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SUPERSYMMETRIC QUASIPOTENTIAL EQUATIONS.

II. Supersymmetric Generalization of the Equations on the Light-Front

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1. INTRODUCTION

In the previous paper [1] (referred to as I) it has been shown that the three-dimensional two-particle relativistic equation of Logunov-Tavkhelidze [2] can be generalized also to the supersymmetric case. Remember that the equation of Logunov-Tavkhelidze can be found from the Bethe-Salpeter equation for vanishing the nonphysical relative-time in the center-of-mass system. Because of noninvariance with respect to the supertransformations of the "equal-time" operation, this is made in a fixed reference frame in the superspace (see I). As it is known, there exist also other methods for finding the three-dimensional two-particle relativistic equation, for instance: vanishing the relative energy in the center-of-mass system (Markov-Yukawa contion) [3-5], or one of the variables of the light-front [6-9], as well as on a hyperboloid in the momentum space [10].

The object of the present paper is to obtain the supersymmetric two-particle equation on the light-front. For simplicity the explicit form of these equations is given only for simple chiral superfields. However, without difficulties, these equations can be written also for the case of extended supersymmetry. From theories of such a kind of a special interest is the supersymmetric N=4 Yang-Mills theory [11,12], as a possible theory with the "confinement". As is well known, the quasipotential approach is a natural scheme for solving the bound-state problem. Moreover, the quasipotential equations on the light-front are a convenient tool for investigation of the form-factors and other high energy processes.

2. SUPERSYMMETRIC "TWO-TIME" GREEN FUNCTION ON THE LIGHT-FRONT

Consider a massless chiral superfields $\Phi^{+}(x,\theta)$ and its hermitian conjugated field $\Phi^{-}(x,\bar{\theta})$ [13] (see also [1]). Here the notation of paper [1] is used. Recall, that the two-component spinor formalism is used. As in paper [1] every possible four-point Green functions for the chiral superfields can be combined in the following matrix

$$
G = \begin{bmatrix} G^{++,++} & G^{++,-+} & G^{++,-+} & G^{++,--} \\ G^{-+,++} & G^{-+,-+} & G^{-+,+-} & G^{-+,--} \\ G^{+-,++} & G^{+-,-+} & G^{+-,+-} & G^{+-,--} \\ G^{--,++} & G^{--,-+} & G^{--,+-} & G^{--,--} \end{bmatrix}, \qquad (2.1)
$$

where

$$
G^{a,\beta,\gamma,\delta} = \langle 0 | T(\Phi^{a}(x_1, \theta_1)\Phi^{\beta}(x_2, \theta_2)\Phi^{\gamma}(x_3, \theta_3)\Phi^{\delta}(x_4, \theta_4) \qquad (2.2)
$$

$$
(a, \beta, \gamma, \delta = + -)
$$

are the four-point Green functions for chiral superfields. In an analogous way the two-particle B-S amplitude can be represented also in the matrix form

$$
\Psi_{p\zeta} = \begin{bmatrix} \Psi^{++}_{p\zeta}(q, \theta_1, \theta_2) \\ \Psi^{-+}_{p\zeta}(q, \bar{\theta}_1, \theta_2) \\ \Psi^{+-}_{p\zeta}(q, \theta_1, \bar{\theta}_2) \\ \Psi^{--}_{p\zeta}(q, \bar{\theta}_1, \bar{\theta}_2) \end{bmatrix}, \qquad (2.3)
$$

where

$$
\Psi^{a,\beta}_{p,\zeta} = \langle 0 | T(\Phi^{a}(x_1, \theta_1)\Phi^{\beta}(x_2, \theta_2)) | p, \zeta \rangle, \quad (a, \beta = + -) \qquad (2.4)
$$

is the B-S amplitude for chiral superfields.

