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RELATIVISTIC WAVE FUNCTIONS OF TWO SPIN 1/2 QUARKS IN A MODEL WITH QCD INTERACTION

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1. INTRODUCTION

In the recent years an important progress has been achieved in the application of results of calculations of various processes with the use of the perturbative quantum chromodynamics (QCD). In the QCD the two-quark interaction constant is a function of the momentum transfer squared $Q^2 = -(p-k)^2$

$$\alpha_s(Q^2) \underset{Q^2 \gg \Lambda^2}{\cong} \alpha_0 / \ln(Q^2/\Lambda^2) \tag{1.1}$$

(its form in the infrared limit $Q^2 \ll \Lambda^2$ is still unknown). In the Born approximation the quark-quark one-gluon exchange amplitude has the form

$$V_{qq}(Q^2) \underset{Q^2 \gg \Lambda^2}{\cong} \alpha_s(Q^2)/Q^2 \approx 1/Q^2 \ln(Q^2/\Lambda^2). \tag{1.2}$$

In the present paper we will find exact solutions to the relativistic three-dimensional two-particle quasipotential equation[1] for the case of two spin quarks interacting through a "chromodynamical" potential with the behaviour (1.2). We shall use two-particle quasipotential equations arising in the covariant Hamiltonian formulation of QFT, proposed by Kadyshevsky[2-4] and coinciding in form with analogous equations, obtained in[5] within a single-time description of relativistic-particle systems of Logunov and Tavkhelidze[1].

In some papers[6] attempts were made to apply the Fourier transform of the amplitude (1.2) (regularized phenomenologically at small Q^2) for describing the mass spectrum of new Ψ and Υ mesons on the basis of the Schrödinger equation. It turns out however, that the potentials thus obtained have rather a complicated form in r-space, and solutions with them can be found only through the computer numerical calculations.

Our further exposition will rest on the fact[7,11] that a simple, Coulomb-like potential in a new relativistic configuration representation first introduced in[4] reproduces, in the momentum representation, an interaction potential with the asymptotics (1.2). Solutions of quasipotential equations with the Coulomb potentials are known for a long time[8,9].

Earlier, solutions with the "chromodynamic" Coulomb potential were found only for the interaction of two spinless quarks and used in this model for calculations of the pion elastic form factor /9,10,7/ and decays $\pi^0 \to 2\gamma$ and $\pi^0 \to \mu\nu$ /11/. Our paper is organized as follows: In the second section, in the framework of the Hamiltonian formulation of QFT we obtain equations for the vertex and wave functions of a bound system of 2 particles with spin 1/2. In Sec.3 the equations for the wave function will be transformed to the relativistic configuration representation (RCR). In Sec.4 we solve equations with a potential with the "asymptotically free" asymptotics (1.2).

2. EQUATIONS FOR THE VERTEX AND WAVE FUNCTIONS IN THE HAMILTONIAN FORMULATION OF QFT

The Hamiltonian formulation of QFT is based on the equation for operator $R(\lambda\tau)$ /2/:

$$R(\lambda\tau) = -\mathcal{H}(\lambda\tau) - \int \mathcal{H}(\lambda\tau - \lambda\tau')\frac{d\tau'}{2\pi(\tau' - i\epsilon)} R(\lambda\tau'), \tag{2.1}$$

where $\mathcal{H}(\lambda\tau)$ is the Fourier transform of the Hamiltonian density, and the operator $R(\lambda\tau)$ at $\tau = 0$ is connected with the S-matrix by the relation $S = 1 + iR(0)$. The nonzero argument $\lambda\tau$, where λ^μ is a unit time-like 4-vector, defines quantities over energy-momentum shell.

