then, as is seen from fig.2, the G^3-contribution to r_n is negative and rather small. To extend the agreement between theory and experiment to higher n, one should take $\langle g^3 G^3 \rangle$ negative, e.g., taking $\langle g^2 G^2 \rangle = (0.94 \text{ GeV})^4$ $\langle g^3 G^3 \rangle = -(0.80 \text{ GeV})^6$ we have obtained a curve which is very close to experimental points up to $n = 10$ (see fig. 2). This fact can be interpreted at least in two ways. First, it is quite possible that DIGA is a rather poor approximation. This view is supported by the observation that the DIGA prediction on the relation between $\langle g^2 G^2 \rangle$ and $\langle m \bar{\psi} \psi \rangle$ is wrong by factor 10 [5,8]. Another viewpoint is that the DIGA estimate for $\langle g^3 G^3 \rangle$ is about right, but the necessary positive contribution is given in fact by the next power corrections due to $\langle g^4 G^4 \rangle$ -terms. Note that if the dominance of the vacuum intermediate states [1] for G^4 gluon operators works with at least 10% accuracy, then $\langle g^4 G^4 \rangle$ can be expressed through $\langle g^2 G^2 \rangle^2$ (see [6]), and there will be no new free parameters in the $O(m_c^{-8})$ -correction. Thus, to get a real estimate of $\langle g^3 G^3 \rangle$ from experimental data one has to compute the G^4-correction. The computations based on the approach outlined in the present paper are in progress now.

Our method can be applied also to other calculations of gluonic power corrections. However, by our own experience we know that many people are prejudiced against using such a singular gauge as that defined by eq. (5). Thus, it seems worth presenting here also a gauge-independent derivation [9] of the formalism discussed above.

5. Analysis in Arbitrary Gauge. Notice first that the usual expansion for the quark propagator in the external field (corresponding to the standard Feynman rules for the quark-gluon vertices) is, in fact, a perturbative solution to the Dirac equation

$$[i\gamma^\mu (\partial/\partial x^\mu - ig\hat{A}_\mu(x)) - m] S^c(x,y;A) = -\delta^4(x-y) , \qquad (12)$$

where $\hat{A} \equiv A^a \tau^a$. Representing $S^c(x,y;A)$ as

$$S^c(x,y;A) = \hat{E}(x,z_0;A) \mathcal{S}^c(x,y;A,z_0) \hat{E}(z_0,y;A) , \qquad (13)$$

where $\hat{E}(x,y,A)$ is the path-ordered exponential in the quark (fundamental) representation *

$$\hat{E}(x,y;A) = P \exp(ig \int_y^x \hat{A}_\mu(z)\, dz^\mu) \qquad (14)$$

---

* Integration in eq. (14) is performed along the straight line.

one finds that eq. (13) is satisfied only if $\hat{S}^c(x,y;A,z_0)$ is a solution to the modified Dirac equation with $\hat{A}_\mu(x)$ substituted by $\hat{\mathcal{G}}_\mu(x,z_0)$

$$\mathcal{G}^a_\mu(x,z_0) = (x^\nu - z_0^\nu)\int_0^1 t\,dt\, G^b_{\mu\nu}(z)\,\tilde{E}^{ba}(z,z_0). \qquad (15)$$

Here $z = z_0 + t(x - z_0)$ and $\tilde{E}$ is a path-order exponential in the gluonic (adjoint) representation. Using the Baker-Hausdorff theorem (see, e.g., refs. [9,10]) gives

$$G^b_{\mu\nu}(z)\,\tilde{E}^{ba}(z,z_0) = \sum_{n=0}^{\infty} G^a_{\mu\nu;\mu_1\ldots\mu_n}(z_0)\,\frac{(z-z_0)^{\mu_1}\ldots(z-z_0)^{\mu_n}}{n!}. \qquad (16)$$

This means that $\mathcal{G}$ satisfies eq. (6) in any gauge. Evidently, in the Schwinger gauge (5) $\hat{\mathcal{G}}$ coincides with A. The last observation is that the $\hat{E}$ -factors present in eq. (13) cancel for $\pi^{\mu\nu} \sim \mathrm{Sp}\{\gamma^\mu S^c(x,y;A)\gamma^\nu S^c(y,x;A)\}$, and we get finally the same prescription for calculation of $\pi^{\mu\nu}$ as in the Schwinger gauge (5).

Acknowledgement. We are grateful to B.M.Barbashov, A.V.Efremov, S.V.Mikhailov, V.A.Nesterenko, Yu.V.Pinelis, M.A.Shifman and A.V.Smilga for stimulating discussions and to V.A.Meshcheryakov and D.V.Shirkov for interest in this work and support.

REFERENCES

1. Shifman M.A., Vainshtein A.I., Zakharov V.I. Nucl.Phys. B, 1979, 147, pp. 385, 447.
2. Schwinger J. Particles, Sources and Fields. Addison-Wesley Publishing Co, New York, 1970.
3. Fateev V.A., Schwartz A.S., Tyupkin Yu.S. Preprint N155, P.N.Lebedev Phys.Inst., M., 1976.
4. Cronström C. Phys.Lett.B, 1980, 90, p. 267.
5. Dubovikov M.S., Smilga A.V. Nucl.Phys., B, 1981, 185, p. 109.
6. Shifman M.A. Nucl.Phys.B, 1980, 173, p. 13.
7. Guberina B., Meckbach R., Peccei R.D., Rückl R. Nucl.Phys. B, 1981, 184, p. 476.
8. Novikov V.A. et al. Nucl.Phys.B, 1980, 174, p. 378.
9. Efremov A.V., Radyushkin A.V. Riv.Nuovo Cimento 1980, vol.3, No.2.
10. Kumar K. Journ.Math.Phys., 1965, 6, pp. 1923, 1928.

Received by Publishing Department
on December 2 1981.