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S.G.Mikhov

NON-SCHWINGER SOLUTION OF THE TWO-DIMENSIONAL MASSLESS SPINOR ELECTRODYNAMICS

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1. INTRODUCTION

Massless spinor electrodynamics in two space-time dimensions, usually identified with the Schwinger model (Schwinger 1962), has been a subject of numerous investigations. The great interest of physicists in this model is motivated by the fact that the electromagnetic field acquires a mass and that the electric charge is screened[1-3]. It is believed that this gives an example for both dynamical generation of masses and confinement. Most of the papers provide an analysis of this situation based on explicit operator solutions of the model. In order to formulate the purpose of the present paper we shall sketch briefly the formulation of the two-dimensional massless spinor electrodynamics. We follow the approach of Nakanishi[4] to write down the following system of equations:

$$i\partial\!\!\!/\psi - gA\!\!\!/\psi = 0, \tag{1}$$

$$\partial^\mu F_{\mu\nu} - \partial_\nu F = gj_\nu ; \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \tag{2}$$

$$\partial^\mu j_\mu = 0, \tag{3}$$

$$\partial^\mu A_\mu + aF = 0, \tag{4}$$

$$\Box F = 0. \tag{5}$$

Here the metrics is chosen to be $g_{\mu\mu}=(+,-)$, and the γ-matrices are defined as follows:

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \gamma_5 = \gamma_0\gamma_1 ,$$

$$\gamma_\mu \gamma_5 = \epsilon_{\mu\nu}\gamma^\nu , \quad \epsilon_{\mu\nu} = -\epsilon_{\nu\mu} = -\epsilon^{\mu\nu} , \quad \epsilon_{01} = 1.$$

The above system of equations needs some commentary. The Maxwell equation is written in the form (2) so that it could be considered as an operator valued equation (see, f.i., refs[5,3]).

As for the gauge fixing conditions (4) and (5), they are written in the above form following Nakanishi [6,7] in order to provide a manifest gauge covariant formulation of the model (eq. (5) just fixes the class of possible gauges).

Of course, these equations cannot fix a unique solution. For the purpose a system of boundary conditions is needed. In particular, at least some relevant commutators are required to be local and canonical. In what follows we shall discuss the rest of the boundary conditions that are related with the symmetries of the equations and the solution. Besides Poincare invariance they are:

i. Gauge invariance. We must note that not only the equations, but also the physically relevant quantities (the Maxwell tensor, the current and the charge) should be invariant under the action of the gauge transformations

$$\psi(x) \to e^{ig\Lambda(x)}\psi(x),$$

$$A_\mu(x) \to A_\mu(x) - \partial_\mu \Lambda(x).$$

Here we distinguish between the case when the gauge function $\Lambda(x)$ is specified by the equation

$$\Box\Lambda(x) = 0 \tag{6}$$

or by the equation

$$\Box\Lambda(x) = \rho F(x), \qquad \Box^2\Lambda(x) = 0. \tag{7}$$

In the first case (eq. (6)) we remain within one and the same gauge, while in the second we can move from one gauge to another, but remaining in the class of gauges determined by eq. (4). In that second case gauge transformations result in the change of the gauge fixing parameter α.

ii. Since the Dirac equation (1) is massless, it then follows that the system of equations exhibits γ_5-symmetry and even more - γ_5-gauge invariance. In that case the Maxwell tensor and the current $j_\mu(x)$ (at least on the classical level, when $j_\mu(x)=\bar\psi(x)\gamma_\mu\psi(x)$ is well defined) are also invariant under the action of the following transformations

$$\psi(x) \to e^{ig\tilde\Lambda(x)\gamma_5}\psi(x),$$

$$A_\mu(x) \to A_\mu(x) - \epsilon_{\mu\nu}\partial^\nu\tilde\Lambda(x),$$

where the gauge function $\tilde\Lambda(x)$ satisfies the equation

$$\Box\tilde\Lambda(x) = 0 \tag{8}$$

which is needed for the invariance of $F_{\mu\nu}(x)$. If one considers classical field theory, where no regularization problem arises, γ_5-invariance leads to the conservation of the quantity

$$j_{5\mu} = \bar{\psi}(x)\gamma_\mu \gamma_5 \psi(x) = \epsilon_{\mu\nu} \bar{\psi}(x)\gamma^\nu \psi(x) = \epsilon_{\mu\nu} j^\nu(x). \tag{9}$$