We define the vertex function $\Gamma(k_1,k_2|\mathcal{P},\lambda\tau)$, following by the relation

$$\begin{aligned}<\vec{k}_1,\vec{k}_2|R(\lambda\tau)|\vec{\mathcal{P}},M,J> &= (2\pi)^4\delta^{(4)}(\mathcal{P}-k_1-k_2+\lambda\tau)\times\\ &\times(2k_{10}\cdot 2k_{20}\cdot 2\mathcal{P}_0)^{-1/2}\Gamma^J_{m_J}(k_1,k_2|\mathcal{P},\lambda\tau),\end{aligned} \tag{2.2}$$

where $\vec{\mathcal{P}}$, M and J , m_J, are respectively, the momentum, mass, spin of a composite particle, and its spin projection; k_1 and k_2 are momenta of constituents. Based on equation (2.1) and using the procedure for deriving the quasipotential equation /2,3,10/, we obtain the equation for the vertex function (indices J,m_J will be omitted)

$$\begin{aligned}\Gamma^{\alpha\beta}(k_1,k_2|\mathcal{P},\lambda\tau) &= \frac{1}{(2\pi)^3}\int d^4k_1' d^4k_2' d\tau' \times\\ V^{\alpha\beta}_{\gamma\delta}&(k_1,k_2;\lambda\tau|k_1',k_2';\lambda\tau')S^{(+)\gamma}_{\ \ \kappa}(k_1';-m)S^{(+)\delta}_{\ \ \theta}(k_2';m)\times\\ &\times\frac{1}{\tau'-i\epsilon}\Gamma^{\kappa\theta}(k_1',k_2'|\mathcal{P},\lambda\tau')\cdot\delta^{(4)}(\mathcal{P}-k_1'-k_2'+\lambda\tau).\end{aligned} \tag{2.3}$$

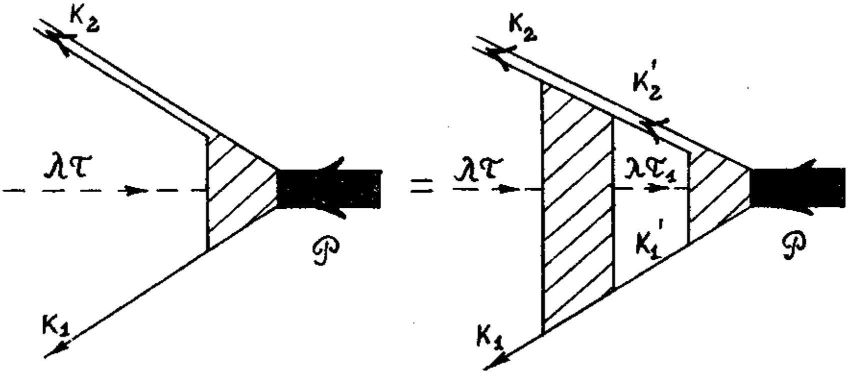

Fig.1. Graphical representation of an equation for the vertex function in Hamiltonian formulation of quantum field theory.

Here $S^{(+)}(k,m)$ are positive-frequency components of the spinor Green functions, and $\alpha,\beta,\ldots,\theta\,(=0,1,2,3)$ are bispinor indices. Equation (2.3) is an analog of the Edwards equation in the Feynman-Dyson formalism. Equation (2.3) in terms of spurion diagrams can be represented as is shown in Fig.1. A variety of diagrams denoted by the tetragon in Fig.1 consists of a set of irreducible (in the sense of cutting particle, antiparticle and spurion lines) graphs. A block of such diagrams denoted by V represents the integral kernel and plays the role of a potential in the Schrödinger equation. The same block of diagrams (or the same potential) is a kernel of the quasipotential equation for the scattering amplitude /1-5/. In what follows, for simplicity we shall assume that the quasipotential in (2.3) has a bispinor structure of the form $I \otimes I$*.

We will look for the solution of (2.3) with a given spinor structure (independent of momentum variables), i.e., represent the vertex function $\Gamma^{\alpha\beta}(k_1,k_2|\mathcal{P},\lambda\tau)$ in the form

$$\Gamma^{\alpha\beta}(k_1,k_2|\mathcal{P},\lambda\tau) = \Gamma(k_1,k_2|\mathcal{P},\lambda\tau)\hat{O}^{\alpha\beta} \quad , \tag{2.4}$$

where for matrix $\hat{O}$ it is assumed that $\mathrm{Sp}\hat{O}^2 \neq 0$. Our model consists in that in addition to the usually assumed simple

*Results obtained for the case of the structure of V of the form $\gamma_5 \otimes \gamma_5$ and $\gamma_\mu \otimes \gamma^\mu$ will be given in subsequent publications. However, in the interesting case of interaction $\gamma_\mu \otimes \gamma^\mu$ the scalar term is decisive /12/.

