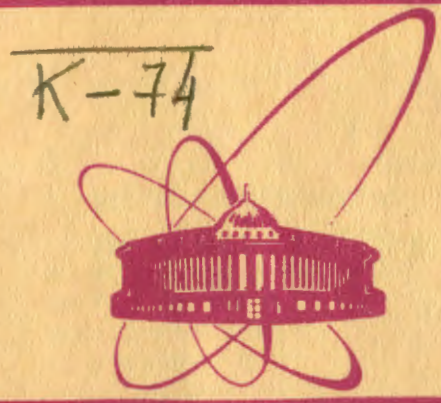


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HADRON-NUCLEUS SCATTERING  
IN CONSTITUENT QUARK MODEL

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## 1. THE EIGENSTATE METHOD

The notion of elastic diffractive scattering due to presence of inelastic channels has come into quantum mechanics from optics. In the works by Feinberg and Pomeranchuk<sup>/1/</sup> it has been demonstrated for the first time that analogous diffractive mechanism leads to the characteristic phenomena of diffractive dissociation. These ideas have been further developed by Good and Walker<sup>/2/</sup>.

The operator of the scattering matrix has nondiagonal form in the basis of hadronic states because real hadrons can be transformed via scattering to new states. In the Glauber-Sitenko approximation<sup>/3/</sup> the nondiagonal transitions are neglected. These contributions correspond to the production of inelastic intermediate states in the multiple rescattering of hadrons in the nucleus<sup>/4/</sup>. The inelastic screening reduces the total hadron-nucleus cross section by approximately 10% for heavy nuclei<sup>/5,6/</sup>. On the contrary, in the process of diffractive dissociation of hadrons on nuclei inelastic screening enlarges the value of the cross section<sup>/7/</sup>. This effect which is left out of consideration in the generally accepted procedure of the analysis of experimental data leads to a paradoxical conclusion about the abnormally small interaction cross section of an unstable system with nucleons.

Considering the diffraction processes it is convenient to use the basis of the scattering amplitude eigenstates denoted by  $|k\rangle$ , where  $k$  labels the states. The physical hadron states  $|a\rangle$  can be expanded in this basis

$$|a\rangle = \sum_k C_k^a |k\rangle. \quad (1)$$

Index  $a$  in  $|a, k\rangle$  denotes those quantum numbers of the hadron which do not depend on the interaction with the target, defined by the index  $k$ :

$$\hat{f}|a, k\rangle = f_k |a, k\rangle. \quad (2)$$

Here and below the diffractive amplitude is assumed to be imaginary and its imaginary part is denoted by  $f$ . The basis of physical states  $|a\rangle$  is orthonormal but the states  $|a, k\rangle$  which are mutually orthogonal over index  $k$ , are not orthogonal in index  $a$ :

$$\langle \beta, \ell | a, k \rangle = \delta_{k\ell} B_k^{\alpha\beta} \quad (3)$$

The matrix element  $B_k^{\alpha\alpha}$  is equal to unity. The following relations are valid for the matrix  $B_k$ :

$$\sum_k C_k^\alpha (C_k^\beta)^* B_k^{\alpha\beta} = \delta_{\alpha\beta}, \quad (4)$$

$$\sum_\beta (C_k^\alpha)^* B_k^{\alpha\beta} B_\ell^{\beta\gamma} C_\ell^\gamma = \delta_{k\ell} B_k^{\alpha\gamma}. \quad (5)$$

From relations (1)-(5) it follows that diffractive amplitude for the transition  $\alpha \rightarrow \beta$  has the following form:

$$f_{\alpha\beta} = \langle \beta | \hat{f} | \alpha \rangle = \sum_k C_k^\alpha (C_k^\beta)^* B_k^{\alpha\beta} f_k. \quad (6)$$

This representation of the diffraction amplitude is particularly convenient for the investigation of the hadron-nucleus processes. Indeed, the time of mixing of states with different values of index  $k$ , which has the order of inverse hadronic mass in a hadron rest system, undergoes Lorentz dilatation in the laboratory system and has a large value of  $t \approx E/m^2$ . If hadron energy  $E$  is so high that the fluctuation time is much higher than the dimension of the nucleus that is  $E/m^2 \gg R_A$  than expansion (1) can be accepted as a stationary one. The interaction amplitude  $f_k$  of the nucleus -  $|k\rangle$  state can be calculated in Glauber approximation which in this case is exact since the  $|k\rangle$  state is subjected only to the elastic rescattering. This calculational method is equivalent to taking into account all the inelastic contribution to the Glauber-amplitude due to the condition of completeness in (5).

