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SUPERSYMMETRIC QUASIPOTENTIAL EQUATIONS.

I. Supersymmetric Extension of the Logunov-Tavkhelidze Approach

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I. INTRODUCTION

Usually, when we are dealing with the relativistic two-particle problem in the framework of QFT it is convenient to use the Bethe-Salpeter equation. However, in this case there arise some difficulties because of the nondefinite sign of the norm of the two-particle amplitude. The origin of these difficulties is the existence in the theory of one nonphysical parameter, relative time or its conjugate, relative energy. An extremely useful procedure for removing these difficulties has been suggested by Logunov and Tavkhelidze /1/. The main idea of Logunov-Tavkhelidze is the equality of times of both particles in the center-of-mass system, i.e., the relative time is put to be zero /1/. The theory developed on the base of this idea as well as its manifestly covariant modifications /2-6/ constitute a powerful method for studying the two-particle problem in QFT /7,8/.

On the other hand, in the last years supersymmetric quantum field theories are intensively developed. Essential characteristic of such theories is the unification of the bosonic and fermionic fields on one multiplet. For this reason some of divergences from the bosonic sector are cancelled with the ones from the fermionic sector. On the whole supersymmetric QFT's have less divergences than the ordinary theories. There is a promise that in some case of extended supersymmetric theories the divergences do not exist. As an example we can point out the supersymmetric SU(4) Yang-Mills theory where there is no the charge renormalization in the three-loop approximation /9,10/. There is a hope that these renormalizations do exist in any order of perturbation theory as well. In that case the supersymmetric SU(4) Yang-Mills theory is a good candidate for the theory which is able to describe the quark confinement phenomenon.

In this sequal of papers we make an attempt to construct the supersymmetric quasipotential equations. In the first paper a supersymmetric extension of the Logunov-Tavkhelidze approach is considered. In the second and third papers the same is made in the case of light-cone variables /11/ and for the approach in which the Markov-Yukawa condition is used /2,5,6/. In all the cases, for simplicity, we restrict ourselves only to simple scalar supermultiplets, i.e., superfields describing one scalar, one pseudoscalar, and one spinor particles. As in the usual case, the quasipotential can be found from perturbation theory.

With the help of these equations the bound states in the case of supersymmetric theories can be found.

2. SUPERSYMMETRIC BETHE-SALPETER EQUATION

Consider the supersymmetric four-point Green function

$$G(z_1,z_2,z_3,z_4)=\langle 0|T(\Phi(z_1)\Phi(z_2)\Phi^+(z_3)\Phi^+(z_4))|0\rangle, \qquad (2.1)$$

where $z=(x_\mu,\Theta_\alpha)$, Θ is, in general, the four-component anticommuting Majorana spinor variable and Φ are superfields. It is supposed that G is invariant with respect to the super-Poincare transformations. For the Green function G the following supersymmetric Bethe-Salpeter equation [12] can be written

$$G(z_1,z_2;w_1,w_2)=G_0(z_1,z_2;w_1,w_2)+\int d^8u_1 d^8u_2 d^8v_1 d^8v_2$$
$$\times D_0(z_1,u_1)D_0(z_2,u_2)K(u_1,u_2,v_1,v_2)G(v_1,v_2;w_1,w_2), \qquad (2.2)$$

where $D_0(z_1,z_2)$ is the supersymmetric free-particle propagator

$$D_0(z_1,z_2)=\langle 0|T(\Phi(z_1)\Phi(z_2))|0\rangle \qquad (2.3)$$

and K is the invariant Bethe-Salpeter kernel.