factorization of spin and orbital parts of the vertex function we also assume $\Gamma^{\alpha\beta}$ to be proportional only to one $\hat{O}^{\alpha\beta}$ matrix, which allows us to find exact solutions of the equation (see below), whereas the expansion of the vertex $\Gamma^{\alpha\beta}$ over the total system of γ_5, γ_μ, $\gamma_5\gamma_\mu$ and $\sigma_{\mu\nu}\mathcal{P}^\nu$ -matrices leads to a system of coupled equations. From (2.3) with (2.4) for the invariant vertex function we get the equation

$$\Gamma(k_1,k_2|\mathcal{P},\lambda\tau)=[\mathrm{Sp}\,\hat{O}^2]^{-1}\frac{1}{(2\pi)^3}\int\frac{d\tau'}{\tau'-i\epsilon}\cdot\frac{d\vec{k}'_1}{2\sqrt{m^2+\vec{k}'^2_1}}\cdot\frac{d\vec{k}'_2}{2\sqrt{m^2+\vec{k}'^2_2}}\times \tag{2.5}$$
$$\times V(k_1,k_2;\lambda\tau|k'_1,k'_2;\lambda\tau')\,\mathrm{Sp}\,[\hat{O}(k'_1-m)\hat{O}(k'_2+m)]$$
$$\times\Gamma(k'_1,k'_2|\mathcal{P},\lambda\tau')\cdot\delta^{(4)}(\mathcal{P}-k'_1-k'_2+\lambda\tau').$$

Note that the integration in the momentum space in eq. (2.5) runs over the upper mass-shell hyperboloid

$$k_0^2-\vec{k}^2=m^2 \tag{2.6}$$

with the invariant measure $d\Omega_k=d^3\vec{k}/2\sqrt{m^2+\vec{k}^2}$.

Let us choose in (2.5) the 4-vector λ^μ directed along the momentum of a composite particle $\lambda^\mu=\mathcal{P}^\mu/\sqrt{\mathcal{P}^2}=\mathcal{P}^\mu/M\equiv\lambda^\mu_{\mathcal{P}}$. We make use of the invariance of δ-function in (2.5) in order to realize by means of the Lorentz boost $\Lambda^{-1}_\lambda(\Lambda_\lambda(1,\vec{0})=\lambda^\mu)$ the transition, like in /10/ to the description in terms of the vector $\Delta^\mu_{k,m\lambda_{\mathcal{P}}}$ /13/ ($\vec{\Delta}_{k,m\lambda_{\mathcal{P}}}=\vec{k}(-)m\vec{\lambda}_{\mathcal{P}}$ by notation of ref. /14/)

$$\vec{\Delta}_{k,m\lambda_{\mathcal{P}}}=(\overrightarrow{\Lambda^{-1}_{\lambda_{\mathcal{P}}}k})=\vec{k}-\vec{\lambda}_{\mathcal{P}}[k_0-\frac{\vec{k}\,\vec{\lambda}_{\mathcal{P}}}{1+\lambda^0_{\mathcal{P}}}], \tag{2.7}$$

$$\Delta^0_{k,m\lambda_{\mathcal{P}}}=(\Lambda^{-1}_{\lambda_{\mathcal{P}}}k)^0=\sqrt{m^2+\vec{\Delta}^2_{k,m\lambda_{\mathcal{P}}}}=k_0\lambda^0_{\mathcal{P}}-\vec{k}\vec{\lambda}_{\mathcal{P}}, \tag{2.8}$$

$$\delta^{(4)}(\mathcal{P}+\lambda\tau'-k'_1-k'_2)=\delta(M+\tau'-\Delta^0_{k'_1,m\lambda_{\mathcal{P}}}-\Delta^0_{k'_2,m\lambda_{\mathcal{P}}})\cdot\delta(\vec{\Delta}_{k'_1,m\lambda_{\mathcal{P}}}+\vec{\Delta}_{k'_2,m\lambda_{\mathcal{P}}}). \tag{2.9}$$