## 2. THE PARTON MODEL

The eigenstate method seems to be particularly useful in the framework of the parton model<sup>8,7,9</sup> which gives a clear space-time interpretation of the interaction.

In the parton model we assume for the incoming relativistic hadron the existence of a parton wave function that is of "prepared" parton fluctuations of some weights. All of these fluctuations have a definite number of partons distributed in longitudinal and transversal momentum. In the momentum region  $p_{||} \leq \mu^2 R$ , where  $R$  is the longitudinal size of the target, the parton number has fluctuation during the interaction time and it has indefinite value. All of these wee partons have a cloud of slower partons whose number changes during the interaction and these partons determine the effective

cross section of wee parton interaction with the target. Generally this cross section depends on  $R$  but this dependence is slow-logarithmic, so below we neglect it.

Since only wee partons interact with the target, their number determines the measure of amplitude  $f_k$  and so it can play the role of index  $k^{8-10}$ . Note that a state without wee partons ( $k=0$ ) can be accepted as a passive one since  $f_0=0$ .

Let us discuss the following question: what is the analogue of the inelastic corrections, decreasing the hadron-nucleus total cross section, in terms of the parton model. In the frame where the nucleus collides with the rest hadron the nucleus suffers Lorentz contraction and the longitudinal overlapping of parton clouds from different nucleons leads to the junction of parton clouds<sup>10</sup> and therefore to the decrease of the number of wee partons in the nuclei and thus to the drop of the interaction cross section. Turning from the parton model to the Reggeon graphs it is easy to find immediately that the junction of parton ladders directly corresponds to the inelastic contributions.

The interpretation is entirely different in lab. frame as the partons of different nucleons are separated in space. If one applies the optical approximation for amplitude  $f_k$  in expression (6)

$$f_k^{(A)}(\vec{b}) = 1 - \exp[-f_k^{(N)} \cdot T(\vec{b})], \quad (7)$$

where

$$T(\vec{b}) = \int_{-\infty}^{+\infty} dz \rho_A(z, \vec{b})$$

is the profile function of the nucleus at given  $\vec{b}$  one can use the inequality

$$\langle \exp[-f_k^{(N)} T(\vec{b})] \rangle \geq \exp[-\langle f_k^{(N)} \rangle T(\vec{b})], \quad (8)$$

where

$$\langle f_k \rangle = \sum_k |C_k^\alpha|^2 f_k.$$

The right side of expression (8) corresponds to the Glauber approximation. It can be seen from (7) and (8) that the amplitude (6) is smaller than that given by the Glauber-Sitenko approximation.

### 3. THE CONSTITUENT QUARKS. THE TWO COMPONENT APPROXIMATION

In the constituent quark model a hadron is considered as an entity of two or three valence quarks (valons) which determine the quantum numbers of hadron and each valon has a

cloud of sea partons. The wee sea partons interact with the target and this corresponds to the pomeron contribution. In addition, the valon, emitting sea partons, can slow down and interact with the target. The probability of finding a valon among the wee partons decreases with energy like a power function. These contributions to the scattering amplitude correspond to secondary reggeons. Later they will be discussed but now they are neglected.