However, in quantum theory a regularization is needed and eq. (9) is by no means evident. On the contrary all available solutions do not satisfy it, and therefore spoil γ_5-invariance. Instead, a less stringent condition is satisfied. Namely, the conservation of the free axial current

$$j^f_{5\mu} = :\bar{\psi}^f \gamma_\mu \gamma_5 \psi^f:(x) = \epsilon_{\mu\nu} j^{f\nu},$$

where $\psi^f(x)$ denoted the solution of the free massless Dirac equation.

There are two conquering reasons for that choice. The first one consists in the observation that eqs. (2) and (9) imply

$$\Box F_{\mu\nu} = 0 \tag{10}$$

and therefore $\Box^2 A_\mu = 0$, which in its turn contradicts the result of Schwinger [1] that the electromagnetic potential A_μ acquires a mass.

The second is due to the procedure of quantization and therefore of regularization of the current. As is well known in quantum field theory the current should be defined as the limit of the corresponding bilinear form when both arguments tend to one and the same value. It is clear that such a procedure is not gauge invariant. Owing to an argument of Schwinger [8] the current in gauge theories should be defined as the limit of the following form:

$$j_\mu(x) = \lim\{\bar{\psi}(x+\epsilon)\gamma_\mu \psi(x) e^{-ig\int_x^{x+\epsilon} A_\mu(t)dt^\mu} - \langle \dots \rangle_0\}.$$

And now it is evident that this expression is not γ_5-gauge invariant. Moreover since the quantity

$$\gamma_5 \int_x^{x+\epsilon} \epsilon_{\mu\nu} A^\nu(t)\, dt^\mu$$

is not gauge invariant, it seems that one cannot think about a regularization that is compatible with both gauge and γ_5-gauge invariance.

However, we have the following possibility. Suppose that $A_\mu(x)$ can be decomposed into

$$A_\mu(x) = A^L_\mu(x) + A^{tr}_\mu(x), \tag{11}$$

where A^L_μ is γ_5-gauge invariant longitudinal part, while A^{tr}_μ is gauge invariant transverse part. Then it is obvious that the following expression

$$\bar{\psi}(x+\epsilon)\gamma_\mu e^{-ig\gamma_5 \int_x^{x+\epsilon} \epsilon_{\lambda\nu} A^{tr\,\nu}(t)\, dt^\lambda} \psi(x) e^{-ig \int_x^{x+\epsilon} A_\rho(t) dt^\rho} \qquad (12)$$

is both gauge and γ_5-gauge invariant, and therefore presents a good ground for a regularization that is compatible with both symmetries.

The aim of the present paper is to prove that one can carry out a consistent quantization of massless spinor electrodynamics in two space-time dimensions and construct an explicit operator solution that is both gauge and γ_5-gauge invariant. Such a solution we call a non-Schwinger solution of the two-dimensional massless electrodynamics. We are interested in it because of its more direct formal analogy with the four-dimensional massless spinor electrodynamics.

2. THE BUILDING BLOCK FIELDS

In this section we fix the set of building block fields that are necessary to construct our operator solution. Since both current and pseudocurrent are conserved and related by eq. (9), then following the analysis of Johnson[9] we must introduce a couple of dual scalar fields $\phi(x)$ and $\tilde{\phi}(x)$ satisfying

$$\partial_\mu \phi(x) + \epsilon_{\mu\nu} \partial^\nu \tilde{\phi}(x) = 0 .$$

Then the current and the pseudocurrent are expressed as

$$j_\mu = \partial_\mu \phi(x), \quad j_{5\mu} = -\partial_\mu \tilde{\phi}(x) = \epsilon_{\mu\nu} \partial^\nu \phi(x). \qquad (13)$$

Now having in mind eq. (11), we can write down for the electromagnetic potential the following general representation:

$$A_\mu(x) = \partial_\mu \Phi(x) + \epsilon_{\mu\nu} \partial^\nu \tilde{\Phi}(x) = A^L_\mu(x) + A^{tr}_\mu(x) . \qquad (14)$$

