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AN ALTERNATIVE CALCULATION OF THE INSTANTON DRIVEN β-FUNCTION IN YANG-MILLS THEORY

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The existence of both the weak /1/ and strong /2/ coupling regime in SU(N) lattice gauge theories has become a well-known fact. They are clearly distinguished by the corresponding branches of the β-function, and the continuous transition between these regimes /3/ seems to take place over a narrow interval in g. Interesting phenomena seem to happen in the transition region /4/, but the typical relevant field configurations are not yet identified sufficiently.

Within the continuum Yang-Mills theory so far the only mechanism to provide a breakdown of perturbative behaviour of the β-function, still at small coupling, is the instanton mechanism, proposed by Callan, Dashen and Gross (CDG) /5/. There are however some ingredients in their analysis which look foreign to the usual field theory formulation: the "instanton medium" and the "vacuum permeability" μ_{vac} are used for renormalizing multiplicatively the fields as well as the coupling constant in the following form

$$g_{ren}^2(a) = \mu_{vac}(\varsigma_c) g^2(a), \quad g^2(a) = \frac{8\pi^2}{b_N \ln(1/a\Lambda)}, \quad b_N = 11N/3. \quad (1)$$

ς_c represents the usual infrared cutoff for the instanton scale ς, whereas a is the lattice constant of an effective lattice gauge theory with a coupling $g_{ren}(a)$. In order to obtain the latter by constrained functional integration, CDG argued for setting $a \simeq \varsigma_c$, the only length scale in the game. Apart from the lack of a proof for this, it is not clear in what respect eq.(1) reflects correctly renormalization group properties as known from perturbation theory. This holds also for other definitions of a running coupling constant with inclusion of instanton effects, e.g., in Ref./6/.

In this paper we propose another, more conventional way to estimate the instanton contributions to the β-function by considering leading radiative as well as semiclassical corrections to the gluon two- and three-point functions of pure Yang-Mills theory. We define the connected Green functions with inclusion of instanton sectors via the generating functional W(J) which can be written within the dilute gas approximation (DGA) /7/ - without any account of instanton interactions and up to corrections of order $O(g_0^2)$ - as follows /8/

$$W(J) = W_0(J) + \sum_{\pm} \int d^4y \int \frac{d\rho}{\rho} \int dR \; d(\rho) \times$$
$$\times \left[e^{-\int A_{inst} J} e^{\frac{1}{2} \int J (D(A_{inst}) - D_0) J} (1 + O(g_0^2)) - 1 \right], \quad (2)$$

where $W_0(J)$ is the generating functional corresponding to the usual perturbation theory and $D(A^{inst})$ and D_0 are the gluon propagators in the one-instanton and A=0 background, respectively. Therefore, in the leading order in g_0, the nonperturbative sectors contribute to the two- and three-point functions only the classical fields $A^{inst} \sim O(1/g_0)$, weighted by the instanton amplitude $d(\rho)$, given in one-loop approximation /9,7/

$$d(\rho) = C_N \, x_0^{2N} e^{-x_0} (M\rho)^{b_N} e^{-A(\bar{\rho})\rho^2} \rho^{-4}, \qquad x_0 = \frac{8\pi^2}{g_0^2}. \quad (3)$$

Here g_0 is understood as the unrenormalized coupling constant; then M plays the role of a regularization mass parameter according to the Pauli-Villars method used by 't Hooft /9/. The exponential cutoff corresponds to a hard core enforcement of diluteness invented in Ref./10/, with $A(\bar{\rho}) = a_N/\bar{\rho}^2$ and $a_N = (b_N-4)/2$. $\bar{\rho}$ is the average size of instantons being in one-to-one correspondence with the diluteness parameter a' which controls the smallness of interactions between two (anti)instantons, $a' < |y_1-y_2|^4/\rho_1^2\rho_2^2$. This parameter is estimated in Ref./10/ from the positivity of the action as $a' \gtrsim O(100)$. Using the Fourier transform of the singular gauge instanton

$$\overset{\pm}{A}{}^a_\mu(k) = 4\pi^2 i \, R^{\alpha a} \, \overset{\mp}{\eta}_{\alpha\mu\nu} \frac{k_\nu}{k^2} \frac{\rho^2}{g_0} F(k\rho)$$
$$F(\rho') = \frac{4}{\rho'^2}\left(1 - \frac{\rho'^2}{2} K_2(\rho')\right) \quad (4)$$