Since the parton wave function can be ascribed to the constituent quark, expansion (1) can be carried out for the constituent quarks<sup>5,6/</sup>. It is convenient here to introduce an approximation neglecting the difference between the amplitude  $f_k$  in the active component of the constituent quark that is for  $k \geq 1$ . If one requires the equality of all amplitudes with  $k \geq 0$  then in accordance with completeness relation (4) the inelastic diffraction amplitude (6) turns to zero. It seems natural that there is an utmost difference between the  $f_0$  scattering amplitude in a passive state and amplitudes  $f_k$  with  $k \geq 1$ . So the above introduced two-component approximation is reasonable for constituent quarks.

Note that at asymptotic energies the two component approximation becomes exact as the relative weight of the active state tends to a constant value<sup>11/</sup> and the density distribution of wee partons in the impact parameter plane has a homogeneous parton density  $\rho_0$  inside a disc of radius R:

$$\rho(\vec{b}) = \rho_0 \theta(R^2 - \vec{b}^2),$$

where  $\theta(x)$  is a step function.

From the analysis of experimental data it will be argued later that the interaction amplitude of two constituent quarks at energies presently available at accelerators now is close to the asymptotical behaviour. Here we describe some simple asymptotic relations.

For the weight of the active component of hadron  $a$  we adopt notation introduced in<sup>11/</sup>:  $P_a = \sum_{k=1}^{\infty} |C_k^a|^2$ . Then on taking into account (8) the amplitude of interaction of two quarks with one of them in the rest frame is as follows

$$f(\vec{b}) = P_q F_0 \theta[R^2(Y) - \vec{b}^2]. \quad (9)$$

Here  $R^2$  depends on the rapidity  $Y = \ln(s/s_0)$ ,  $F_0$  is the amplitude of the wee parton-target interaction which does not depend on  $b$  when  $b < R(y)$ . Comparing (9) with the expression of the c.m. amplitude:

$$f(\vec{b}) = P_q^2 \theta[4R^2(Y/2) - \vec{b}^2]$$

we obtain that  $R(Y) = 2R(Y/2)$ , that is  $R$  is a linear function of  $Y$ . Besides one gets that  $F_0 = P_q < 1$ . It is natural that the disc of wee partons is not black. If the mutual screening of partons has a Glauber form, then:

$$F_0 = 1 - e^{-\rho_0 \sigma}, \quad (10)$$

where  $\sigma$  is a wee parton-rest quark cross section. Due to the junction process of partons the quantity  $\rho_0$  has a limited value, so  $F_0 < 1$ . This can be regarded as an argument for the existence of passive component of the quark. Now we consider the constituent quark-nucleus interaction. The partial quark-nucleus scattering amplitude in optical approximation has the following form<sup>5,6/</sup>.

$$f_{qA}(\vec{b}) = P_q \left[ 1 - \exp\left(-\frac{1}{2} \frac{\sigma_{qN}^{tot}}{P_q} T(\vec{b})\right) \right]. \quad (11)$$

Here  $\sigma_{qN}^{tot}/P_q$  is a total active quark-nucleon cross section. It is interesting that for infinitely large nucleus, that is for  $T(\vec{b}) \rightarrow \infty$ , the nucleus does not become black and  $f_{qA}(\vec{b})$  tends to the value  $f = P_q$ . The same result can be easily obtained in the system where a nucleus collides with the rest quark. As the clouds of wee partons of the nucleus quarks which are distributed only by longitudinal coordinate are overlapped and junctioned, a balance is formed with the parton density  $\rho_0$  as in the case of one quark. Therefore the quark-target amplitude is equal to  $P_q$ .

#### 4. THE AMPLITUDE OF HADRON-NUCLEUS ELASTIC SCATTERING

Let us consider the nucleon-nucleus and pion-nucleus scattering. As long as we take into account the vacuum pole contribution only, the difference in the interactions of  $u$  and  $d$  quarks and antiquarks, denoted below by symbol  $q$ , can be neglected. The strange quark scattering will be discussed separately.