Then eq. (4) and the definition of the Maxwell tensor imply that the fields $\Phi(x)$ and $\tilde{\Phi}(x)$ satisfy the following equations:

$$\Box \Phi(x) = -\alpha F(x) , \qquad (15)$$

$$\Box \tilde{\Phi}(x) = {}^*F(x) , \qquad (16)$$

where

$${}^*F(x) = \frac{1}{2} \epsilon^{\mu\nu} F_{\mu\nu}(x), \quad F_{\mu\nu}(x) = -\epsilon_{\mu\nu}\, {}^*F(x) . \qquad (17)$$

Having in mind eqs. (5) and (10) we see that we are left with two Froissart[10] (1959) systems of equations for the massless dipole ghost fields $\Phi(x)$ and $\tilde{\Phi}(x)$.

It is easy to show that $F(x)$, $^*F(x)$ and $\phi(x)$ cannot be completely independent, since the Maxwell equation (2), in view of eqs. (13) and (17), can be rewritten in the form

$$\partial_\mu (F(x) + g\phi(x)) - \epsilon_{\mu\nu}\partial^\nu {}^*F(x) = 0 . \qquad (18)$$

It is obvious that one can satisfy eq. (18) in many different ways. This arbitrariness can result in the type of behaviour of the solution at $g \to 0$* as well as in the set of independent fields involved in the solution. In what follows we confine ourselves to the very simple case when

$$\phi(x) = \frac{1}{g_0} F(x), \quad {}^*F(x) = -(1 + \frac{g}{g_0})\tilde{F}(x) = -\lambda \tilde{F}(x) , \qquad (19)$$

where $g_0 \neq 0$ is an arbitrary constant with mass dimension, and $\tilde{F}(x)$ is the dual field of $F(x)$. With this choice we restrict ourselves to the case of a minimal set of building block fields, and at the same time our system of equations exactly coincides with that considered in a previous paper/11/. The latter makes it possible to use all the results of this paper without any modification. At the same time, we do not make any hypothesis about the behaviour of the fields $F(x)$, $^*F(x)$ and $\phi(x)$ at $g \to 0$. Thus we leave some room for further speculations on this point.

Now we go further and discuss the solution of the Dirac equation. For the purpose we shall make use of the representation of a spinor field in two space-time dimensions by means of non-linear scalar fields/12,13,14/, which is now traditionally called bozonization. The latter proved to be extremely convenient in two-dimensional models. So, we write down

$$\psi(x) = e^{i\tilde{K}^-(x)\gamma_5}\, e^{-iK^-(x)}\, e^{-iK^+(x)}\, e^{i\tilde{K}^+(x)\gamma_5}\, u , \qquad (20)$$

where u_α, $\alpha = 1,2$ are two complex numbers. Substituting the Ansatz (20) into the Dirac equation (1) and having in mind the representation (14) and the properties of the γ-matrices, we can identify the fields $K^{\pm}(x)$ and $\tilde{K}^{\pm}(x)$ in the following way:

*Remark. We must note that whatever the differences in the behaviour at $g \to 0$ are, one cannot expect to obtain the usual perturbation theory limit, since it is well known that there is a perturbation theory anomaly which gives rise to a mass term for the electromagnetic field. The latter contradicts the main idea of our present solution.

$$K^{\pm}(x) = \frac{d}{g_0} F^{\pm}(x) - g\Phi^{\pm}(x),$$

$$\tilde{K}^{\pm}(x) = \frac{d}{g_0} \tilde{F}^{\pm}(x) - g\tilde{\Phi}^{\pm}(x),$$

where $F^{\pm}(x)$, $\tilde{F}^{\pm}(x)$, $\Phi^{\pm}(x)$ and $\tilde{\Phi}^{\pm}(x)$ are the positive and negative frequency parts of the corresponding fields, d is an arbitrary real constant, and g_0 and g are necessary in order to make $K^{\pm}(x)$ and $\tilde{K}^{\pm}(x)$ dimensionless*. Since, as we have already noted, the quantum problem for the fields $F^{\pm}(x)$, $\tilde{F}^{\pm}(x)$, $\Phi^{\pm}(x)$ and $\tilde{\Phi}^{\pm}(x)$ is solved in a previous paper/11/, we have therefore found a solution of the two-dimensional massless spinor electrodynamics, that does not contain a massive electromagnetic potential. This is true iff a regularization of the type (12) exists and leads to the expression (13). The latter will be treated in the following section.

