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ON THE ELASTIC SCATTERING OF NEUTRINO AND ANTINEUTRINO ON PROTONS IN THE WEINBERG-SALAM THEORY

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1. As is well known, all the available data on neutral currents are is good agreement with the standard $SU(2)\times U(1)$ gauge theory of electroweak interactions, the Weinberg-Salam theory. The important parameter of this theory $\sin^2\theta_W$ (θ_W, the Weinberg angle) has also far reaching implications for grand unified models and its highly accurate determination is of extreme interest.

In this work we shall consider in detail the processes:

$$\nu_\mu + p \to \nu_\mu + p, \tag{1}$$

$$\tilde{\nu}_\mu + p \to \tilde{\nu}_\mu + p. \tag{2}$$

These reactions are the simplest neutral-current interactions between neutrinos and nucleons. In the framework of the Weinberg-Salam theory (and neglecting the contributions of s, c and other heavier quarks) the matrix elements of processes (1) and (2) are expressed in terms of the parameter $\sin^2\theta_W$ and of the electromagnetic and axial-vector nucleon form factors only. Therefore, further information about the value of $\sin^2\theta_W$ can be obtained by studying processes (1) and (2). For this purpose new more precise experiments on elastic neutrino-nucleon scattering are planned at Serpukhov[3] and Brookhaven[4].

The electromagnetic nucleon form factors are determined from elastic electron-nucleon scattering data. Information about the axial-vector nucleon form factor can be obtained from the quasielastic charged-current reactions:

$$\nu_\mu + n \to \mu^- + p, \tag{3}$$

$$\tilde{\nu}_\mu + p \to \mu^+ + n, \tag{4}$$

and from neutron β-decay data.

* At present the most precise experimental determination of the parameter $\sin^2\theta_W$ comes from deep-inelastic ν_μ-nucleon scattering[1] and from measurements of the P-odd asymmetry in the deep-inelastic scattering of polarized electrons on nucleons[2].

Up to now no consistent theory of the form factors exists and the description of their q^2-behaviour is achieved by phenomenological parametrizations. We shall rely upon the general analysis of the existing data on the elastic scattering of electrons on protons and deuterons, performed in papers /5,6/. In these papers two different parametrizations of the nucleon form factors have been used and it has been shown that both of them give a satisfactory description of all the available data on elastic e-p and e-d scattering.

In the present work the cross sections of processes (1) and (2) have been calculated with both the parametrizations of the electromagnetic nucleon form factors used in refs. /5,6/. It turned out that these cross sections are quite sensitive to the choice of the parametrization. To obtain a sufficiently accurate determination of the value of $\sin^2\theta_W$ from the experimental data on neutral current processes (1), and (2), it is necessary, therefore to use the kinematical region of small $q^2 (q^2 \leq 1\ [\mathrm{GeV/c}]^2)$ where the different parametrizations give the same values for electromagnetic nucleon form factors. We investigated the cross sections of reactions (1) and (2) in this region and determined their dependence on $\sin^2\theta_W$ in the interval $0.20 \leq \sin^2\theta_W \leq 0.24$. It turned out that the cross sections of the elastic antineutrino-proton scattering is strongly dependent on the value of $\sin^2\theta_W$, while the process of elastic neutrino-proton scattering does not reveal such a sensitivity to the value of $\sin^2\theta_W$.

2. In the lowest order in the weak coupling constant G the cross sections for processes $\nu_\mu(\tilde{\nu}_\mu)p \to \nu_\mu(\tilde{\nu}_\mu)p$ are given by the following expressions /7,8/:

$$\left(\frac{d\sigma}{dq^2}\right)^{NC}_{\nu_\mu(\tilde{\nu}_\mu)} = \frac{G^2}{4\pi}\Big\{(F_V^\circ \pm F_A^\circ)^2 + \left(1-\frac{q^2}{2ME}\right)^2 (F_V^\circ \mp F_A^\circ)^2 + \tag{5}$$

$$+\frac{q^2}{2E^2}\left((F_A^\circ)^2-(F_V^\circ)^2\right) + \frac{q^2}{2M^2E^2}F_M^\circ\left[\left(-\frac{q^2}{4}+E^2-\frac{q^2E}{2M}\right)F_M^\circ + q^2F_V^\circ \pm (4ME-q^2)F_A^\circ\right]\Big\}.$$