Transition to the three-dimensional formalism for the Green function (2.1) and B-S amplitude (2.3) is achieved by making the relative coordinate on the light-front (see [8]) to vanish

$$
x_{+}^{12} = \frac{1}{2}(x_0^{12} + x_3^{12}) = 0, \qquad (2.5)
$$

where

$$
x_{12} = x_1 - x_2 .
$$

As has been pointed out in [1], the condition (2.5) is not invariant with respect to supertransformations. Therefore the condition (2.5) is applied to the Green function (2.1) or to the B-S amplitude (2.3) in a fixed reference frame on light-front in the superspace, i.e., for which the parameters of the supertransformations are zero. This reference frame is called the supersystem on the light-front. In an arbitrary reference frame the condition (2.5) can be written in an invariant form

$$n^{\mu}\,(x_{\mu}^{12}+i\bar{\epsilon}\gamma_{\mu}\,\theta^{12}) = 0, \tag{2.6}$$

where $x_{12}=x_1-x_2$, $\theta_{12}=\theta_1-\theta_2$, n ($n^2=0$) is a light-like four-vector, components of which in the supersystem on the light-front are given by

$n=(1,0,0,-1)$

and ϵ is an anticommuting spinor parameter of the supertransformations.

Then, in an arbitrary reference frame the "equal-time" operation is given by:

$$\bar{\Psi} = \int dx_{+}^{12}\,\delta[n^{\mu}\,(x_{\mu}^{12}+i\bar{\epsilon}\gamma_{\mu}\,\theta^{12})\Psi(X,x_{12},\theta_1,\theta_2), \tag{2.7}$$

$$\bar{G} = \int dx_{+}^{12}dx_{+}^{34}\,\delta[n^{\mu}\,(x_{\mu}^{12}+i\bar{\epsilon}\gamma_{\mu}\theta_{12})]\,\delta[n^{\nu}\,(x_{\nu}^{34}+i\epsilon\gamma_{\nu}\theta_{34})]\times$$

$$\times\, G(X,Y,x_{12},x_{34},\theta_1,\ldots,\theta_4), \tag{2.8}$$

where

$$X=\frac{1}{2}(x_1+x_2),\quad Y=\frac{1}{2}(x_3+x_4)$$

are the corresponding center-of-mass coordinates for the equal mass case. In the momentum space from (2.7) and (2.8) in the supersystem on the light-front, we have

$$\bar{\Psi}_p(\underline{q},\theta_1,\theta_2) = \int_{-\infty}^{\infty} dq_{-}\Psi_p\,(q,\,\theta_1,\theta_2) \tag{2.9}$$

and

$$\bar{G}(p,\underline{q},\underline{q}') = \int dq_{-}dq'_{-}\,G(p,q,q',\theta_1,\ldots,\theta_4), \tag{2.10}$$

where

$$p=p_1+p_2=p_3+p_4=(p_{\pm},0),\quad q=\frac{1}{2}(p_1-p_2),\quad q'=\frac{1}{2}(p_3-p_4),$$

$$\underline{q}=(q_{+},q_{\perp}),\quad q_{\pm}=\frac{1}{2}(q_0\pm q_3),\quad q_{\perp}=(q_1,q_2).$$

Then the quasipotential V is determined from the equation

$$\bar{G}^{-1}=\bar{G}_0^{-1}-\frac{1}{2\pi i}V, \tag{2.11}$$

where $\overline{G}_0$ is the "two-time" disconnected two-particle Green function.

3. SUPERSYMMETRIC QUASIPOTENTIAL EQUATION ON THE LIGHT-FRONT FOR MASSLESS CHIRAL SUPERFIELDS

In the free case the disconnected Green function is

$$G_0^{\alpha\beta,\gamma\delta} = D_0^{\alpha\gamma}(x_1 - x_3; \theta_1, \theta_3) D_0^{\beta\delta}(x_2 - x_4; \theta_2, \theta_4), \quad (\alpha, \beta, \gamma, \delta = +,-), \tag{3.1}$$

where

$$\begin{aligned} D_0^{++}(p, \theta_1, \theta_2) &= m\delta^{\Gamma}(\theta_1 - \theta_3)(p^2 - m^2 + i\epsilon)^{-1}, \\ D_0^{+-}(p, \theta_1, \theta_3) &= \frac{1}{2}\exp(2\theta_1 \underset{\sim}{p} \bar{\theta}_3)(p^2 - m^2 + i\epsilon)^{-1}, \end{aligned} \tag{3.2}$$