Now we must briefly discuss the gauge transformation properties. It is obvious that since the Maxwell tensor and the current are both gauge and γ_5-gauge invariants, then the scalar fields $F(x)$ and $\tilde{F}(x)$ (eq. (19) is implicit) are invariants of these transformations too. Having in mind that the longitudinal part of A_μ is γ_5-gauge invariant, while its transverse part is gauge invariant, we see that the only field that should suffer gauge transformations is the dipole ghost $\Phi(x)$, while γ_5-transformations should act on the dipole ghost field $\tilde{\Phi}(x)$ only. Now it is not difficult to see that the equations (15), (16) and (18) are compatible with these transformations provided eqs. (6) and (8) hold (in the case of eq. (7) the parameter α is replaced by $\alpha + \rho$). Thus, we see that the representation (14) of the electromagnetic potential has the proper gauge and γ_5-gauge transformation properties that are necessary in order to make use of the definition (12) for the current.

3. CORRECT REGULARIZATION PROCEDURE

In this section we shall prove that the regularization procedure based on formula (12) is self-consistent. For the pur-

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* Remark. We write g_0 in the denominator of the first term in order to avoid irregularity at $g \to 0$; at the same time we write g in the numerator of the second term in order that at $g \to 0$ we obtain a solution of the free massless Dirac equation without any reference to the behaviour of the field $F^{\pm}(x)$, $\tilde{F}^{\pm}(x)$, $\Phi^{\pm}(x)$ and $\tilde{\Phi}^{\pm}(x)$ at $g \to 0$.

pose we use the modification of the Johnson's[9] definition of the current introduced by Aneva et al.[15]. All necessary commutators of $F^{\pm}(x)$, $\tilde{F}^{\pm}(x)$, $\Phi^{\pm}(x)$ and $\tilde{\Phi}^{\pm}(x)$ are listed in the Appendix.

We start considering the following gauge invariant quantities:

$$
\begin{aligned}
J_{\mu;rs}(x,y) &\equiv e^{ig\Phi^{-}(x)}(\bar{\psi}(x)\gamma_{\mu})_r\, e^{ig\Phi^{+}(x)}\, e^{-ig\Phi^{-}(y)}\psi_s(y)\, e^{-ig\Phi^{+}(y)} = \\
&= u_r^{+}u_s(-1)^{(r-1)\mu}\, e^{(-1)^{r+s}\{[2d\lambda(\lambda-1)+g^2c_1+\frac{g^2\lambda^2x^2}{4}]D^{+}(x-y)+\frac{g^2\lambda^2x^2}{2\pi}\}} \times \\
&\times e^{-[(-1)^r+(-1)^s]d\lambda(\lambda-1)\tilde{D}^{+}(x-y)}\, e^{-i\{(-1)^r[\frac{d}{g_0}\tilde{F}^{-}(x)-g\tilde{\Phi}^{-}(x)]-(-1)^s[\frac{d}{g_0}\tilde{F}^{-}(y)-g\tilde{\Phi}^{-}(y)]\}} \times \\
&\times e^{+i\frac{d}{g_0}(F^{-}(x)-F^{-}(y))}\, e^{i\frac{d}{g_0}(F^{+}(x)-F^{+}(y))}\, e^{-i\{(-1)^r[\frac{d}{g_0}\tilde{F}^{+}(x)-g\tilde{\Phi}^{+}(x)]-(-1)^s(\frac{d}{g_0}\tilde{F}^{+}(y)-g\tilde{\Phi}^{+}(y)]\}}.
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
\bar{J}_{\mu;rs}(x,y) &\equiv e^{-ig\Phi^{-}(x)}\psi_r(x)\, e^{-ig\Phi^{+}(x)}\, e^{ig\Phi^{-}(y)}(\bar{\psi}(y)\gamma_{\mu})_s\, e^{ig\Phi^{+}(y)} = \\
&= u_r u_s^{+}(-1)^{(s-1)\mu}\, e^{(-1)^{r+s}\{[2d\lambda(\lambda-1)+g^2c_1+\frac{g^2\lambda^2x^2}{4}]D^{+}(x-y)+\frac{g^2\lambda^2x^2}{2\pi}\}} \times \\
&\times e^{-\{(-1)^r+(-1)^s]d\lambda(\lambda-\tilde{D}^{+}(x-y)}\, e^{i\{(-1)^r[\frac{d}{g_0}\tilde{F}^{-}(x)-g\tilde{\Phi}^{-}(x)]-(-1)^s[\frac{d}{g_0}\tilde{F}^{-}(y)-g\tilde{\Phi}^{-}(y)]\}} \times \\
&\times e^{-i\frac{d}{g_0}(F^{-}(x)-F^{-}(y))}\, e^{-i\frac{d}{g_0}(F^{+}(x)-F^{+}(y))}\, e^{i\{(-1)^r[\frac{d}{g_0}\tilde{F}^{+}(x)-g\tilde{\Phi}^{+}(x)]-(-1)^s[\frac{d}{g_0}\tilde{F}^{+}(y)-g\tilde{\Phi}^{+}(y)]\}}.
\end{aligned}
\tag{22}
$$