As in the usual case, we can introduce a complete system of intermediate states. Then G can be represented in the following form

$$G=\sum_n \langle 0|T(\Phi(z_1)\Phi(z_2))|n\rangle\langle n|T(\Phi^+(w_1)\Phi^+(w_2))|0\rangle$$
$$=\sum_n \Psi_n(z_1,z_2)\Psi_n^+(w_1,w_2), \qquad (2.4)$$

where by

$$\Psi_n(z_1,z_2)=\langle 0|T(\Phi(z_1)\Phi(z_2))|n\rangle \qquad (2.5)$$

the Bethe-Salpeter amplitude is denoted and $x_0^a>y_0^b$ $(a,b=1,2)$ is assumed. Then, substituting (2.4) in (2.2) we obtain the corresponding homogeneous supersymmetric Bethe-Salpeter equation for the two-particle amplitude

$$\int d^8u_1 d^8u_2 G_0^{-1}(z_1,z_2,u_1,u_2)\Psi_n(u_1,u_2)=\int d^8u_1 d^8u_2 d^8v_1 d^8v_2 \times$$

$$\times D_0(z_1,u_1)D_0(z_2,u_2)K(u_1,u_2,v_1,v_2)\Psi_n(v_1,v_2). \tag{2.6}$$

In eqs. (2.2) and (2.6) it is convenient to introduce the collective coordinates. In the equal-mass case, we restrict ourselves to

$$Z = \frac{1}{2}(z_1 + z_2) = \frac{1}{2}(x^1_\mu + x^2_\mu, \Theta^1_\alpha + \Theta^2_\alpha) \tag{2.7}$$

and

$$z = z_1 - z_2 = (x^1_\mu - x^2_\mu, \Theta^1_\alpha - \Theta^2_\alpha) \tag{2.8}$$

are the super-center-of-mass coordinate and the super-relative coordinates, respectively. It is easy to see that with respect to the supertransformations the center-of-mass coordinate (2.7) is transformed as a coordinate in the superspace but the transformation law of the relative coordinate (2.8) is

$$z \to z' = \{ x^1_\mu - x^2_\mu + i\bar{\epsilon}\gamma_\mu(\theta^1 - \Theta^2)\}, \tag{2.9}$$

where ϵ is the anticommuting spinor parameter of the supertransformations.

Transition to the momentum space, with respect to x is performed as in the ordinary case. Then the Bethe-Salpeter eq. (2.2) can be written symbolically in the following way:

$$G = G_0 + G_0 \check{K} \check{G}, \tag{2.10}$$

where G_0 is the two-particle supersymmetric disconnected Green function, and by the integration over intermediate spinor variables is denoted by $\vee$, the integration over intermediate momentum variables also being taken into account. The solution of eq. (2.10) can be found by iteration, i.e.,

$$G = G_0 + G_0 \check{K} \check{G}_0 + G_0 \check{K} \check{G}_0 \check{K} \check{G}_0 + \dots . \tag{2.11}$$

In the supersymmetric case in (2.11) there are, in general, less singular terms than in the ordinary case. However, there also exist unphysical parameters: the relative coordinate or its conjugate relative energy.

3. SUPERSYMMETRIC TWO-TIME GREEN FUNCTION

To make free the theory from the difficulties caused by the relative time (energy), we following Logunov-Tavkhelidze/1/ put the relative time in (2.1) and (2.5) to be zero in the c.m.s., i.e.,

$$x_0^1 - x_0^2 = 0. \tag{3.1}$$

However, from (2.8) it follows that this operation is not invariant with respect to the supertransformations. As is well known, the equal-time operation also is not invariant with respect to the Lorentz transformations; the operation (3.1) can be made meaningful in a fixed reference frame, i.e., c.m.s. In the supersymmetric case the operation (3.1) also has sense in the fixed reference frame in the superspace. Such supercenter-of-mass system is introduced by the conditions $\vec{P}=0$ and

$$\bar{\epsilon}\gamma_0(\theta_1 - \theta_2) = 0. \tag{3.2}$$

The last equation is satisfied if:

a) $\epsilon = 0$, i.e., the parameter of the supertransformations is zero.

b) $\theta_1 - \theta_2 = 0$, i.e., the Grassman spinor variables coincide, and

c) $\epsilon = \lambda(\theta_1 - \theta_2)$.

The case b) annihilates some of the spin-states in the B-S amplitude, and we do not consider it here. The cases a) and c) in some sense are equivalent. For definiteness here the supercenter-of-mass will be fixed by the conditions ($\vec{P}=0$, $\epsilon=0$).