Partial amplitude of the hadron-nucleus elastic scattering where hadron  $h$  contains  $k$  constituent quarks and the mass number of nucleus is  $A$  has the following form:

$$f_{hA}(\vec{b}) = \int \prod_{i=1}^k d^2 \vec{b}_i \prod_{j=1}^A d^2 \vec{r}_j \delta^2 \left( \frac{1}{k} \sum_{i=1}^k \vec{b}_i - \vec{b}_h \right) \times \quad (12)$$

$$\times \delta^2 \left( \frac{1}{A} \sum_{j=1}^A \vec{r}_j - \vec{r}_A \right) \rho_h(\vec{b}_1, \dots, \vec{b}_k) \rho_A(\vec{r}_1, \dots, \vec{r}_A).$$

$$\times \sum_{\ell=0}^k C_{\ell}^k P_q^{\ell} (1-P_q)^{k-\ell} \left\{ 1 - \prod_{m=1}^{\ell} \prod_{n=1}^A \left[ 1 - \frac{f_{qN}}{P_q} (\vec{b}_m - \vec{r}_n) \right] \right\}.$$

Here  $\vec{b} = \vec{b}_h - \vec{r}_A$ , where  $\vec{b}_h$  and  $\vec{r}_A$  are the coordinates of centers masses of hadron and nucleus, respectively,  $\vec{b}_i$  and  $\vec{r}_j$  are the coordinates in the  $\vec{b}$ -plane of quarks in the hadron  $h$  and of nucleons in the nucleus, respectively;  $\rho_h(\vec{b}_1, \dots, \vec{b}_k)$  is the quark distribution function in hadron  $h$ , which is normalized by the following condition

$$\int \rho_h(\vec{b}_1, \dots, \vec{b}_k) \delta^2 \left( \frac{1}{k} \sum_{i=1}^k \vec{b}_i - \vec{b}_N \right) \prod_{i=1}^k d^2 \vec{b}_i = 1. \quad (13)$$

Nucleon distribution function in the nucleus  $\rho_A(\vec{r}_1, \dots, \vec{r}_A)$  is normalized in the same manner.

The sum over  $\ell$  runs through a different combination of quark numbers having the quarks in active and passive states.  $f_{hN}(\vec{b}_m - \vec{r}_n) / P_q$  is the interaction amplitude of the  $m$ -th quark in the active state with the  $n$ -th nucleon of nucleus.

It is easy to see that if  $P_q = 1$  then expression (12) changes to a common Glauber like formula of the nucleus-nucleus scattering but its form does not agree with the Glauber amplitude for hadron-nucleus scattering since the introduction of the quark structure of hadron takes already into account a part of inelastic corrections (the *PPR* type terms).

For the sake of simplicity we accept the usual factorization assumption of nuclear density

$$\rho_A(\vec{r}_1, \dots, \vec{r}_A) = \prod_{i=1}^A \rho_A(\vec{r}_i - \vec{r}_A). \quad (14)$$

If one assumes that  $\rho_A(\vec{r}_i)$  has a Gaussian form that is  $\rho_A(\vec{r}_i - \vec{r}_A) = \rho_A(0) \exp[-(\vec{r}_A - \vec{r}_i)^2 / R_A^2]$ , then the  $\delta$ -function in (12), taking into account the nucleus center of mass motion, can be substituted in the amplitude  $F_{hA}(\vec{q})$  (in the  $\vec{q}$ -momentum transfer-representation) by the factor

$$K(\vec{q}) = \exp(\vec{q}^2 R_A^2 / 4A). \quad (15)$$

This factor does not give contribution to the total cross section and should be taken into account in the calculation of differential cross section only. Note that the Gaussian form of  $\rho_A(\vec{r}_i)$  for heavy nuclei is unrealistic but in this case the correction due to the nucleus center of mass motion is small so this approximation does not give an observable error. In further calculations we use the Woods-Saxon parametrization for  $\rho_A(\vec{r}_i)$  in heavy nucleus.