In the Weinberg-Salam theory (and neglecting the contribution of isoscalar current, built up by s,c and other heavier quarks) the form factors $F_V^\circ(q^2)$, $F_M^\circ(q^2)$, $F_A^\circ(q^2)$ are expressed in terms of the charge $G_E^{p(n)}(q^2)$ and magnetic $G_M^{p(n)}(q^2)$ proton (neutron) form factors and the axial-vector nucleon form factor $F_A(q^2)$ as follows /8/:

$$F_V^{\circ}(q^2) = \frac{1}{1+\tau}\left[\left(\frac{1}{2} - 2\sin^2\theta_W\right)(G_E^p + \tau G_M^p) - \frac{1}{2}(G_E^n + \tau G_M^n)\right] \tag{6}$$

$$F_M^{\circ}(q^2) = \frac{1}{1+\tau}\left[\left(\frac{1}{2} - 2\sin^2\theta_W\right)(G_M^p - G_E^p) - \frac{1}{2}(G_M^n - G_E^n)\right]$$

$$F_A^{\circ}(q^2) = \frac{1}{2}F_A(q^2).$$

Here $\tau = \frac{q^2}{4M^2}$ (q^2 is the square of the momentum transfered), M is the proton mass, E is the energy of the incoming neutrino (antuneutrino) in the lab. system.

In experiment the ratios

$$R_{\nu(\tilde{\nu})} = \left(\frac{d\sigma}{dq^2}\right)^{NC}_{\nu(\tilde{\nu})} \Big/ \left(\frac{d\sigma}{dq^2}\right)^{CC}_{\nu(\tilde{\nu})} \tag{7}$$

are investigated. Here $\left(\frac{d\sigma}{dq^2}\right)^{CC}_{\nu}$ and $\left(\frac{d\sigma}{dq^2}\right)^{CC}_{\tilde{\nu}}$ are the differential cross sections of the charged-current processes $\nu_\mu + n \to \mu^- + p$ and $\tilde{\nu}_\mu + p \to \mu^+ + n$. They are given by /7/:

$$\left(\frac{d\sigma}{dq^2}\right)^{CC}_{\nu_\mu(\tilde{\nu}_\mu)} = \frac{G^2}{4\pi}\cos^2\theta_c\left\{(F_V \pm F_A)^2 + \left(1 - \frac{q^2}{2ME}\right)^2(F_V \mp F_A)^2 + \right.$$
$$\left. + \frac{q^2}{2E^2}(F_A^2 - F_V^2) + \frac{q^2}{2M^2E^2}F_M\left[\left(\frac{q^2}{4} + E^2 - \frac{q^2E}{2M}\right)F_M + q^2F_V \pm (4ME - q^2)F_A\right]\right\} \tag{8}$$

(θ_c is the Cabibbo angle).
The form factors $F_V(q^2)$ and $F_M(q^2)$ (according to the conserved vector current hypothesis) are connected with the electromagnetic nucleon form factors as follows:

$$F_V(q^2) = \frac{1}{1+\tau}[G_E^p - G_E^n + \tau(G_M^p - G_M^n)] \tag{9}$$

$$F_M(q^2) = \frac{1}{1+\tau}[G_M^p - G_E^p - G_M^n + G_E^n].$$

Note that in the expression (8) the terms proportional to the square of the muon mass are neglected. In the energy region under consideration the contribution of these terms does not exceed ≈ 1%.

3. All the existing data on the elastic scattering of electrons on protons and deuterons have been analysed in refs. /5,6/ under the following assumptions.