are propagators for the free chiral superfields. Here $\underset{\sim}{p} = p^{\mu}\sigma_{\mu}$, $\sigma_0 = I$ is the identity matrix and σ_j $(j = 1,2,3)$ are the Pauli matrices. It is evident that in the massless case $D^{++} = D^{--} = 0$ and, consequently, the corresponding four-point disconnected Green function in the free fields case has the simple form, i.e.,

$$G_0 = \begin{bmatrix} 0 & 0 & 0 & G_0^{++,--} \\ 0 & 0 & G_0^{-+,+-} & 0 \\ 0 & G_0^{+-,-+} & 0 & 0 \\ G_0^{--,++} & 0 & 0 & 0 \end{bmatrix} \tag{3.3}$$

For this reason, consider first the massless case. Substituting (3.3) into (2.10) and taking into account (3.2) for $m = 0$ we have

$$\begin{aligned} \overline{G}_0^{++,--} = \frac{1}{4}\exp(\theta_1 \underset{\sim}{P} \bar{\theta}_3 + \theta_2 \underset{\sim}{Q} \bar{\theta}_4)[\overline{J}_0 + 2(\theta_1 \underset{\sim}{\sigma}_+ \bar{\theta}_3 + \theta_2 \underset{\sim}{\sigma}_+ \theta_4)\overline{J}_1 + \\ + 2(\theta_1 \underset{\sim}{\sigma}_+ \bar{\theta}_3 + \theta_2 \underset{\sim}{\sigma}_+ \bar{\theta}_4)^2 \overline{J}_2], \end{aligned}$$

$$\overline{G}^{-+,+-} = \frac{1}{4}\exp(\bar\theta_1 \tilde{P}\theta_3 + \theta_2 \underset{\sim}{Q}\bar\theta_4)[J_0 + 2(\bar\theta_1\tilde\sigma_+\theta_3 + \theta_2\underset{\sim}{\sigma}_+\bar\theta_4)J_1 + + 2(\bar\theta_1\underset{\sim}{\sigma}_+\theta_3 + \theta_2\underset{\sim}{\sigma}_+\bar\theta_4)^2 J_2], \tag{3.4}$$

with the following notation

$$P(p,\underset{\sim}{q}) = (p_+ + 2q_+,\ p_-,\ p_\perp + 2q_\perp),$$

$$Q(p,\underset{\sim}{q}) = P(p,-\underset{\sim}{q}) = (p_+ - 2q_+,\ p_-,\ p_\perp - 2q_\perp), \tag{3.5}$$

$$\tilde{P} = P^\mu \tilde\sigma_\mu, \quad \tilde\sigma_\mu = \epsilon^{-1}\underset{\sim}{\sigma}^+_\mu\epsilon, \quad \epsilon = i\sigma_2,$$

and

$$\overline{J}_k = \int_{-\infty}^{\infty} dq_- q_-^k [p^2 + 2pq + q^2 - i\epsilon]^{-1} [p^2 - 2pq + q^2 + i\epsilon]^{-1}. \tag{3.6}$$

In view of that $q^2 = 2q_+q_- - q_\perp^2$ it is evident that the integrals $\overline{J}_k (k=1,2)$ are divergent. The integral J_0 is the corresponding "two-time" free two-particle Green function on light-front for scalar particles [6]. On the massless case J_0 is given by

$$\overline{J}_0 = \Theta(x)\Theta(1-x)\tilde{J}_0, \tag{3.7}$$

where

$$\tilde{J}_0(p,q_+,q_\perp,q'_+,q'_\perp) = \frac{4\pi i\,\delta(q_+ - q'_+)\delta^{(2)}(q_\perp - q'_\perp)}{p_+[x(1-x)p^2 - q_\perp^2]} = \tilde{J}_0(p,q_+,q_\perp)\delta(q_+ - q'_+)\delta^{(2)}(q_\perp - q'_\perp). \tag{3.8}$$