Taking into account (14) one performs integration over  $\vec{r}_j$  in (12), and obtains

$$\begin{aligned}
h_{hA}(\vec{b}) &= \int \prod_{i=1}^k d^2 \vec{b}_i \delta^2 \left( \frac{1}{k} \sum_{i=1}^k \vec{b}_i - \vec{b}_h \right) \rho_h(\vec{b}_1, \dots, \vec{b}_k) \times \\
&\times \sum_{\ell=0}^k C_{\ell}^k P_q^{\ell} (1-P_q)^{k-\ell} \{ 1 - [ \int d\vec{r} \rho_A(\vec{r} - \vec{r}_A) \times \\
&\times \prod_{m=1}^{\ell} (1 - f_{qN}(\vec{b}_m - \vec{r}) / P_q) ]^A \}.
\end{aligned} \tag{16}$$

For further transformation of this expression we assume that  $\rho_h(\vec{b}_1, \dots, \vec{b}_k)$  have a factorized form (13), too, and the  $\rho_h(\vec{b})$  single particle density has a Gaussian distribution with root mean square  $R_h^2$ . We assume as well Gaussian dependence on  $\vec{b}$  for the quark-nucleon scattering amplitude:

$$f_{qN}(\vec{b}) = f_{qN}(0) \exp(-\vec{b}^2 / R_{qN}^2).$$

Further transformations of expression (16) are presented in the Appendix.

The final formula for the calculation of the partial amplitude of the  $\pi$ -A scattering has the following form:

$$f_{\pi A}(\vec{b}) = 2P_q (1-P_q) F_{\pi A}^{(1)}(\vec{b}) + P_q^2 F_{\pi A}^{(2)}(\vec{b}), \tag{17}$$

where

$$F_{\pi A}^{(1)} = 1 - \left[ 1 - \frac{\sigma_{qN}^{tot}}{2P_q} I_1(\vec{b}) \right]^A \tag{18}$$

and

$$F_{\pi A}^{(2)} = 1 - \left[ 1 - \frac{\sigma_{qN}^{tot}}{P_q} I_1(\vec{b}) L_1 + \frac{(\sigma_{qN}^{tot})^2}{P_q^3} \frac{I_2(\vec{b}) L_2^{\pi N}}{32\pi(R_{\pi}^2 + R_{qN}^2)} \right]^A \tag{19}$$

Here  $\sigma_{qN}^{tot}$  is the total cross section of the quark-nucleon interaction;

$$I_n(\vec{b}) = \frac{1}{2\pi} \int e^{-R_{qN}^2 k^2 / n} \rho_A(k) J_0(kb) k dk, \tag{20}$$

$J_0(x)$  is the zero order Bessel function,

$$\rho_A(\vec{k}) = \int e^{i\vec{k}\vec{b}} \rho_A(\vec{b}) d^2 \vec{b} \tag{21}$$

is the single particle form factor of the nucleus, while  $\rho_A(0) = 1$ .

The pion size is taken into account by factors

$$L_1^{\pi N} = \exp(-R_{\pi}^2 / R_A^2), \tag{22}$$

$$L_2^{\pi N} = \exp(-R_{\pi}^2 / R_{qN}^2). \tag{23}$$



In the two component approximation the constituent strange quark  $s$  is different from  $u$  and  $d$  quarks only in the weight of the active component  $P_s$ . The cross section of the interaction in the active state does not depend on the sort of the quark:  $\sigma_{sh}^{tot}/P_s = \sigma_{qh}^{tot}/P_q$ . Therefore the  $K$ -meson-nucleus scattering amplitude has the form:

$$f_{KA}(\vec{b}) = (P_q + P_s - 2P_q P_s) F_{KA}^{(1)}(\vec{b}) + P_q P_s F_{KA}^{(2)}(\vec{b}). \quad (24)$$

The quantities  $F_{KA}^{(1)}(\vec{b})$  and  $F_{KA}^{(2)}(\vec{b})$  can be obtained from (18) and (19) by substituting  $R_\pi^2$  by  $R_K^2$ .