A. Scaling relation holds between the charge and magnetic nucleon form factors:

$$\frac{G_M^p(q^2)}{\mu_p} = \frac{G_M^n(q^2)}{\mu_n} = G_E^p(q^2); \quad G_E^n(q^2) = 0 \qquad (10)$$

(μ_p and μ_n are the total magnetic moments of the proton and neutron in Bohr magnetons). The magnetic proton form factor is taken as a sum of two poles:

$$G_M^p(q^2) = \mu_p \left(\frac{a_3}{1+a_1q^2} + \frac{1-a_3}{1+a_2q^2}\right), \qquad (11)$$

where a_i are free parameters. The expression (11) is a generalization of the well-known dipole formula. The available data on e–p elastic scattering* can be described by (11) with the values of parameters:

$$a_1 = 0{,}67\ (\mathrm{GeV/c})^2, \quad a_2 = 2{,}23\ (\mathrm{GeV/c})^2, \quad a_3 = -0{,}45. \qquad (12)$$

B. Scaling holds between the isovector $G^V_{E,M}(q^2)$ and isoscalar $G^S_{E,M}(q^2)$ form factors of the nucleon/9/:

$$G_M^V(q^2) = \frac{\mu_V}{1/2} G_E^V(q^2),$$
$$G_M^S(q^2) = \frac{\mu_S}{1/2} G_E^S(q^2), \qquad (13)$$

where

$$\mu_V = \frac{1}{2}(\mu_p - \mu_n), \quad \mu_S = \frac{1}{2}(\mu_p + \mu_n). \qquad (14)$$

From eqs. (13) and (14) we obtain the following relations between proton and neutron form factors:

$$G_M^n(q^2) = \mu_n G_E^p(q^2) + \frac{\mu_p}{\mu_n}[G_M^p(q^2) - \mu_p G_E^p(q^2)],$$
$$G_E^n(q^2) = \frac{1}{\mu_n}[G_M^p(q^2) - \mu_p G_E^p(q^2)]. \qquad (15)$$

As has been shown in ref./5,6/, the existing data on the elastic e–p and e–d scattering can be satisfactorily described by using (15) if one assumes a form for $G_M^p(q^2)$ given by (11) and the following parmatrization for $G_E^p(q^2)$

$$G_E^p(q^2) = \frac{b_3}{1+b_1q^2} + \frac{1-b_3}{1+b_2q^2} \qquad (16)$$

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* Note that the dipole formula is compatible with the experimental data for small q^2 ($q^2 \lesssim 0.5\ (\mathrm{GeV/c})^2$) only /5/.

with

$a_1=0,58\ (GeV/c)^2$ $\qquad$ $b_1=0,37\ (GeV/c)^2$

$a_2=2,42\ (GeV/c)^2$ $\qquad$ $b_2=2,50\ (GeV/c)^2$ $\qquad$ (17)

$a_3=-0,33$ $\qquad$ $b_3=-0,24$

The expression for the axial-vector nucleon form factor is adopted from the recent work /10/. It has been pointed out there that the best description of the data (the process $\nu_\mu+n\to\mu^-+p$ had been considered) is granted by the dipole parametrization for $F_A(q^2)$

$$F_A(q^2)=\frac{1.23}{(1+q^2/M_A^2)^2}, \tag{18}$$

where $M_A=(1.07\pm0.06)$ GeV. (19)

4. Let us proceed now to the discussion of the results obtained. The cross sections of processes /1-4/ and the ratios R_ν, $R_{\tilde\nu}$ have been calculated at the initial neutrino (antineutrino) energies in the interval $0.5\le E\le 10$ GeV and at q^2 varying from 0 to $q^2_{max}=\frac{4ME^2}{M+2E}$ for every fixed value of the energy. The parameter $\sin^2\theta_W$ was supposed to vary in the interval $0.20\le\sin^2\theta_W\le 0.24$ in consistency with the existing experimental data /1,2/ and with the theoretical predictions of the grand unified models /11/. We have investigated the dependence of the cross sections and $R_{\nu(\tilde\nu)}$ on $\sin^2\theta_W$ splitting for convenience the energy interval into two parts: $0.5\le E\le 1$ GeV and $1\le E\le 10$ GeV.