Equation (2.5) is then rewritten in the form

$$\Gamma(\Delta^0_{k,m\lambda})=\frac{[\mathrm{Sp}\,\hat{O}^2]^{-1}}{(2\pi)^3}\int\frac{d^3\vec{\Delta}_{k',m\lambda}}{2\sqrt{m^2+\vec{\Delta}^2_{k',m\lambda}}}\times \tag{2.10}$$
$$\times\frac{\mathrm{Sp}[\hat{O}(k'_1-m)\hat{O}(k'_2+m)]}{2\Delta^0_{k',m\lambda}[M-2\Delta^0_{k',m\lambda}+i\epsilon]}V(\vec{\Delta}_{k,m\lambda};\vec{\Delta}_{k',m\lambda})\Gamma(\Delta^0_{k',m\lambda})$$

with the notation* $\vec{\Delta}_{k,m\lambda} = \vec{\Delta}_{k_1,m\lambda\wp} = -\vec{\Delta}_{k_2,m\lambda\wp}$ and
$\Gamma(\Delta^{\circ}_{k,m\lambda}) = \Gamma(k_1,k_2|\mathcal{P},\lambda r)$

$V(\vec{\Delta}_{k,m\lambda};\vec{\Delta}_{k',m\lambda}) = V(k_1,k_2;\lambda r|k_1',k_2';\lambda r')$.

While choosing the matrix $\hat{O}^{\alpha\beta} = \gamma_5$, γ_μ , $\gamma_5\gamma_\mu$ we find

$$Sp[\gamma_5(k_1'-m)\gamma_5(k_2'+m)] = -4(\Delta_{k',m\lambda}\cdot\Delta_{k_2',m\lambda}+m^2) = -8(\Delta^{\circ}_{k',m\lambda})^2,$$

$$Sp[\gamma_\mu(k_1'-m)\gamma^\mu(k_1'+m)] = -4[2(\Delta^{\circ}_{k',m\lambda})^2 - m^2], \tag{2.11}$$

$$Sp[\gamma_5\gamma_\mu(k_1'-m)\gamma_5\gamma_\mu(k_2'+m)] = -4[3m^2 - 2(\Delta^{\circ}_{k',m\lambda})^2].$$

We define the wave function $\Psi(\Delta^{\circ}_{k,m\lambda})$ as follows [10]

$$\Psi(\Delta^{\circ}_{k,m\lambda}) = [2\Delta^{\circ}_{k,m\lambda}(M-2\Delta^{\circ}_{k,m\lambda})]^{-1}\cdot\Gamma(\Delta^{\circ}_{k,m\lambda}). \tag{2.12}$$

Then from (2.10) we obtain the following equation

$$\frac{2\Delta^{\circ}_{k,m\lambda}}{2m}[M-2\Delta^{\circ}_{k,m\lambda}]\Psi(\Delta^{\circ}_{k,m\lambda}) = \tag{2.13}$$

$$= (2\pi)^{-3}\int \frac{d\vec{\Delta}'_{k',m\lambda}}{\sqrt{1+\frac{\vec{\Delta}^2_{k',m\lambda}}{m^2}}}\cdot V(\vec{\Delta}_{k,m\lambda};\vec{\Delta}_{k',m\lambda})A(\Delta^{\circ}_{k',m\lambda})\Psi(\Delta^{\circ}_{k',m\lambda}),$$

where

$$A(\Delta^{\circ}_{k',m\lambda}) = m^{-2}(\Delta^{\circ}_{k',m\lambda})^2 \quad \text{for} \quad \hat{O} = \gamma_5 \tag{2.14a}$$

$$A(\Delta^{\circ}_{k',m\lambda}) = m^{-2}[2(\Delta^{\circ}_{k',m\lambda})^2 - m^2] \quad \text{for} \quad \hat{O} = \gamma_\mu \tag{2.14b}$$

$$A(\Delta^{\circ}_{k',m\lambda}) = m^{-2}[3m^2 - 2(\Delta^{\circ}_{k',m\lambda})^2] \quad \text{for} \quad \hat{O} = \gamma_5\gamma_\mu. \tag{2.14c}$$

It is not difficult to see that in the nonrelativistic limit equation (2.13) for all $A(\Delta^{\circ}_{k',m\lambda})$ given by (2.14) transforms into the Schrödinger equation in the momentum representation.