For the nucleon-nucleus scattering amplitude one obtains

$$f_{NA}(\vec{b}) = 3P_q (1 - P_q)^2 F_{NA}^{(1)}(\vec{b}) + 3P_q^2 (1 - P_q) F_{NA}^{(2)}(\vec{b}) + P_q^3 F_{NA}^{(3)}(\vec{b}). \quad (25)$$

The quantities  $F_{NA}^{(1)}$  and  $F_{NA}^{(2)}$  can be obtained from (18) and (19) by a change of  $R_\pi^2$  to  $R_N^2$ . The quantity  $F_{NA}^{(3)}$  has the form

$$F_{NA}^{(3)}(\vec{b}) = 1 - \left[ 1 - \frac{3}{2} \frac{\sigma_{qN}^{tot}}{P_q} I_1(\vec{b}) M_1^{NN} + 3 \frac{(\sigma_{qN}^{tot})^2}{P_q^3} \frac{I_2(\vec{b}) M_2^{NN}}{32\pi(R_N^2 + R_{qN}^2)} - \frac{(\sigma_{qN}^{tot})^3}{3P_q^5} \frac{I_3(\vec{b}) M_3^{NN}}{2^7 \pi^2 (R_N^2 + R_{qN}^2)^2} \right] A.$$

Here

$$M_1^{NN} = e^{-\frac{2}{3} \frac{R_N^2}{R_A^2}}, \quad M_2^{NN} = \exp\left(-\frac{R_N^2}{6R_A^2} - \frac{R_N^2}{R_{qN}^2}\right), \quad M_3^{NN} = e^{-2 \frac{R_N^2}{R_{qN}^2}}.$$

One can easily obtain the expression for the hyperon-nucleus scattering amplitude ( $Y = \Lambda, \Sigma$ )

$$f_{YA}(\vec{b}) = [2P_q (1 - P_q)(1 - P_s) + P_s (1 - P_q)^2] F_{NA}^{(1)}(\vec{b}) - [P_q^2 (1 - P_s) + 2P_q P_s (1 - P_q)] F_{NA}^{(2)}(\vec{b}) + P_q^2 P_s F_{NA}^{(3)}(\vec{b}). \quad (26)$$

Here we assume the same hyperon radius as for the nucleon.

## 5. COMPARISON WITH EXPERIMENT

The formulae of the previous section for the hadron-nucleus elastic scattering amplitude have two independent parameters:  $P_q$ , which is the weight of the active component of the constituent  $u$  or  $d$  quarks and  $\sigma_{qN}^{tot}$ , which is the total cross section of the quark-nucleon interaction. The values of these pa-

parameters can be easily established using nucleon-nucleus cross sections at high energies <sup>/12/</sup>.

As total cross sections for sufficiently heavy nuclei have no energy dependence within the experimental errors at energies higher than 100 GeV we have used for the analysis the 240 GeV <sup>/12/</sup> data. A Woods-Saxon form with parameters of work <sup>/12/</sup> was chosen for the nuclear density function. The result of analysis gives with  $\chi^2 \approx 1$  per degree of freedom (see Table 1).

Table 1

A	12	27	63	208
$\sigma_{tot}^{NA} (fm^2)$ exp	32.82 $\pm$ 0.21	62.95 $\pm$ 0.37	122.5 $\pm$ 1.1	291.9 $\pm$ 4.8
$\sigma_{tot}^{NA} (fm^2)$ theor	32.79	63.04	122.51	284.64
$\sigma_{tot}^{YA} (fm^2)$ theor (Y = $\Lambda, \Sigma$ )	29.23	56.44	110.28	258.97

$$P_q = 0.5, \quad \sigma_{tot}^{qN} = 16 \text{ mb.} \quad (27)$$

Formula (25) is different from the expression which was used for the analysis of neutron-nucleus total cross sections in work <sup>/6,13/</sup>. In these works the size of the nucleon compared with the size of the nucleus has been neglected. The terms of 2nd and 3rd order in factor  $\sigma_{qN}^{tot} / (R_N^2 + R_{qN}^2)$  in expressions (19) and (26) were also omitted. Numerically this gives a 10-20% difference in parameters  $P_q$  and  $\sigma_{tot}^{qN}$ .