where $\lambda = 1+g/g_0$ and the functions $D^{\pm}(x)$ and $\tilde{D}^{\pm}(x)$ are defined in the Appendix. It is obvious that the above quantities still are not γ_5-gauge invariant. That is why we go further defining

$$
\begin{aligned}
J_{\mu}(x;\epsilon) &= \sum_{r=1}^{2}(-\epsilon^2)^{\frac{1}{4\pi}[2d\lambda(\lambda-1)+g^2c_1]-\frac{1}{2}} \times \\
&\times \{ e^{-ig(-1)^r(\tilde{\Phi}^{-}(x+\epsilon)-\tilde{\Phi}^{-}(x))} J_{\mu;rr}(x+\epsilon;x)\, e^{-ig(-1)^r(\tilde{\Phi}^{+}(x+\epsilon)-\tilde{\Phi}^{+}(x))} - \\
&- e^{ig(-1)^r(\tilde{\Phi}^{-}(x)-\tilde{\Phi}^{-}(x-\epsilon))}\, \bar{J}_{\mu;rr}(x;x-\epsilon)\, e^{ig(-1)^r(\tilde{\Phi}^{+}(x)-\tilde{\Phi}^{+}(x-\epsilon))} \}.
\end{aligned}
\tag{23}
$$

Having in mind the explicit expressions (21) and (22), it is evident that the above quantity is both gauge and γ_5-gauge invariant. Thus it can be used to define a proper current. The factor $(-\epsilon^2)^{\frac{1}{4\pi}(2d\lambda(\lambda-1)+g^2c_1)-\frac{1}{2}}$ is needed in order that the singularity of each term of eq. (23) becomes a first order pole (this is easily seen from eqs. (21) and (22) and the explicit expressions for the functions $D^+(x)$ and $\tilde{D}^+(x)$).

Next, following Johnson[9] we introduce the quantities

$$J_\mu(x) = \lim_{\substack{\epsilon^\circ=0 \\ \epsilon'\to 0}} J_\mu(x;\epsilon)$$

$$\tilde{J}_\mu(x) = \lim_{\substack{\epsilon^\circ=0 \\ \epsilon^1\to 0}} \tilde{J}_\mu(x;\epsilon) = \lim_{\substack{\epsilon^\circ=0 \\ \epsilon^1\to 0}} J_\mu(x;\tilde{\epsilon}) \ ,$$

where $\tilde{\epsilon}_\mu = \epsilon_\mu{}^\nu \epsilon_\nu$. In order to eliminate the dependence on the dimensional parameter μ that appears in the function $D^+(x)$ we choose the constant u_r to be

$$|u_1|^2 = |u_2|^2 = \frac{1}{4d}(\mu^2)^{\frac{1}{4\pi}[2d\lambda(\lambda-1)+g^2c_1]} .$$