In an arbitrary reference frame the equal-time condition (3.1) can be written in the following invariant form

$$(L_P)_0^{\nu}\,[x_\nu^1 - x_\nu^2 + i\bar{\epsilon}\gamma_\nu\,(\theta_1 - \Theta_2)] = 0, \tag{3.3}$$

where $(L_P)_\mu^\nu$ are matrix elements of the boost operator, for which

$$(L_P)_0^\nu = n^\nu = \frac{P^\nu}{\sqrt{P^2}}, \quad n^2 = 1.$$

Here P is the center-of-mass momentum of the two-particle system. Note that the momentum P is invariant with respect to the supertransformations.

Transition from the four-time Green functions to the two-time ones and from the two-time B-S amplitudes to the one-time wave function can be made in a covariant manner according to the formulas

$$\begin{aligned}\tilde{G}(z_1, z_2; w_1, w_2) &= \int dx_0^2 dy_0^2\, \delta\{n^\mu[x_\mu^1 - x_\mu^2 + i\bar{\epsilon}\gamma_\mu\,(\theta_1 - \theta_2)]\} \\ &\times G(z_1, z_2; w_1, w_2)\cdot\delta\{n^\mu\,[y_\mu^1 - y_\mu^2 + i\bar{\epsilon}\gamma_\mu\,(\theta_1 - \theta_2)]\},\end{aligned} \tag{3.4}$$

and

$$\tilde{\Psi}_n(z_1, z_2) = \int dx_0^2 \delta\{n^\mu[x_\mu^1 - x_\mu^2 + i\bar{\epsilon}\gamma_\mu(\theta_1 - \theta_2)]\}\Psi_n(z_1, z_2). \quad (3.5)$$

Going to the momentum space from (3.4) and (3.5) in the super-center-of-mass system we have

$$\tilde{G}(E, \vec{q}, \vec{q}', \theta_1, \theta_2, \bar{\theta}_1, \bar{\theta}_2) = \int_{-\infty}^{\infty} dq_0 dq_0' \, G(E, q, q', \theta_1, \theta_2, \bar{\theta}_1, \bar{\theta}_2) \quad (3.6)$$

and

$$\tilde{\Psi}_E(\vec{q}, \theta_1, \theta_2) = \int_{-\infty}^{\infty} dq_0 \Psi(E, q, \theta_1, \theta_2). \quad (3.7)$$

Consequently in the momentum space the "equal-time" operation (3.1) is replaced by the integration over the relative energies, as in the ordinary case [1].

For the two-time Green function (3.4) or (3.6) we have the following equation

$$\tilde{G} = \tilde{G}_0 + \underset{\vee}{G_0} K \underset{\vee}{G}, \quad (3.8)$$

which can be found from the B-S eq. (2.10) by the "equal-time" operation (3.1). Then, as in the ordinary case, in the supersymmetrical case the quasipotential is determined from the equation

$$[\tilde{G}]^{-1} = [\tilde{G}_0]^{-1} - \frac{1}{2\pi i} V. \quad (3.9)$$

Here the inverse operator is determined by the following condition

$$\int d^3\vec{q}'' d^4\Theta_1'' d^4\Theta_2'' \, \tilde{G}(E, \vec{q}, \vec{q}'', \Theta_1, \theta_2; \theta_1'', \Theta_2'') \quad (3.10)$$
$$\times \tilde{G}^{-1}(E, \vec{q}'', q', \Theta_1'', \Theta_2''; \theta_1', \Theta_2') = \delta^{(3)}(\vec{q} - \vec{q}')\delta(\theta_1 - \theta_1')\delta(\theta_2 - \theta_2'),$$

where $\delta(\theta)$ is the Grassmannian δ-function [13].