Variable x in (3.7) and (3.8) denotes

$$x = 1/2 + q_+/p_+. \tag{3.9}$$

Because of the Θ-functions in (3.7), $\overline{G}_0$ is nonvanishing only in the interval

$$0 < x < 1 \quad \text{or} \quad -p_+/2 < q_+ < p_+/2. \tag{3.10}$$

Following paper [9] the terms in the "two-time" Green finction (3.4) containing divergent integrals cancel out by the projection operators. To be found such projection operators point out that the structures, which coefficients are divergent integrals $\overline{J}_k (k=1,2)$, have the following form:

$$\theta_1 \underset{\sim}{\sigma}_+ \bar{\theta}_3 , \quad \theta_2 \underset{\sim}{\sigma}_+ \bar{\theta}_4 , \quad \bar{\theta}_1 \tilde{\sigma}_+ \theta_3 , \quad \bar{\theta}_2 \tilde{\sigma}_+ \theta_4 \tag{3.11}$$

and their products. Here

$$\underset{\sim}{\sigma}_+ = \frac{1}{2}(\underset{\sim}{\sigma}_0 + \underset{\sim}{\sigma}_3) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \tilde{\sigma}_+ = \frac{1}{2}(\tilde{\sigma}_0 + \tilde{\sigma}_3) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

and, consequently,

$$\theta_1 \sigma_+ \bar{\theta}_3 = (\theta_1)_1 (\bar{\theta}_3)_{\dot{1}} , \quad \bar{\theta}_1 \tilde{\sigma}_+ \theta_3 = (\bar{\theta}_1)^{\dot{2}} (\theta_3)^2 = (\bar{\theta}_1)_{\dot{1}} (\theta_3)_1 .$$

Then, using the anticommutation (nilpotent) properties of θ, i.e., $((\theta_j)_1)^2=0$, the following projection operators are introduced

$$\pi = \begin{bmatrix} (\tilde{\sigma}_+ \theta_1)(\tilde{\sigma}_+ \theta_2) & & & 0 \\ & (\underset{\sim}{\sigma}_+ \bar{\theta}_1)(\tilde{\sigma}_+ \theta_2) & & \\ & & (\tilde{\sigma}_+ \theta_1)(\underset{\sim}{\sigma}_+ \bar{\theta}_2) & \\ 0 & & & (\underset{\sim}{\sigma}_+ \bar{\theta}_1)(\underset{\sim}{\sigma}_+ \bar{\theta}_2) \end{bmatrix} \tag{3.12}$$

which cancel the structures of divergent integrals J_k $(k=1,2)$ in the "two-time" Green function (3.4). The projected wave function is denoted by

$$\underset{\smile}{\Psi}_p (q, \theta_2^1, \theta_2^2) = \int [d(\theta_1)_1][d(\theta_2)_1] \, \pi \overline{\Psi}_p (q, \theta_1, \theta_2) , \tag{3.13}$$

where $[d\theta_3]$ denoted integration over $\bar{\theta}_{\dot{1}}$ or θ_1 for − or + components of the wave function, respectively. Consequently, the wave function depends only on $\bar{\theta}_{\dot{1}}$ and θ_1 components of the anticommuting spinor variables θ. From (3.4) for the matrix elements of the "two-time" Green function after projection

$$\underset{\smile}{G}_0 = \int [d(\theta_1)_1] \ldots [d(\theta_4)_1] \, \pi \overline{G}_0 \, \pi \tag{3.14}$$

we have

$$\begin{aligned} \underset{\smile}{G}_0^{++,--} &= \frac{1}{4} \exp\{P_+ \theta_1 \underset{\sim}{\sigma}_- \bar{\theta}_3 + Q_+ \theta_2 \underset{\sim}{\sigma}_- \bar{\theta}_4\} \overline{J}_0 , \\ G_0^{-+,+-} &= \frac{1}{4} \exp\{P_+ \bar{\theta}_1 \tilde{\sigma}_- \theta_3 + Q_+ \theta_2 \sigma_- \bar{\theta}_4\} \overline{J}_0 . \end{aligned} \tag{3.15}$$