The analysis of  $K_L^-$ -A total cross section data from ref. <sup>/12/</sup> gives for the weight of the active component of the strange quark  $P_s = P_q / 2$ .

Now we can predict the hyperon-nucleus cross section values. Using formula (26) and parameters  $P_s, P_q$  and  $\sigma_{qN}^{tot}$  we obtain the values presented in Table 1. for the  $\Lambda$  and  $\Sigma$  hyperons.

Having the parameters (27) one can calculate the differential cross sections of elastic scattering. The results of calculations are compared with the proton-nucleus data of work <sup>/14/</sup> in Fig. 1. In the calculation of the cross section the Coulomb contribution is taken into account and the contribution of the quasifree nuclear scattering is calculated in accordance with formula <sup>/15/</sup>

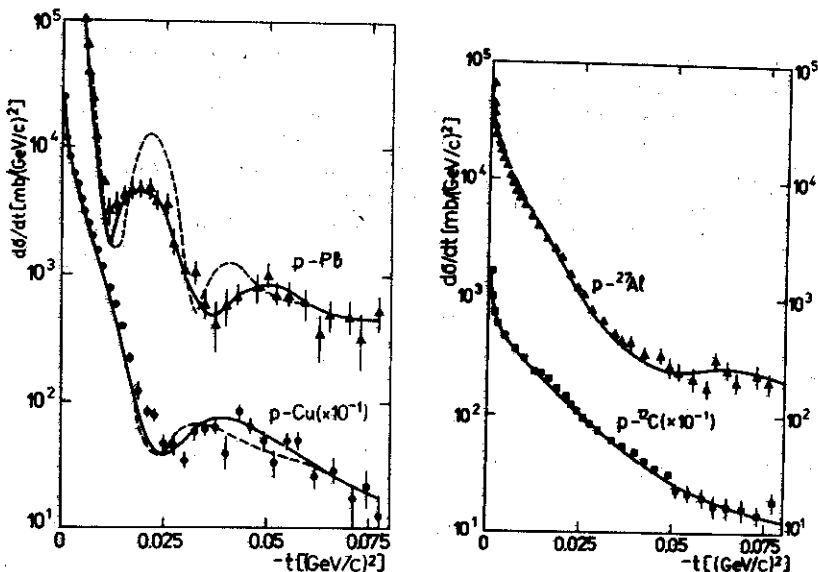


Fig. 1. The proton-nucleus differential cross sections for the  $^{63}\text{Cu}$  and  $^{207}\text{Pb}$  (Fig. 1a)  $^{12}\text{C}$ ,  $^{27}\text{Al}$  (Fig. 1b) nuclei at 175 GeV. The experimental points are taken from work<sup>14/</sup>. The solid lines are calculated in the quark parton model, the dotted lines correspond to the usual Glauber approximation. The Coulomb and the quasifree nuclear scattering contributions are taken into account, see text.

$$d\sigma_{\text{Q}}^{\text{NA}}/dt = \frac{d\sigma_{\text{el}}^{\text{NA}}}{dt} \cdot A_{\text{eff}} \quad (28)$$

where

$$A_{\text{eff}} = \int d^2b \left[ e^{-\sigma_{\text{in}}^{\text{NN}} T(b)} - e^{-\sigma_{\text{el}}^{\text{NN}} T(b)} \right] \quad (29)$$

Figures 1a, 1b contain also the results of the Glauber-Sitenko calculation. For the nucleus  $^{207}\text{Pb}$  there is a strong difference between the results of the two methods, which is decreased for smaller atomic number nuclei where the quasielastic background fills up the diffractive minima.