Then taking the corresponding limits, we have the following explicit expressions:

$$J_0(x) = -\frac{i}{2g_0}\{(-1)^{\frac{d\lambda(\lambda-1)}{4\pi}}(\partial_1 F(x) + \partial_1 \tilde{F}(x)) + (-1)^{-\frac{d\lambda(\lambda-1)}{4\pi}}(\partial_1 F(x) - \partial_1 \tilde{F}(x))\},$$

$$J_1(x) = -\frac{i}{2g_0}\{(-1)^{\frac{d\lambda(\lambda-1)}{4\pi}}(\partial_1 F(x) + \partial_1 \tilde{F}(x)) - (-1)^{-\frac{d\lambda(\lambda-1)}{4\pi}}(\partial_1 F(x) - \partial_1 \tilde{F}(x))\},$$

$$\tilde{J}_0(x) = \frac{1}{g_0}\partial_0 F(x) ,$$

$$\tilde{J}_1(x) = \frac{1}{g_0}\partial_0 \tilde{F}(x) .$$

It is evident that in order to make $J_\mu(x)$ to be a real function it is necessary to fix the arbitrary constant d by means of the following condition:

$$d\lambda(\lambda-1) = 2(2k+1)\pi, \quad k = 0, \pm 1, \pm 2, \dots .$$

Then we can finally define the regularized current by means of the following expression:

$$j_\mu(x) = \frac{1}{2}[\tilde{J}_\mu(x) - \epsilon_{\mu\nu} J^\nu(x)] = \frac{1}{g_0}\partial_\mu F(x) ,$$

which exactly coincides with expression (13). Thus, we have proved that our regularization procedure is compatible with both gauge and γ_5-gauge invariance and lead to the standard relation between the regularized current and pseudocurrent, that are both conserved. In fact, this proves the self-consistency of our formulation of the problem.

At the end of this section we must note that both currents imply the existence of the corresponding charge and pseudocharge operators. Namely,

$$Q^{\pm} = \int_{-\infty}^{\infty} dx^1 j_0^{\pm}(x) = \frac{1}{g_0} \int_{-\infty}^{\infty} dx^1 \partial_0 F^{\pm}(x),$$
$$\tilde{Q}^{\pm} = \int_{-\infty}^{\infty} dx^1 j_{5\,0}^{\pm}(x) = \frac{1}{g_0} \int_{-\infty}^{\infty} dx^1 \partial_0 \tilde{F}^{\pm}(x). \tag{24}$$

In fact this is a direct corollary from the properties of the infrared regularization of the fields $F^{\pm}(x)$ and $\tilde{F}^{\pm}(x)$ that follow from the analysis of Hadjiivanov and Stoyanov/6/ and Mikhov/11/. However, despite of the existence of the charge operator (24), quite a peculiar situation arises when one looks for the charge of the solution of the Dirac equation (1). For the purpose we consider the commutators of the charge operators and the operator solution for the Dirac field and the electromagnetic potential. Having in mind eqs. (14), (20) and the formulae from the Appendix, we obtain the following commutators:

$$[A_\mu^{\pm}(x), Q^{\mp}] = [A_\mu^{\pm}(x), \tilde{Q}^{\mp}] = 0,$$
$$[\psi(x), \tilde{Q}^{\mp}] = \frac{2\lambda(1-\lambda)}{\sqrt{2\pi}} \gamma^5 \psi(x),$$
$$[\psi(x), Q^{\mp}] = \frac{2a(1-\lambda)}{\sqrt{2\pi}} \psi(x). \tag{25}$$

The first two of the above commutators seem quite natural and need not any comment. As for the last one, it might seem at first, that we have obtained the eigenvalue of the charge operator, but this is not the case. The quantity a is the gauge fixing parameter, and therefore the last commutator depends on the gauge. The situation is even worse, since in the Landau gauge (a =o) we have in fact a zero "charge". So we are forced to conclude that either the charge of the solution is zero or the charge operator $Q^{\mp}$ does not in fact define the electric charge of the obtained solution. This can be a manifestation of the charge screening mechanism, that is known to take place in the Schwinger model.