($R^{\alpha a}$ - adjoint representation of SU(N), $\overset{\pm}{\eta}$ - the 't Hooft tensor /9/) we find the unrenormalized, unamputated three-point function taken at the symmetric point $k_i k_j = -k^2(1-3\delta_{ij})/2$ (in Landau gauge)

$$G^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)\Big|_{s.p.} = i(2\pi)^4\delta^4(\textstyle\sum k_i)\Big[1 + \frac{61N}{12} x_0^{-1} \ln\frac{M}{k} +$$
$$+ \frac{\pi^2 C_N}{N(N^2-1)} x_0^{2N+2} e^{-x_0} \left(\frac{M}{k}\right)^{b_N} \left(1 + O(g_0^2)\right)\Big] \frac{1}{k^6} \times \quad (5)$$
$$\times g_0 f^{a_1a_2a_3} \left\{ \delta_{\mu_1\mu_2}(k_1-k_2)_{\mu_3} + \text{cycl.} \right\} + kkk\text{-terms}$$

We have denoted

$$I_m(k\bar{\rho}) = \int_0^\infty \frac{d\rho'}{\rho'} \rho'^{\,b_N+m} F^3(\rho') \exp\left(-a_N\left(\frac{\rho'}{k\bar{\rho}}\right)^2\right)$$

and took into account the divergent contributions from the perturbative one-loop corrections. In the same way one proceeds to calculate the leading instanton contribution to the gluon propagator. To renormalize the Green functions one could choose a minimal subtraction procedure by defining Z-factors that contain only the perturbative logarithmic divergence. Via coupling constant renormalization the instanton contribution became well-defined then; however, the corresponding β-function would be identical with the usual, perturbative one-loop result /11/. We choose to apply a momentum subtraction prescription, which is related to the minimal one by a finite renormalization. We demand the renormalized three-gluon vertex to exhibit the zeroth order vertex structure, with the bare coupling constant replaced by the renormalized one at some subtraction point $k^2 = \mu^2$. Analogously we deal with the gluon propagator. Then eq.(5) serves to identify the Z-factor combination $Z_3^{\frac{3}{2}} Z_1^{-1}$ with instanton contribution. The knowledge of Z_3 from the propagator (gluon wave function) renormalization leads finally to

$$Z_3^{3/2} Z_1^{-1}\left(\frac{M}{\mu}, \mu\bar{\varsigma}\right) = 1 + \frac{b_N}{2} x_0^{-1} \ln\frac{M}{\mu} + O(g_0^4) + \\ + \frac{\pi^2 C_N}{N(N^2-1)} x_0^{2N+2} e^{-x_0} \left(\frac{M}{\mu}\right)^{b_N} I_2(\mu\bar{\varsigma}) \left(1 + O(g_0^2)\right) \qquad (6)$$

which renormalizes the coupling constant as $g_{ren} = Z_3^{3/2} Z_1^{-1} g_0$. The instanton contribution to Z_3 does not explicitly appear in eq.(6), since it is suppressed by $O(g_0^2)$. We obtain the β-function from