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* The fact of dependence of $\Gamma(k_1,k_2|\mathcal{P},\lambda r)$ only on one variable $\Delta^{\circ}_{k,m\lambda\wp}$ is proved in [10], and the quasipotential dependence on the vector $\vec{\Delta}_{k,m\lambda\wp}$ $(-)\vec{\Delta}_{k',m\lambda\wp}$ (when constructing V out of Feynman matrix elements) is proved in ref. [12].

3. TRANSITION TO THE RELATIVISTIC CONFIGURATION REPRESENTATION

Relativistic configuration representation (RCR) has been first introduced in/4/. The RCR for the wave function has the form

$$\Psi(\vec{\Delta}_{p,m\lambda}) = \int d\vec{r}\, \xi^*(\vec{\Delta}_{p,m\lambda};\vec{r})\Psi(\vec{r}) \tag{3.1a}$$

$$\Psi(\vec{r}) = (2\pi)^{-3} \int \frac{d\vec{\Delta}_{p,m\lambda}}{\sqrt{m^2+\vec{\Delta}^2_{p,m\lambda}}} \xi(\vec{\Delta}_{p,m\lambda};\vec{r})\Psi(\vec{\Delta}_{p,m\lambda}), \tag{3.1b}$$

where the functions/15/

$$\xi(\vec{\Delta}_{p,m\lambda};\vec{r}) = \left[\frac{\Delta^{\mu}_{p,m\lambda}\cdot n_\mu}{m}\right]^{-1-irm} \tag{3.2}$$

$$n_\mu n^\mu = 0,\quad n_\mu = (1,\vec{n});\quad \vec{n}^2 = 1,\quad \vec{r} = r\vec{n};\quad 0 \le r < \infty$$

realize unitary irreducible representations of the Lorentz group - group of motions of the mass hyperboloid (2.6). The invariant parameter r in (3.2) numerates eigenvalues of the Casimir operator of the Lorentz group and plays the role of a relativistic analog of the modulus of the relative coordinate/4/ in the Schrödinger equation. In the nonrelativistic limit $\xi(\vec{p},\vec{r}) \to e^{i\vec{p}\vec{r}}$.

In ref./4/ it has been shown that the operator

$$\hat{H}_0 = m\cosh\left(\frac{i}{m}\frac{\partial}{\partial r}\right) + \frac{i}{r}\sinh\left(\frac{i}{m}\frac{\partial}{\partial r}\right) - \frac{\Delta_{\theta,\phi}}{2mr^2} e^{\frac{i}{m}\frac{\partial}{\partial r}} \tag{3.3}$$

plays the role of the free Hamiltonian ($\Delta_{\theta,\phi}$ is the Laplace operator on the sphere)

$$\hat{H}_0\, \xi(\vec{\Delta},\vec{r}) = \Delta^\circ \xi(\vec{\Delta},\vec{r}). \tag{3.4}$$

We shall assume that the potential $V(\vec{\Delta}_{k,m\lambda};\vec{\Delta}_{k',m\lambda})$ in (2.13) is local in the Lobachevsky space realized on the upper sheet of the mass hyperboloid (2.6), i.e., $V(\vec{\Delta}_{k,m\lambda};\vec{\Delta}_{k',m\lambda}) = V(\vec{\Delta}_{k,m\lambda}(-)\vec{\Delta}_{k',m\lambda})$. In this case the r.h.s. of equation (2.13) is a convolution of functions in the Lobachevsky space, and the equation itself in the RCR becomes local

$$\frac{2\hat{H}_0}{2m}[M - 2\hat{H}_0]\,\Psi(\vec{r}) = V(\vec{r})\hat{A}\Psi(\vec{r}), \tag{3.5}$$

where

$$\hat{A} = m^{-2}\hat{H}_0^2 \; ; \quad \text{for } \hat{O} = \gamma_5 , \tag{3.6a}$$

$$\hat{A} = m^{-2}(2\hat{H}_0^2 - m^2); \quad \text{for } \hat{O} = \gamma_\mu , \tag{3.6b}$$

$$\hat{A} = m^{-2}(3m^2 - 2\hat{H}_0^2); \quad \text{for } \hat{O} = \gamma_5\gamma_\mu . \tag{3.6c}$$