4. QUASIPOTENTIAL EQUATION FOR SCALAR CHIRAL SUPERMULTIPLETS

In this section we restrict our consideration to scalar chiral superfields (see Appendix A). The four-particle Green function for these fields can be represented in the following form

$$G = \begin{bmatrix} G^{++,++} & G^{++,-+} & G^{++,+-} & G^{++,--} \\ G^{-+,++} & G^{-+,-+} & G^{-+,+-} & G^{-+,--} \\ G^{+-,++} & G^{+-,-+} & G^{+-,+-} & G^{+-,--} \\ G^{--,++} & G^{--,-+} & G^{--,+-} & G^{--,--} \end{bmatrix}, \qquad (4.1)$$

where

$$G^{\alpha,\beta,\gamma,\delta} = \langle 0 | T(\Phi^{\alpha}(x_1,\theta_1)\Phi^{\beta}(x_2,\theta_2)\Phi^{\gamma}(x_3,\theta_3)\Phi^{\delta}(x_4,\theta_4)) | 0 \rangle \qquad (4.2)$$

are the four-point Green functions (4.2) for the chiral scalar superfields. Here the following notation is used:

$$\Phi^{+}(x,\theta) = \Phi(x,\theta) \qquad \text{and} \qquad \Phi^{-}(x,\theta) = \bar{\Phi}(x,\bar{\Theta}),$$

where $-$ is the complex conjugation. For the two-particle wave function we have

$$\Psi(x_1,x_2;\theta_1,\theta_2) = \begin{bmatrix} \Psi^{++}(x_1,x_2;\Theta_1,\Theta_2) \\ \Psi^{-+}(x_1,x_2;\bar{\Theta}_1,\Theta_2) \\ \Psi^{+-}(x_1,x_2;\Theta_1,\bar{\Theta}_2) \\ \Psi^{--}(x_1,x_2;\bar{\Theta}_1,\bar{\Theta}_2) \end{bmatrix}, \qquad (4.3)$$

where

$$\Psi^{\alpha,\beta} = \langle 0 | T(\Phi^{\alpha}(x_1,\Theta_1)\Phi^{\beta}(x_2,\Theta_2)) | p,j,j_3 \rangle . \qquad (4.4)$$

Superfields Φ^{+-} contain components with spin 0 and 1/2 and consequently the states $|p,j,j_3\rangle$ have the spin

$$j = \ell\ ,\ \ell \pm 1/2\ ,\ \ell \pm 1, \qquad (4.5)$$

where ℓ is the orbital momentum with respect to the center-of-mass system. The transformation law of the states $|p,j,j_3\rangle$ with respect to the supertransformations is not discussed here.

From (3.9) it follows that determination of the quasipotential requires the inverse Green function $\tilde{G}_0^{-1}$ to be found. The corresponding supersymmetric four-particle two-time Green function according to (3.6) is given by

$$\tilde{G}_0(E,\vec{q},\vec{q}\,',\theta_1,\ldots,\Theta_4) = \int_{-\infty}^{\infty} dq_0 dq_0' G_0(E,q,q',\theta_1,\ldots,\Theta_4). \qquad (4.6)$$

Here G_0 has a matrix form (4.1) with matrix elements in the free case

$$G_0^{\alpha,\beta,\gamma,\delta} = D_0^{\alpha\gamma}(E,q,\theta_1,\theta_3)\, D_0^{\beta,\delta}(E,q,\theta_2,\theta_4)\,\delta^{(4)}(q-q'), \tag{4.7}$$

where D_0 are free supersymmetric propagators given in Appendix A. Substituting (4.7) in (4.6), after integration over q_0 and q_0', we have:

$$\tilde{G}_0^{++,++} = m^2 J_0\, \delta^{\Gamma}(\theta_1-\theta_3)\,\delta^{\Gamma}(\theta_2-\theta_4)\,\delta^{(3)}(\vec{q}-\vec{q}'),$$

$$\tilde{G}_0^{++,-+} = \frac{m}{2}\,\delta^{\Gamma}(\theta_2-\theta_4)\,\delta^{(3)}(\vec{q}-\vec{q}')\exp(\theta_1 \underset{\sim}{\mathcal{P}}\bar{\theta}_3)[J_0+2\xi^2 J_2]\,,$$