As the calculations show at energies in the first region the cross sections of the neutral current processes (1) and (2) and the ratios $R_{\nu(\tilde\nu)}$ are practically independent of the nucleon form-factors parametrization. In case A we have used also the dipole formula for $G_M^p(q^2)$:

$$G_M^p(q^2)=\frac{\mu_p}{(1+q^2/0.71)^2}.$$

The change of the cross sections and of $R_{\nu(\tilde\nu)}$ in this energy region does not exceed 5% when the type of the parametrization is changed. That is not astonishing because for $q^2\lesssim 1\,(GeV/c)^2$ all parametrizations of the nucleon electromagnetic form factors considered here (and many others) give almost the same values for $G_{E,M}^{p(n)}(q^2)$.

It turned out also that the cross sections of the considered processes and the ratios R have a quite different dependence on $\sin^2\theta_W$ (in the interval $0.20\le\sin^2\theta_W\le 0.24$) for the neutrino and antineutrino. The cross section of the antineutrino proton elastic scattering (and, consequently, $R_{\tilde\nu}$) is

Table 1

Neutrino cross sections $\left(\frac{d\sigma}{dq^2}\right)^{NC}$ and $\left(\frac{d\sigma}{dq^2}\right)^{CC}$ and ratio R_ν at E_ν =0.6 GeV

q^2 $(GeV/c)^2$	$\left(\frac{d\sigma}{dq^2}\right)^{CC} \times 10^{38}$	$(d\sigma/dq^2)^{NC} \times 10^{38}$			R_ν		
		$\sin^2\theta_W$ = 0.20	$\sin^2\theta_W$ = 0.22	$\sin^2\theta_W$ = 0.24	$\sin^2\theta_W$ = 0.20	$\sin^2\theta_W$ = 0.22	$\sin^2\theta_W$ = 0.24
0	I.764	0.287	0.282	0.280	0.I63	0.I60	0.I59
0.I	I.775	0.280	0.269	0.260	0.I58	0.I52	0.I46
0.2	I.588	0.247	0.235	0.224	0.I55	0.I48	0.I4I
0.3	I.365	0.2I0	0.I99	0.I88	0.I54	0.I46	0.I38
0.4	I.I55	0.I77	0.I67	0.I57	0.I53	0.I44	0.I36
0.5	0.974	0.I49	0.I40	0.I3I	0.I53	0.I43	0.I34
0.6	0.82I	0.I25	0.II7	0.II0	0.I52	0.I43	0.I34
0.632	0.778	0.II8	0.III	0.I04	0.I52	0.I43	0.I33

Table 2

Antineutrino cross sections $(d\sigma/dq^2)^{NC}$ and $(d\sigma/dq^2)^{CC}$ and ratio $R_{\tilde\nu}$ at $E_{\tilde\nu}$=0.6 GeV

q^2 $(GeV/c)^2$	$\left(\frac{d\sigma}{dq^2}\right)^{CC} \times 10^{39}$	$(d\sigma/dq^2)^{NC} \times 10^{39}$			$R_{\tilde\nu}$		
		$\sin^2\theta_W$= 0.20	$\sin^2\theta_W$= 0.22	$\sin^2\theta_W$= 0.24	$\sin^2\theta_W$= 0.20	$\sin^2\theta_W$ = 0.22	$\sin^2\theta_W$= 0.24
0	I7.640	2.872	2.824	2.8I0	0.I63	0.I60	0.I59
0,I	8.842	I.569	I.57I	I.587	0.I77	0.I78	0.I79
0.2	4.337	0.872	0.895	0.928	0.20I	0.206	0.2I4
0.3	2.0II	0.493	0.523	0.560	0.245	0.260	0.279
0.4	0.830	0.286	0.3I7	0.353	0.344	0.382	0.452
0.5	0.264	0.I75	0.203	0.235	0.660	0.769	0.890
0.6	0.032	0.II7	0.I42	0.I69	3.6I2	4.380	5.227
0,632	0.00I	0.I06	0.I30	0.I56	8I.080	99.I20	II9.000