4. COMMUTATION AND WIGHTMAN FUNCTIONS

In this section we discuss briefly some of the relevant commutation and Wightman functions, which gives the possibility to fix the arbitrary constants c_1 and c_2 that appear in the commutators of the fields $\Phi^{\pm}(x)$ and $\tilde{\Phi}^{\pm}(x)$. The necessary functions are simply evaluated by using the formulae from the Appendix and having in mind the explicit expressions (14) and (10) for the solutions.

Let us first consider the commutation function of two electromagnetic potentials

$$[A_\mu(x), A_\nu(0)] =$$

$$= ig_{\mu\nu}\frac{\lambda^2-\alpha^2}{2}D(x) - i\frac{(\alpha-\lambda)^2}{2}[x_\mu\partial_\nu + x_\nu\partial_\mu + \frac{x^2}{2}\partial_\mu\partial_\nu]D(x) + \tag{26}$$

$$+ \frac{\lambda-\alpha}{\alpha\lambda}(c_1\lambda - c_2\alpha)\partial_\mu\partial_\nu D(x) .$$

It is evident that function has a gauge independent term contributing to its transverse part and therefore it is not trivial. The expression (26) is local provided the following equation

$$c_1\lambda - c_2\alpha = 0 \tag{27}$$

holds. Under that condition the Wightman function of two electromagnetic potentials has the following form:

$$<0|A_\mu(x)A_\nu(0)|0> = [A^+_\mu(x), A^-_\nu(0)] =$$

$$= g_{\mu\nu}[\frac{\lambda^2-\alpha^2}{2}D^+(x) - \frac{(\alpha-\lambda)^2}{8\pi}] - \frac{(\alpha-\lambda)^2}{4}[x_\mu\partial_\nu + x_\nu\partial_\mu]D^+(x) =$$

$$= g_{\mu\nu}[\frac{\alpha^2-\lambda^2}{8\pi}\ln(-\mu^2x^2 + i0x^0) - \frac{(\alpha-\lambda)^2}{8\pi}] + \frac{(\alpha-\lambda)^2}{8\pi}\frac{x_\mu x_\nu}{x^2-i0x^0} .$$

At the same time we have trivial Wightman functions of two currents, since $F^{\pm}(x)$ commute trivially. This is a quite unusual feature of our solution. It is clear that even if we consider a larger set of building block fields (in order to avoid eq. (19)), the same situation would appear provided eqs. (15), (16), (18) and translational invariance hold. However, the commutator of the current and the Dirac field is almost standard

$$[\, j^{\pm}_{\mu}(x),\ \psi(0)] = i(1-\lambda)\{\alpha\,\partial_{\mu}\, D^{+}(x) + \lambda\gamma^{5}\partial_{\mu}\,\bar{D}^{+}(x)\}\psi(0)\ . \tag{28}$$

The only difference of this expression and the corresponding standard commutator, obtained after the analysis of Johnson[9] (we remind that in our formulation both current and pseudocurrent are conserved and obey eq. (13) as it is in the Thirring model), consists in the coefficients $\alpha(1-\lambda)$ and $\lambda(1-\lambda)$ that are present in eq. (28). This results in the pathological gauge dependence of the commutator of the charge. Here again we must note that the introduction of a large set of building block fields would not affect the general features of eq. (28).

At the end of this section we write down the Wightman function of two Dirac fields

$$\begin{aligned}
<0|\psi_{r}(x)\bar{\psi}_{s}(0)|0> &= u_{r}\bar{u}_{s}\exp\{\frac{g^{2}x^{2}}{4\pi}[\alpha^{2} + (-1)^{s+r+1}\lambda^{2}\,]\}\times\\
&\times\exp\{[\frac{2}{\lambda}(2\pi(2k+1) + \frac{g^{2}c_{2}}{2})(\alpha + (-1)^{s+r+1}\lambda) + \frac{g^{2}x^{2}}{4}(\alpha^{2}+(-1)^{r+s+1}\lambda^{2})]D^{+}(x)\}\times\\
&\times\exp\{[\frac{\alpha+\lambda}{\lambda}(2\pi(2k+1) + \frac{g^{2}c_{2}}{2}) + \frac{\alpha\lambda g^{2}x^{2}}{4}][(-1)^{r} - (-1)^{s}]\bar{D}^{+}(x)\}\,.
\end{aligned}$$