$$\tilde{G}_0^{++,+-} = \frac{m}{2}\,\delta^{\Gamma}(\theta_1-\theta_3)\,\delta^{(3)}(\vec{q}-\vec{q}')\exp(\theta_2 \underset{\sim}{Q}\bar{\theta}_4)[J_0+2\eta^2 J_2],$$

$$\tilde{G}_0^{++,--} = \frac{1}{4}\,\delta^{(3)}(\vec{q}-\vec{q}')\exp(\theta_1 \underset{\sim}{\mathcal{P}}\bar{\theta}_3+\theta_2 \underset{\sim}{Q}\bar{\theta}_4)[J_0+2(\xi+\eta)^2 J_2+4\xi^2\eta^2 J_4],$$

$$\tilde{G}^{-+,++} = \frac{m}{2}\,\delta^{\Gamma}(\theta_2-\theta_4)\,\delta^{(3)}(\underline{q}-\underline{q}')\exp(\bar{\theta}_1\tilde{\mathcal{P}}\theta_3)[J_0+2\bar{\xi}^2 J_2],$$

$$\tilde{G}_0^{-+,-+} = -m^2\,\delta^{\Gamma}(\bar{\theta}_1-\bar{\theta}_3)\,\delta^{\Gamma}(\theta_2-\theta_4)\,\delta^{(3)}(\underline{q}-\underline{q}'), \tag{4.8}$$

$$\tilde{G}_0^{-+,+-} = -\frac{1}{4}\,\delta^{(3)}(\underline{q}-\underline{q}')\exp(\bar{\theta}_1\tilde{\mathcal{P}}\theta_3+\theta_2 \underset{\sim}{Q}\bar{\theta}_4)[J_0+2(\bar{\xi}+\eta)J_2+4\bar{\xi}^2\eta^2 J_4],$$

$$G^{+-,++} = \frac{m}{2}\,\delta^{(3)}(q-q')\,\delta^{\Gamma}(\theta_1-\theta_3)\exp(\bar{\theta}_2\tilde{Q}\theta_4)[J_0+2\bar{\eta}^2 J_2].$$

The remaining elements of $\tilde{G}_0$ can be found from (4.8) by complex conjugation of the coefficients of J_0, J_2 and J_4. In the formulas (4.8) the following notation is used

$$\xi = \theta_1\sigma_0\bar{\theta}_3, \quad \eta = \theta_2\sigma_0\bar{\theta}_4,$$
$$\mathcal{P} = (E, 2\underline{q}), \quad Q = \mathcal{P}(E,-\underline{q}) = (E,-2\underline{q})$$

and

$$J_k = \int_{-\infty}^{\infty} dq_0\, q_0^k\, [(\tfrac{1}{2}E+q_0)^2 - w^2 + i\epsilon]^{-1}\,[(\tfrac{1}{2}E-q_0)^2 - w^2 + i\epsilon]^{-1},$$

where $w = \sqrt{\underline{q}^2+m^2}$. After integration over q_0, we have

$$J_1 = 0,$$

$$J_0 = i\pi\,[\,2w(\tfrac{1}{4}E^2 - w^2 + i\epsilon\,)]^{-1}, \qquad (4.9)$$

$$J_2 = i\pi/2w$$

Consequently, J_0 is just the two-time Green function for the scalar fields [1].

It can be verified that the two-fermion component of $\tilde{G}_0$ coincides with the corresponding two-time Green function for the free spin 1/2 particles [2,14]. This Green function can be found from G_0 (4.8) as the coefficient of the first power in Θ_1, Θ_2, Θ_3 and Θ_4. However, it is known [2,14] that the two-time fermionic Green function has no inverse in the whole 16-component spinor space. The resolvent operator can be found only in the 8-dimensional subspace only with equal sign of energy of both the particles*. This subspace can be separated using the projection operator $\Lambda_\pm$ onto subspaces with the positive and negative energy. Operators $\Lambda_\pm$ have a simple form in the Foldy-Worthnysen representation. The transition to the F-W transformation is made by operators