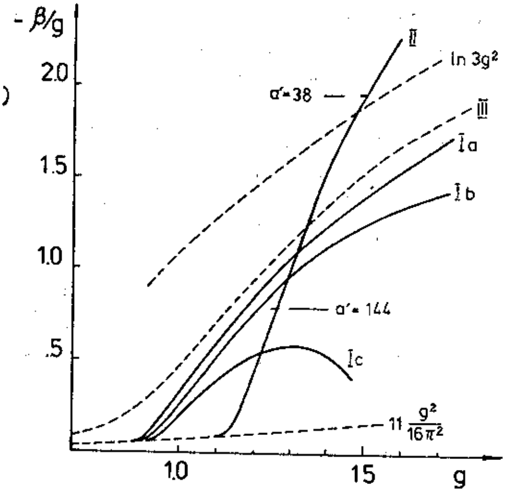
$$\beta = g_0 \mu \frac{\partial}{\partial\mu} Z_3^{3/2} Z_1^{-1} = -\frac{b_N}{2} \frac{g^3(\mu)}{8\pi^2} \Big\{ 1 + O(g^2(\mu)) + \frac{2\pi^2 C_N}{N(N^2-1)} x(\mu)^{2N+3} e^{-x(\mu)} \times \\ \times \left(I_2(\mu\bar{\varsigma}) - 2 a_N I_4(\mu\bar{\varsigma}) / b_N(\mu\bar{\varsigma})^2 \right) \left(1 + O(g^2(\mu))\right) \Big\} \qquad (7)$$

where the one-loop running coupling constant

$$x(\mu) = \frac{8\pi^2}{g(\mu)^2} \equiv x_0 - b_N \ln\frac{M}{\mu} = b_N \ln\frac{\mu}{\Lambda}$$

has been used in order to eliminate x_0, and we represent β parametrically as $g_{ren} = g_{ren}(g(\mu), \bar{\varsigma}\Lambda)$, $\beta = \beta(g(\mu), \bar{\varsigma}\Lambda)$. Here $\bar{\varsigma}(a')\Lambda$ appears as a free parameter bounded only by the above-mentioned diluteness criterion. In order to compare with Euclidean lattice calculations we have to choose a corresponding regularization scheme which is tantamount to a change of the Λ parameter and the overall constant C_N: $\Lambda_{PV}/\Lambda_{Latt} = 31.3$ /12/, $C_N^{PV}/C_N^{Latt} = (\Lambda_{PV}/\Lambda_{Latt})^{-b_N}$. For some values of a' we have plotted the resulting β-function in the Figure. Still tolerable seems to be the curve a'=114, whereas a' = 30 is already in a region where the DGA cannot be trusted any-

SU(3) gauge theory β-function with instanton contributions, according to (I) momentum space subtraction (described in this work) for different degrees of diluteness: (a) a'=30, $\bar{\varsigma}\Lambda_{Latt}$=.0055, (b) a'=114, $\bar{\varsigma}\Lambda_{Latt}$=.0048, (c) a'=691, $\bar{\varsigma}\Lambda_{Latt}$=.0040, (II) CDG /5/ renormalization (eq.(1)) with a= $\bar{\varsigma}$ (a')/10/ (without instanton interactions), Curve III shows a Padé extrapolation of the Euclidean strong coupling expansion /2/. The dashed curves show the leading terms of the strong and weak coupling expansions, respectively.



more /10/. There are two salient features visible: the departure from the perturbative β-function happens at $g \approx 0.9$ almost independently of the diluteness, and the slope fits well with Padé extrapolated strong coupling result /2/.

The renormalization scheme chosen here is conceptionally very far from the CDG /5/ procedure, the result of which is shown for comparison, too. According to eq.(1), diluteness changes while one approaches the strong coupling branch of the β-function. Nevertheless, the instanton gas is dilute enough to upset any hope that instanton interaction might lead to a smooth bend-over into the strong coupling regime. In our earlier papers /10/ we have pointed out that a cross-over is inevitable within the CDG coupling constant renormalization prescription. Rather we have used this in order to determine there the maximal space-time packing fraction $f \simeq .01$ of the dilute instanton gas, relying on another mechanism which should suddenly take over. In the present formulation however, there is no cross-over at all, and a reasonably dilute gas does well over the whole intermediate coupling region. It remains to be investigated whether account of instanton interactions and higher order (in g^2) terms will spoil this nice picture.

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Note added in proof:

After completion of this paper we received the Tohoku University preprint TU/80/212 by M.Honda "Vacuum Stability of QCD and Constraint on β -Function" where a bound for the rise of $-\beta/g$ is derived. The result obtained in the present letter (curves I) fulfils this bound while curve II does not.

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