Table 3

Ratios R_ν and $R_{\tilde\nu}$ at $E_{\nu(\tilde\nu)}$ =0.9 GeV

q^2 $(GeV/c)^2$	Neutrino			Antineutrino		
	$\sin^2\theta_W$= 0.20	$\sin^2\theta_W$= 0.22	$\sin^2\theta_W$= 0.24	$\sin^2\theta_W$= 0.20	$\sin^2\theta_W$= 0.22	$\sin^2\theta_W$= 0.24
0	0.I63	0.I60	0.I59	0.I63	0.I60	0.I59
0,I	0.I57	0.I5I	0.I46	0.I67	0,I66	0,I65
0.2	0.I54	0.I47	0.I40	0.I73	0.I73	0.I74
0.3	0.I52	0.I44	0.I37	0.I8I	0.I83	0.I87
0.4	0.I50	0.I42	0.I34	0.I9I	0.I97	0.205
0.5	0.I49	0.I40	0.I32	0.207	0.2I7	0.23I
0.6	0.I48	0.I39	0.I3I	0.230	0.249	0.27I
0.7	0.I48	0.I39	0.I30	0,270	0,30I	0,338
0.8	0.I47	0.I38	0.I29	0.342	0.397	0.460
0.9	0.I47	0.I37	0.I28	0.489	0.593	0.7II
I,0	0,I47	0.I37	0.I28	0.835	I.054	I.302
I.I	0,I46	0.I37	0.I27	I.593	2.066	2.60I
I.II	0.I46	0.I37	0.I27	I.679	2.I80	2.747

highly sensitive to the value of $\sin^2\theta_W$. This dependence gets stronger with increasing q^2. In the case neutrino there is no such sensitivity to the value of $\sin^2\theta_W$.

The values of cross sections $(\frac{d\sigma}{dq^2})^{NC}_{\nu(\tilde\nu)}$ and $(\frac{d\sigma}{dq^2})^{CC}_{\nu(\tilde\nu)}$ and the ratios $R_{\nu(\tilde\nu)}$ at the neutrino (antineutrino) energy E = =0.6 GeV and for q^2 changing in the interval $0 \le q^2 \le q^2_{max}$ are listed in Table 1 and 2. The parameter $\sin^2\theta_W$ is taken to be 0.20; 0.22; 0.24. The dependence of $R_{\nu(\tilde\nu)}$ on q^2 at the energy 0.6 GeV and for $\sin^2\theta_W$ =0.20 and 0.24 is illustrated in Fig.1. In Table 3 the values of $R_{\nu(\tilde\nu)}$ at E=0.9 GeV and for $0 \le q^2 \le q^2_{max}$ are listed; the parameter $\sin^2\theta_W$ is again 0.20; 0.22; 0.24. As is seen from Tables 1, 2, 3 and Fig.1, the difference in the values of $(\frac{d\sigma}{dq^2})^{NC}_{\tilde\nu}$ and $R_{\tilde\nu}$ corresponding to the change of $\sin^2\theta_W$ from 0.20 to 0.24 reaches 40% for q^2 near q^2_{max}. The tables show also that the cross sections of the process $\tilde\nu_\mu + p \to \mu^+ + n$ decreases sharply with increasing q^2 and for q^2 near $\sim q^2_{max}$ the cross section of the neutral current process

$\tilde{\nu}_\mu + p \to \nu_\mu + p$ is much larger than the cross section of the charged-current process (4) (nevertheless, the neutral current total cross section is about 20% of the charged current total cross section).

The value of R_ν is practically fully insensitive to the change of $\sin^2\theta_W$, as illustrated also in Fig.1. In the second energy region ($E \geq 1$ GeV) the cross sections considered and the ratios $R_{\nu(\tilde{\nu})}$ become strongly dependent on the type of parametrization. The difference between their values calculated in cases A and B (see section 3) increases with q^2. This is seen in Fig.2, where the dependence of R_ν and $R_{\tilde{\nu}}$ on q^2 at the neutrino (antineutrino) energy 4 GeV is illustrated.