$$T^{(1)}(\underline{q}) = \frac{m + w + \underline{\gamma}^{(1)}\cdot\underline{q}}{\sqrt{2w(m+w)}} = \frac{1}{\sqrt{2w(m+w)}}\begin{pmatrix} m+w & \underline{\sigma}^{(1)}\cdot\underline{q} \\ -\underline{\sigma}_0^{(1)}\underline{q} & m+w \end{pmatrix}$$

$$T^{(2)}(\underline{q}) = \frac{m + w - \underline{\gamma}^{(2)}\cdot\underline{q}}{\sqrt{2w(m+w)}} = \frac{1}{\sqrt{2w(m+m)}}\begin{pmatrix} m+w & -\underline{\sigma}^{(2)}\cdot\underline{q} \\ \underline{\sigma}^{(2)}\cdot\underline{q} & m+w \end{pmatrix}$$

(4.10)

For the matrix γ_0 we use the representation $\gamma_0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$. The superscripts 1,2 in (4.10) indicate the particle on which $T(\underline{q})$ acts.

In the supersymmetric case the wave function (4.3) is decomposed in Θ_1 and Θ_2. The corresponding coefficients, the components of the superwave functions are denoted by $\Psi(a,b)$, where $(a,b=0,1,2)$.

Then, the Foldy-Wouthuysen transformation in the supersymmetric case is defined in the following way

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*Which is the case of Majorana spinors.

$$\tilde{\Psi}_F(\underline{q},1,1) = T^{(1)}(\underline{q})T^{(2)}(\underline{q})\tilde{\Psi}(\underline{q},1,1),$$

$$\tilde{\Psi}_F(\underline{q},1,\alpha) = T^{(1)}(\underline{q})\tilde{\Psi}(\underline{q},1,\alpha),$$

$$\tilde{\Psi}_F(\underline{q},\alpha,1) = T^{(2)}(\underline{q})\tilde{\Psi}(\underline{q},\alpha,1), \tag{4.11}$$

$$\tilde{\Psi}_F(q,\alpha,\beta) = \tilde{\Psi}(q,\alpha,\beta) \qquad (\alpha,\beta=0,2).$$

The corresponding projection operators on the state with definite sign of energy in the F.W. representation are given by

$$\Lambda_{\pm}^{(1,2)} = \frac{I^{(1,2)} \pm \gamma_0^{(1,2)}}{2} = \frac{1}{2}\begin{bmatrix} I^{(1,2)} & \pm\sigma_0^{(1,2)} \\ \pm\sigma_0^{(1,2)} & I^{(1,2)} \end{bmatrix}, \tag{4.12}$$

which act on the fermionic components. For the Majorana spinor $\Psi(x)$ we have $\Lambda_-\Psi=0$ and consequently

$$\Lambda_-^{(1,2)}\Psi(1,1)=0. \tag{4.13}$$

Then, without loss of invariance with respect to the spatial reflections, the following projection operators

$$\Pi_1 = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}, \tag{4.14}$$

can be introduced, where I is a 2x2-identity matrix. Applying these operators to the components of the wave function (4.11), we have

$$\hat{\Psi}_F^{(1,2)}(\underline{q},a,b) = \Pi_{(1,2)}\tilde{\Psi}_F(\underline{q},a,b). \tag{4.15}$$

Then the following super-wave functions can be formed

$$\hat{\Psi}_F^{(1)}(\underline{q},\Theta_1,\Theta_2) = \begin{bmatrix} \tilde{\Psi}_F^{++}(\underline{q},\Theta_1,\Theta_2) \\ \tilde{\Psi}_F^{-+}(\underline{q},\bar{\Theta}_1,\Theta_2) \end{bmatrix} \tag{4.16}$$

and

$$\hat{\Psi}_F^{(2)}(\underline{q},\Theta_1,\Theta_2) = \begin{bmatrix} \tilde{\Psi}_F^{+-}(\underline{q},\Theta_1,\bar{\Theta}_2) \\ \tilde{\Psi}_F^{--}(\underline{q},\bar{\Theta}_1,\bar{\Theta}_2) \end{bmatrix}. \tag{4.17}$$