The inverse Green function G_0^{-1} can be found from the following condition

$$\int [d(\theta_3')_i][d(\theta_4')_1] \int_{-p_+/2}^{p_+/2} dq_+'' \int d^2 q_\perp'' \underset{\smile}{G}_0(p,\underline{q},\underline{q}',\theta_1,\theta_2,\theta_3',\theta_4') \times \tag{3.16}$$
$$\times \underset{\smile}{G}_0^{-1}(p,\underline{q}'',\underline{q}',\theta_3',\theta_4',\theta_3,\theta_4) = \delta(q_+ - q_+')\delta^{(2)}(q_\perp - q_\perp')\delta^{(1)}(\theta_2^1-\theta_2^3) \times$$
$$\times \delta^{(1)}(\theta_2^2 - \theta_4^2).$$

where $[d\theta_2]$ denotes integration over $\sigma_-\bar{\theta}$ or $\vec{\sigma}_-\theta$, and $\delta^{(1)}(\vec{\sigma}_+\theta) = \delta^{(1)}(\theta_1) = \theta_1$ is the one-dimensional Grassman δ-function /13/.

Substituting the free "two-time" Green function (3.14) in (3.16) we find

$$(\underset{\smile}{G}_0^{-1})^{++,--} = \exp(P_+\theta_1\sigma_-\bar{\theta}_3 + Q_+\theta_2\sigma_-\bar{\theta}_4)A, \tag{3.17}$$
$$(\underset{\smile}{G}_0^{-1})^{-+,+-} = \exp(P_+\bar{\theta}_1\tilde{\sigma}_-\theta_3 + Q_+\theta_2\sigma_-\theta_4)A,$$

where

$$A = \frac{4}{P_+Q_+\bar{J}_0} = -\frac{i}{4\pi p_+}\left[p^2 - \frac{q_\perp^2}{x(1-x)}\right]. \tag{3.18}$$

Then, we have the following equation for the wave function

$$(\underset{\smile}{G}_0^{-1}\Psi_p)(q_+, q \underset{\vee}{\ }, \theta_1, \theta_2) = \int_{-p_+/2}^{p_+/2} dq_+ \int d^2 q\, (V \underset{\vee}{\Psi}_p), \tag{3.19}$$

where $\vee$ denotes the integration over the intermediate Grassmannian variables θ. The quasipotential V can be found within QFT. Because of a cumbersome structure eq. (3.19) is not given here for the components of the superwave function. It can be pointed out that the corresponding equation for the scalar component of the wave function coincides with the equation found in paper /6/ for the massless case.

4. SUPERSYMMETRIC QUASIPOTENTIAL EQUATION FOR MASSIVE CHIRAL FIELDS

From (3.1) and (3.2) it follows that the four-point free-field Green functions, when the mass of particles is nonzero, have the general form (2.1). Applying to this function the "two-time" operation (2.10) and projection operation (3.14) we have

$$\underset{\smile}{G}_0 = g \cdot \bar{J}_0(p,q,m), \tag{4.1}$$

where g is a 4x4 antidiagonal matrix with matrix elements

$$g^{++,--} = \exp(P_+ \theta_1 \underset{\sim}{\sigma}_- \bar{\theta}_3 + Q_+ \theta_2 \underset{\sim}{\sigma}_- \theta_4),$$
$$g^{-+,+-} = \exp(P_+ \bar{\theta}_1 \tilde{\sigma}_- \theta_3 + Q_+ \theta_2 \underset{\sim}{\sigma}_- \theta_4) \tag{4.2}$$

and J_0 is the corresponding "two-time" Green function for the free scalar particles with masses m_1 and m_2 /6/, i.e.,

$$\bar{J}_0 = \Theta(x)\,\Theta(1-x)\tilde{J}_0 \tag{4.3}$$

and

$$\tilde{J}_0 = \frac{4\pi i\,\delta(q_+ - q_+')\,\delta^{(2)}(q_\perp - q_\perp')}{p_+ x(1-x)\,[p^2 - (q_\perp^2 + m_1^2)/x - (q_\perp^2 + m_2^2)/(1-x)]}. \tag{4.4}$$