It what follows we shall be interested in the spherically symmetric potential $V(\vec{r}) = V(r)$. It is convenient to introduce a new function $\phi(r)$ connected with $\Psi(r)$ by

$$\Psi(r) = r^{-1}\Phi(r). \tag{3.7}$$

For the zero orbital moment $\ell=0$ eq. (3.5) is rewritten to the form

$$\frac{2\hat{H}_0^{rad}}{2m}[M - 2\hat{H}_0^{rad}]\Phi(r) = V(r)\hat{A}\Phi(r), \tag{3.8}$$

where

$$\hat{A} = m^{-2}(\hat{H}_0^{rad})^2 \; ; \qquad \text{for } \hat{O} = \gamma_5 , \tag{3.9a}$$

$$\hat{A} = m^{-2}[2(\hat{H}_0^{rad})^2 - m^2]; \quad \text{for } \hat{O} = \gamma_\mu , \tag{3.9b}$$

$$\hat{A} = m^{-2}[3m^2 - 2(\hat{H}_0^{rad})^2]; \quad \text{for } \hat{O} = \gamma_5\gamma_\mu , \tag{3.9c}$$

$$\hat{H}_0^{rad} = m\cosh\left(\frac{i}{m}\frac{\partial}{\partial r}\right). \tag{3.10}$$

Our task is to solve equations (3.8) for the Coulomb interaction

$$V(r) = -\frac{\alpha_0}{r} \tag{3.11}$$

whose transform in the momentum representation /7/ $\left(\chi = \mathrm{Ar}\cosh\left(1 + \frac{Q^2}{2m^2}\right)\right)$

$$V(Q^2) = \int d\vec{r}\,\xi(\vec{\Delta}_{p,k};\vec{r})\,V(r) =$$

$$= 4\pi\int_0^\infty \frac{\sin rm\chi}{rm\sinh\chi}\left(-\frac{\alpha_0}{r}\right)r^2 dr = -\frac{\alpha_0}{\chi\cdot\sinh\chi} \underset{Q^2 \gg m^2}{\cong} -\frac{\alpha_0}{Q^2\cdot\ln\frac{Q^2}{m^2}} ;$$

has the same asymptotics at large Q^2 as (1.2).

4. SOLUTION OF QUASIPOTENTIAL EQUATIONS FOR $\ell = 0$

At first, consider the case $\hat{O}=\gamma_5$. It is convenient to replace $(\hat{H}_0^{rad}/m)\Phi(r))$ by a new unknown function denoted also by $\tilde{\Phi}(r)$. For the new function equation (3.8) takes on the form

$$[M-2\hat{H}_0^{rad}]\tilde{\Phi}(r) = V(r)\cdot \frac{\hat{H}_0^{rad}}{m}\cdot\tilde{\Phi}(r). \qquad (4.1)$$

We will write eq. (4.1) for the Coulomb attraction potential (3.11), with allowing for the explicit form of the operator $\hat{H}_0^{rad}$ (3.10), in the following form

$$2mr[\cos x-\cosh(\frac{i}{m}\frac{\partial}{\partial r})]\tilde{\Phi}(r) = -\alpha_0\cosh(\frac{i}{m}\frac{\partial}{\partial r})\tilde{\Phi}(r) \qquad (4.2)$$

$$M = 2m\cos x. \qquad (4.3)$$

It is known that solutions to finite-difference equations are convenient to seek by using the Laplace transformation method (see, e.g., ref./17/). This method was applied in ref./18/ to eq. (3.8) with $\hat{A}=\hat{H}_0/m$ (spinless case) for the linearly growing confining potential*.