where components of the super-wave functions $\hat{\Psi}^{\pm}$ are given by (4.15). Corresponding two-time Green functions are given by

$$\hat{G}^{1}_{0F} = \begin{bmatrix} {}^{1}\hat{G}^{++}_{0F} & {}^{1}\hat{G}^{+-}_{0F} \\ {}^{1}\hat{G}^{-+}_{0F} & {}^{1}\hat{G}^{--}_{0F} \end{bmatrix} = \begin{bmatrix} \tilde{G}^{++,++}_{0F} & \tilde{G}^{++,-+}_{0F} \\ \tilde{G}^{-+,++}_{0F} & \tilde{G}^{-+,-+}_{0F} \end{bmatrix} . \qquad (4.18)$$

and

$$\hat{G}^{2}_{0F} = \begin{bmatrix} {}^{2}\hat{G}^{+,+}_{0F} & {}^{2}\hat{G}^{+,-}_{0F} \\ {}^{2}\hat{G}^{-,+}_{0F} & {}^{2}\hat{G}^{--}_{0F} \end{bmatrix} = \begin{bmatrix} \tilde{G}^{+-,+-}_{0F} & \tilde{G}^{+-,--}_{0F} \\ \tilde{G}^{--,+-}_{0F} & G^{--,--}_{0F} \end{bmatrix} . \qquad (4.19)$$

Here $\tilde{G}^{\alpha,\beta,\gamma,\delta}_{0F}$ can be obtained from (4.8) by the substitution

$$\begin{aligned} &\exp 2\theta_j \underline{q}\theta_k \rightarrow 1 + 4\underline{q}^2 \delta^{\Gamma}(\theta_j)\,\delta^{\Gamma}(\bar{\theta}_k), \\ &m\,\delta^{\Gamma}(\theta_j - \theta_k) \rightarrow m\,\delta^{\Gamma}(\theta_j) + m\,\delta^{\Gamma}(\theta_k) + w\theta_j \epsilon \theta_k , \end{aligned} \qquad (4.20)$$

where $w = \sqrt{q^2 + m^2}$, i.e., the F.W. transformation is performed.

From the condition (3.10) we can determine the corresponding to $\hat{G}^{(1,2)}_{0,F}$ resolvents. The explicit form of ${}^{1}\hat{G}^{-1}_{0,F}$ is given by

$$\begin{aligned} ({}^{1}\hat{G}^{-1}_{0,F})^{+,+} = \frac{i}{\pi}\{ & w[\delta^{\Gamma}(\theta_1)\delta^{\Gamma}(\theta_2) + \delta^{\Gamma}(\theta_1)\delta^{\Gamma}(\theta_4) + \delta^{\Gamma}(\theta_2)\delta^{\Gamma}(\theta_3) + \delta^{\Gamma}(\theta_3)\delta^{\Gamma}(\theta_4)] + \\ & + \frac{2w^2}{m}\theta_1\epsilon\theta_3[\delta^{\Gamma}(\theta_2) + \delta^{\Gamma}(\theta_4)] + m\theta_2\epsilon\theta_4[\delta^{\Gamma}(\theta_1) + \\ & + \delta^{\Gamma}(\theta_3)] - 2w\theta_1\epsilon\theta_3\theta_2\epsilon\theta_4\}, \end{aligned}$$

$$\begin{aligned} ({}^{1}\hat{G}^{-1}_{0,F})^{-+} = \frac{i}{\pi}\{ & \frac{w}{2m}[\delta^{\Gamma}(\theta_2) + \delta^{\Gamma}(\theta_4)] - \frac{w}{2m}[m^2 + 2(\frac{E^2}{4} - w^2)] \times \\ & \times \delta^{\Gamma}(\bar{\theta}_1)\delta^{\Gamma}(\theta_3)[\delta^{\Gamma}(\theta_2) + \delta^{\Gamma}(\theta_4)] + \frac{1}{2}\theta_2\epsilon\theta_4 \\ & - \frac{Ew}{m}(\bar{\theta}_1\theta_3)[\delta^{\Gamma}(\theta_2) + \delta^{\Gamma}(\theta_4) + E(\bar{\theta}_1\theta_3)\theta_2\epsilon\theta_4 \\ & + 2[m^2 + 2(\frac{E^2}{4} - w^2)]\theta_2\epsilon\theta_4\,\delta^{\Gamma}(\bar{\theta}_1)\delta^{\Gamma}(\theta_3)\}. \end{aligned} \qquad (4.21)$$