Note that by the projection (3.14) which cancels down the divergent terms all elements of $\tilde{G}_0$ containing the Grassmanian δ-function vanish. The latter is a consequence of the following identity

$$\theta_a \boldsymbol{\theta}_a \delta(\theta - \boldsymbol{\theta}) = \frac{1}{2}\,\theta_a \boldsymbol{\theta}_a(-\theta\epsilon\theta - \boldsymbol{\theta}\epsilon\boldsymbol{\theta} + 2\theta\epsilon\boldsymbol{\theta}) =$$
$$= \theta_a \boldsymbol{\theta}_a\,[\,\theta_2\theta_1 + \boldsymbol{\theta}_2\boldsymbol{\theta}_1 + \theta_1\boldsymbol{\theta}_2 - \theta_2\boldsymbol{\theta}_1] = 0\,.$$

Then, the inverse "two-time" Green function determined by the condition (3.16) has the following matrix elements

$$(\underset{\smile}{G}_0^{-1})^{++,--} = \exp\{P_+ \theta_1 \underset{\sim}{\sigma}_- \bar{\theta}_3 + Q_+ \theta_2 \underset{\sim}{\sigma}_- \bar{\theta}_4\}\,A(m_1, m_2),$$
$$(\underset{\smile}{G}_0^{-1})^{-+,+-} = \exp\{P_+ \bar{\theta}_1 \tilde{\sigma}_- \theta_3 + \theta_+ \theta_2 \sigma_- \bar{\theta}_4\}\,A(m_1, m_2), \tag{4.5}$$

where

$$A(m_1, m_2) = \frac{4}{P_+ Q_+ \tilde{J}_0(m_1, m_2)} =$$
$$= -\frac{i}{4\pi p_+}\,[p^2 - \frac{q_\perp^2 + m_1^2}{x} - \frac{q_\perp^2 + m_2^2}{1-x}]. \tag{4.6}$$

Substutiting $m_1 = m_2 = 0$ in (4.5) and (4.6) we get the free inverse "two-time" Green function for the massless cases (3.6) and (3.8). Then the supersymmetric equation for the two-particle wave function, has the following form

$$\underset{\vee}{\underline{G}}_0^{-1}\underset{\vee}{\underline{\Psi}} = V\underset{\vee}{\underline{\Psi}}, \qquad (4.7)$$

where the integration over the intermediate momentum and Grassmann variables is taken into account. The quasipotential V can be determined within the quantum field theory.

In conclusion it can be pointed out that since the scalar component of the free "two-time" Green function coincides with the propagator in the free parton model [6], then by using (4.1) the supersymmetric extension of the parton model can be found.

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REFERENCES

1. Zaikov R.P. JINR, E2-81-666, Dubna, 1981.
2. Logunov A.A., Tavkhelidze A.N. Nuovo Cimento, 1963, 29, p.380; Кадышевский В.Г., Тавхелидзе А.Н. В сб.: Проблемы теоретической физики, посвященном Н.Н.Боголюбову в связи с его 60-летием. "Наука", М., 1969.
3. Matveev V.A., Muradyan R.N., Tavkhelidze A.N. JINR, E2-3498, Dubna, 1967.
4. Боголюбов П.Н. ТМФ, 1970, 5, с.244.
5. Ризов В.А., Тодоров И.Т. ЭЧАЯ, 1975, 6, с.669.
6. Гарсеванишвили В.Р. и др. ТМФ, 1975, 23, с.310.
7. Weinberg S. Phys.Rev., 1966, 150, p.1313; Leutwyler H. Nucl.Phys., 1974, B76, p.413.
8. Матвеев В.А., Мурадян Р.Н., Тавхелидзе А.Н. ТМФ, 1979, 40, с.329.
9. Матвеев В.А., Соболев И.К. ОИЯИ, Р2-80-742, Дубна, 1980.
10. Kadyshevsky V.G. Nucl.Phys., 1968- B6, p.125.
11. Tarasov O.V., Vladimirov A.A. JINR, E2-80-483, Dubna, 1981.
12. Caswell W.E., Zanon D. Nucl.Phys., 1981, B182, p.125.
13. Огиевецкий В.И., Мезинческу Л. УФН, 1975, с.637.

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