Let us represent the solution, we look for, in the form of the Laplace contour integral

$$\tilde{\Phi}(r) = \int_\alpha^\beta dy\, e^{-mry} f(y). \qquad (4.4)$$

Inserting (4.4) into (4.2) we get the differential equation for function $f(y)$

$$\frac{d}{dy}[(\cos x-\cos y)f(y)] = -\frac{\alpha_0}{2}\cos y\cdot f(y). \qquad (4.5)$$

We choose the limits of integration α and β so as to fulfil the relation

$$\exp(-mry)[\cos x-\cos y]\cdot f(y)\Big/_\alpha^\beta = 0. \qquad (4.6)$$

As a result, $\tilde{\Phi}(r)$ takes the form

$$\tilde{\Phi}(r) = c\cdot\int_\gamma dy\cdot\exp[-mry+\frac{\alpha_0 y}{2}]\cdot[\sin\frac{y+x}{2}]^{\frac{\alpha_0}{2}\frac{\cos x}{\sin x}-1}\times$$
$$\times[\sin\frac{y-x}{2}]^{-\frac{\alpha_0}{2}\frac{\cos x}{\sin x}-1}, \qquad (4.7)$$

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*For the first time the Laplace transformation method has been applied to a quasipotential equation with Hamiltonian (3.3) by V.M.Vinogradov.

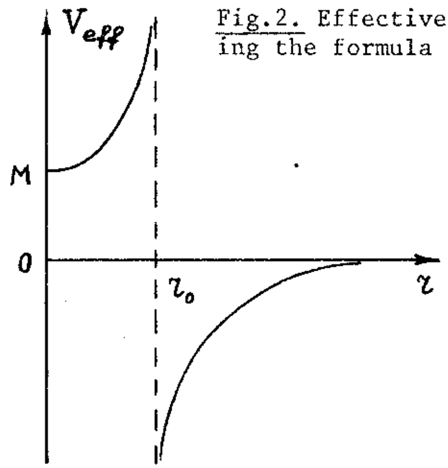

Fig.2. Effective potential, calculated according the formula (4.12).

and the quantization of energy levels is defined by the condition

$$\frac{\alpha_0}{2}\cdot\frac{\cos x}{\sin x}=n;\quad n=1,2,3\ldots \qquad (4.8)$$

For obtaining nontrivial solutions the contour γ in (4.7) should encircle a singular point of the integrand $y=x$, which is in virtue of the condition of quantization (4.8) a pole of $n+1$ order. The contour integral in (4.7) can be calculated with the use of the theory of residues. So, for $n=1$ the wave function $\tilde{\Phi}(r)$ has the form

$$\tilde{\Phi}_{n=1}(r)=c\left(r-\frac{\alpha_0}{2m}\right)\exp(-rmx) \qquad (4.9)$$
$$x=\arccos\frac{M}{2m}.$$

It is an interesting fact that in our model, when the interaction does not depend on spins, the ground-state wave function (4.9) of the pion has zero at a finite distance. This result does not seem so surprising if we rewrite the equation for $\tilde{\Phi}(r)$ with $\ell=0$

$$[M-2\hat{H}_0^{rad}]\,\tilde{\Phi}(r)=V(r)\cdot\frac{\hat{H}_0^{rad}}{m}\tilde{\Phi}(r) \qquad (4.10)$$

in the form customary from the point of view of disposition of kinetic and potential terms /4,8,16/

$$[M-2\hat{H}_0^{rad}]\tilde{\Phi}(r)=V_{eff}(r)\tilde{\Phi}(r) \qquad (4.11)$$

$$V_{eff}(r)=V(r)M[2m+V(r)]^{-1}=-\alpha_0\frac{M}{2m}\left[r-\frac{\alpha_0}{2m}\right]^{-1}. \qquad (4.12)$$

The behaviour of the effective interaction for the Coulomb potential is shown in Fig.2.

Thus, the effective potential is a discontinuous function with a singularity at a finite distance*, and the point of discontinuity coincides with zero of the ground-state wave function.