Remaining elements of ${}^{1}\hat{G}_{0,F}^{-1}$ can be found from (4.21) by complex conjugation of variables θ_1 and θ_3. The Green function ${}^{2}G_{0F}^{-1}$ can be obtained from ${}^{1}\hat{G}_{0F}^{-1}$ by complex conjugation of all variables Θ.

Now we can write a supersymmetric quasipotential equation of the Logunov-Tavkhelidze type for the two-particle super-wave function. In the super-center-of-mass systen it is given by

$$ {}^{(1,2)}\hat{G}_{0,F}^{-1} \, \hat{\Psi}_{F}^{(1,2)} = \hat{V}_{(1,2)} \hat{\Psi}_{F}^{(1,2)}, \qquad (4.22) $$

where integration over the intermediate momentum and spinor variables Θ should be taken into account. Here the quasipotential $\hat{V}$ can be determined in perturbative way from quantum field theory, as in the ordinary case. These potentials have the matrix structure as the Green functions (4.18) and (4.19), respectively. The explicit form of the potential depends on the interaction Lagrangian. Because of a cumbersome structure of the equations corresponding to (4.22) for the components of the super-wave function they are not written here. Note only that the equations for the scalar and spinor components coincide with the corresponding quasipotential eqs. in the ordinary theory [1,2].

APPENDIX A

The simplest scalar chiral superfields are determined by the equations

$$ \bar{D}_{\dot{a}} \Phi^{+}(x,\theta)=0, \qquad (a,\dot{a}=1,2). \qquad (A.1) $$
$$ D_{a} \Phi^{-}(x,\bar{\theta})=0. $$

Here D_a, $\bar{D}_{\dot{a}}$ are supercovariant derivatives (see [13]). For our purposes it is convenient to use the two-component spinor formalism. In the nonsymmetric representations the fields Φ^+ and Φ^- are given by

$$ \Phi^{+}(x,\theta) = \frac{1}{2}(A(x)-iB(x)) + \theta^{a}\phi_{a}(x) + \frac{1}{2}\theta\epsilon\theta\,(F(x)+iG(x)) \qquad (A.2) $$
$$ \Phi^{-}(x,\bar{\theta}) = \frac{1}{2}(A(x)+iB(x)) + \theta_{\dot{a}}\,\phi^{\dot{a}}(x) + \frac{1}{2}\bar{\theta}\epsilon\bar{\theta}(F(x) -iG(x)), $$

where A and F are real scalar fields, B and G are real pseudo-scalar fields and ϕ two-component spinor fields. The corresponding supersymmetric propagators are given by:

$$D^{++}(x_1-x_2;\theta_1,\theta_2) = <0|T(\Phi^+(x_1,\theta_1)\Phi^+(x_2,\theta_2))|0> = $$
$$= m\delta^\Gamma(\theta_1-\theta_2)\Delta_c(x_1-x_2;m),$$
$$\text{(A.3)}$$
$$D^{+-}(x_1-x_2;\theta_1,\bar\theta_2) = <0|T(\Phi^+(x_1,\theta_1)\Phi^-(x_1,\bar\theta_2))|0>$$
$$= \frac{1}{2}e^{-2i\theta_1\tilde\partial\bar\theta_2}\quad\Delta_c(x_1-x_2;m),$$

where $\Delta_c(x,m)$ is the Feynman propagator and $\partial = \sigma_\mu\partial^\mu$, σ_0 is an identity 2x2 matrix and $\sigma_j\,(j=1,2,3)$ are the Pauli matrices.

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