It is evident that apart from the nonstandard diagonal terms (a result of the bosonization) the off-diagonal terms are also unusual. However in the Landau gauge ($\alpha=0$) fixing the constant c_2 by the condition

$$c_{2} = -\frac{\pi}{g^{2}}(4k+3) \tag{29}$$

we can write down the standard matrix form for the off-diagonal terms. Namely we have

$$<0|\psi_{r}(x)\bar{\psi}_{s}(0)|0>_{r\neq s} = i\mu\,|u_{1}|^{2}\exp\frac{g^{2}\lambda^{2}x^{2}}{8\pi}\,x^{\mu}\gamma_{\mu}\,\frac{1}{(-\mu^{2}x^{2}+i0x^{\circ})^{\frac{g^{2}\lambda^{2}x^{2}}{16\pi}}}\,.$$

We conclude this section with the observation that eqs. (27) and (29) completely fix the constants c_1 and c_2. So in fact our solution depends on the initial parameters α and λ only.

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APPENDIX

In this Appendix we list the necessary commutators, as they are obtained in a previous paper/11/. We, first of all, have the trivial commutators

$$[F(x), F(y)] = [F(x), \tilde{F}(y)] = [\tilde{F}(x), \tilde{F}(y)] = 0 .$$

Then we have the following nontrivial commutators:

$$\frac{1}{\alpha}[\Phi^{\pm}(x), F^{\mp}(y)] = \frac{1}{\lambda}[\tilde{\Phi}^{\pm}(x), \tilde{F}^{\mp}(y)] = - D^{\pm}(x-y) ,$$

$$\frac{1}{\alpha}[\Phi^{\pm}(x), \tilde{F}^{\mp}(y)] = \frac{1}{\lambda}[\tilde{\Phi}^{\pm}(x), F^{\mp}(y)] = - \tilde{D}^{\pm}(x-y) ,$$

$$[\Phi^{\pm}(x), \Phi^{\mp}(y)] = \alpha^2 H_0^{\pm}(x-y) + c_1 D^{\pm}(x-y) ,$$

$$[\tilde{\Phi}^{\pm}(x), \tilde{\Phi}^{\mp}(y)] = \lambda^2 H_0^{\pm}(x-y) + c_2 D^{\pm}(x-y) ,$$

$$[\Phi^{\pm}(x), \tilde{\Phi}^{\mp}(y)] = \alpha\lambda \tilde{H}_0^{\pm}(x-y) + \frac{1}{2}(c_1\frac{\lambda}{\alpha} + c_2\frac{\alpha}{\lambda})\tilde{D}^{\pm}(x-y) ,$$

where the functions $D^{\pm}(x)$, $\tilde{D}^{\pm}(x)$, $H_0^{\pm}(x)$ and $\tilde{H}_0^{\pm}(x)$ are defined as follows:

$$D^{\pm}(x) = \mp \frac{1}{4\pi}\ln(-\mu^2 x^2 \pm i0x^{\circ}),$$

$$\tilde{D}^{\pm}(x) = \pm\frac{1}{4\pi}\ln\frac{x_0 + x_1 \mp i0}{x_0 - x_1 \mp i0} ,$$

$$H_0^{\pm}(x) = \frac{x^2}{4}(D^{\pm}(x) \pm \frac{1}{2\pi}).$$

$$\tilde{H}_0^{\pm}(x) = \frac{x^2}{4}\tilde{D}^{\pm}(x) .$$

At the end for the charges $Q^{\pm}$ and $\tilde{Q}^{\pm}$ defined by eqs. (24) we have the following nontrivial commutators

$$[Q^{\pm}, \Phi^{\mp}(x)] = \frac{2i\alpha}{\sqrt{2\pi}} ,$$

$$[\tilde{Q}^{\pm}, \tilde{\Phi}^{\mp}(x)] = \frac{2i\lambda}{\sqrt{2\pi}} .$$

The formulae listed above are suffucient in order to carry out all necessary calculations in the present paper.

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