Equation (4.11) with the potential (4.12) can be explicitly solved without the use of the Laplace method. Indeed, using the parametrization $M=2m\cos x$ we rewrite eq. (4.11) in the form

$$[\cos x-\cosh(\frac{i}{m}\frac{\partial}{\partial r})]\tilde{\Phi}(r)=\cos x\cdot\frac{-\alpha_0/2}{mr-\alpha_0/2}\cdot\tilde{\Phi}(r)$$

or passing to the new variable $\rho=mr-\alpha_0/2$,

$$[\cos x-\cosh(\frac{i}{m}\frac{\partial}{\partial r})]\tilde{\Phi}(\rho)=\cos x\cdot\frac{-\alpha_0/2}{\rho}\cdot\tilde{\Phi}(\rho).$$

This form of the equation is similar to the form of the usual (for the scalar theory /8/) equation with the Coulomn potential $1/\rho$ but with the coupling constant $\alpha'=\alpha_0/2\cdot\cos x$ dependent on the eigenvalues

$$\Phi(\rho)=c_1\cdot\rho\cdot\exp(-\rho x)\cdot{}_2F_1(1-\frac{\alpha_0\cos x}{2\sin x};\,1-i\rho;\,2;\,2i\sin x\cdot\exp(-ix))=$$

$$=c_1(mr-\frac{\alpha_0}{2})\exp(-mrx). \tag{4.13}$$

$$\times\exp(-\alpha_0 x/2)\;{}_2F_1(1-\frac{\alpha_0}{2}\frac{\cos x}{\sin x};\,1+\frac{i\alpha_0}{2}-imr;\,2;\,2i\sin x\cdot\exp(-ix)).$$

Requiring that the hypergeometric function ${}_2F_1$ in (4.13) degenerate into a polynomial, we arrive again at the quantization condition (4.8). It is easy to see that at $n=1$ from (4.13) the ground-state wave function (4.9) follows up to normalizing factor c_1.

Consider now the general form of quasipotential equations arising in the given approach

$$\frac{\hat{H}_0}{m}[M-2\hat{H}_0]\Psi(r)=V(r)[a(\frac{\hat{H}_0}{m})^2+b]\Psi(r). \tag{4.14}$$

Numbers a and b in (4.14) satisfy the condition $a+b=1$, under which the quasipotential is normalized to coincide in a nonrelativistic limit with the corresponding nonrelativistic potential. Recall that for $\hat{O}=\gamma_\mu$, $a=\frac{2}{3}$, $b=\frac{1}{3}$ and for $\hat{O}=\gamma_5\gamma_\mu$ $a=-2$, $b=3$.

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* Dynamical models with discontinuous potentials were considered in ref./19/.

From (4.14) for the quasipotential given by (3.11) we obtain the radial wave function in the form of the Laplace contour integral

$$\Phi(r) = c \int_{\gamma} dy \exp[-y(rm - a\frac{a_0}{2})] \times [\sin(\frac{y+x}{2})]^{n-1} \cdot [\sin(\frac{y-x}{2})]^{-n-1} \times$$
$$\times [\sin \frac{\pi/2+y}{2}]^{-\frac{a_0}{2}\cdot\frac{b}{\cos x}-1} \cdot [\sin \frac{\pi/2-y}{2}]^{\frac{a_0}{2}\cdot\frac{b}{\cos x}-1}, \tag{4.15}$$

where the integer n is defined by the quantization condition

$$\frac{a_0}{\sin 2x}[1 - a \sin^2 x] = n; \quad n = 1, 2, 3.. \tag{4.16}$$

From the general expression it is easy to obtain the ground-state wave function

$$\Phi_{n=1}(r) = c(rm - a_0 a) \exp[-mx] . \tag{4.17}$$

So, the ground-state wave function has zero at a finite distance at positive a (in the considered examples these cases are $\hat{O} = \gamma_5$ and $\hat{O} = \gamma_\mu$) and has no zero at negative a (the case $\hat{O} = \gamma_5 \gamma_\mu$).

5. CONCLUSION

The following results are obtained in this work:

1) The equation for the vertex function for a system of two spin 1/2 particles in the cases when the spin structure of this function has the form $\Gamma_{\alpha\beta} = \Gamma O_{\alpha\beta}$, where Γ is the scalar part, and $O = \gamma_5$, $\gamma_5 \gamma_\mu$, γ_μ ;

2) By the Fourier analysis on the Lorentz group we have found for the scalar "chromodynamical" potential exact solutions of the relativistic two-particle equations for the above spin structures;

3) It is shown that within the given model the π-meson wave function has zero at a finite distance corresponding to the point of discontinuity of the effective potential;

4) The results have the invariant character as it follows from results of refs.[9,10].

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