At energies $E \geq 1$ GeV the cross section of the process $\tilde{\nu}_\mu + p \to \tilde{\nu}_\mu + p$ continues to be quite sensitive to the change of the parameter $\sin^2\theta_W$. The dependence on $\sin^2\theta_W$ is much weaker in the case of neutrino ($\nu_\mu p \to \nu_\mu p$). This can be seen also in Fig.2, where the values of R_ν and $R_{\tilde{\nu}}$ for $\sin^2\theta_W = 0.20$ and 0.24 are shown. The sensitivity of $\left(\frac{d\sigma}{dq^2}\right)^{NC}$ to the change of $\sin^2\theta_W$ is lar-

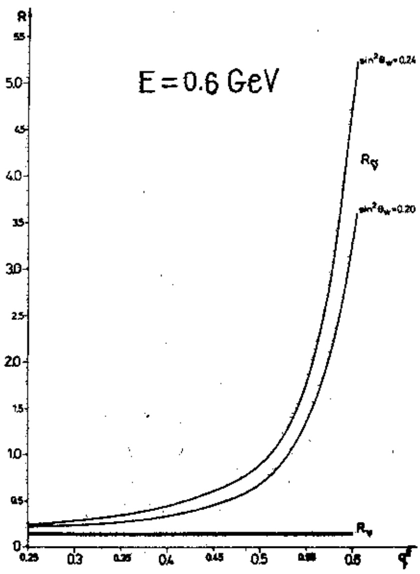

Fig.1. The ratios R_ν and $R_{\tilde{\nu}}$ at $\sin^2\theta_W = 0.20$; 0.24 at neutrino (antineutrino) E = = 0.6 GeV.

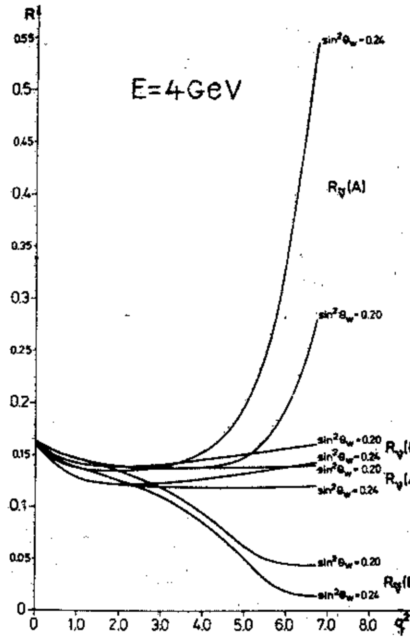

Fig.2. The ratios R_ν and $R_{\tilde{\nu}}$ in parametrizations A and B at $\sin^2\theta_W = 0,20; 0.24$ at neutrino (antineutrino) energy E=4 GeV.

ger for high energies. However, the uncertainties of the cross sections due to the indefiniteness of the nucleon form factors don't allow us to determine the value of $\sin^2\theta_W$ with a sufficient accuracy from the data in this energy region.

5. In conclusion we summarize the results of this paper. The determination of the parameter $\sin^2\theta_W$ with high accuracy is of fundamental importance for the theory. We have considered processes $\nu_\mu(\tilde{\nu}_\mu)+p \to \nu_\mu(\tilde{\nu}_\mu)+p$ which are the simplest interactions caused by nucleon neutral currents. Based on the analysis performed, we may recommend the investigation of the elastic antineutrino-proton scattering at energies $E \leq 1$ GeV and at q^2 near q^2_{max}. Our calculations show that in this kinematical region the cross sections are quite sensitive to the value of $\sin^2\theta_W$ and are only slightly affected by the uncertainies of the electromagnetic nucleon form factors. So, the measurements in this region offer an opportunity to obtain new information about the value of $\sin^2\theta_W$.

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