In work<sup>16/</sup> the differential cross section of the elastic  $p\text{-}^4\text{He}$  scattering has been measured without quasielastic background. The calculations with and without inelastic shadowing corrections presented in Figure 2 show the important role of inelastic screening. It is true that in this case the agreement

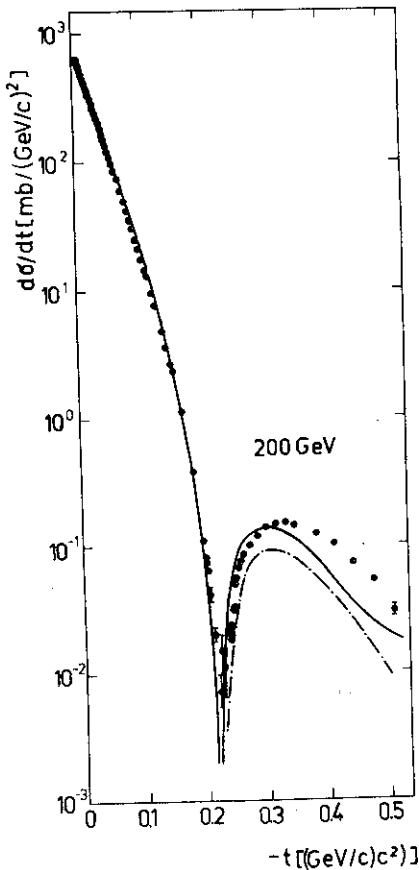


Fig.2. The proton -  $^4\text{He}$  differential cross section at 200 GeV. The experimental points are taken from work <sup>/18/</sup>. The solid lines are calculated in the quark-parton model, the dotted lines correspond to the usual Glauber approximation.

between the data and the calculation is not very good. This can be explained by the oversimplified form of the  $^4\text{He}$  wave function of ref. <sup>/17/</sup>.

#### 6. THE MIXTURE OF PARTON COMPONENTS. THE ENERGY DEPENDENCE OF CROSS SECTIONS AND THE SHRINKAGE OF DIFFRACTIVE CONE ON NUCLEI

During transmission through the nucleus the active state of the incident quark can convert into a passive one and vice versa. The contributions of these transitions, neglected above, can lead to observable effects as for instance energy dependence and additional real part of the elastic hadron-nucleus scattering <sup>/18/</sup>.

Let us investigate for illustration two channel problem with more rigid assumptions than earlier. We retain in expansion (1) only two components  $\partial|0\rangle$  and  $|1\rangle$ . Then the equation, describing the evolution of quark wave function during the nucleus transmission, has the following form <sup>/18/</sup>

$$\frac{d\psi}{dz} = i\hat{Q}\psi, \quad (30)$$

where the momentum operator is equal to

$$\hat{Q} = \begin{pmatrix} q + |C_1|^2 \Delta q & -C_0 C_1^* \Delta q \\ -C_0^* C_1 \Delta q & q + |C_0|^2 \Delta q - i f \end{pmatrix} \quad (31)$$

Here  $q$  is the quark momentum,  $f$  is the forward elastic quark-nucleon amplitude which, as we suppose, for the sake of simplicity does not depend on the longitudinal coordinate  $z$ . The mixture parameter  $\Delta q$  of states  $|0\rangle$  and  $|1\rangle$  has the order  $\mu^2/E$ , where  $\mu$  is some characteristic mass. The scattering amplitude on the layer of nuclear matter with thickness  $u$  has the form:

$$-iF(u) = 1 - \langle \psi_{out}(u) | \psi_{in}(u) \rangle, \quad (32)$$

where  $\psi_{out}(u)$  and  $\psi_{in}(u)$  are the solutions of equation (30) and the incoming wave at point  $z=u$ , respectively. From (30) and (32) we obtain /18/

$$-iF(u) = 1 - \exp\left(\frac{fu}{2} - i\Delta q \frac{u}{2}\right) \left[ \cos\left(\frac{\lambda u}{2}\right) - \frac{i\Delta q + (2P_q - 1)f}{\lambda} \sin\left(\frac{\lambda u}{2}\right) \right]. \quad (33)$$

Here  $P_q = |C_1|^2$

$$\lambda = [(\Delta q)^2 - f^2 - 2if\Delta q(2P_q - 1)]^{1/2}. \quad (34)$$

At high energies  $\Delta q \rightarrow 0$ , the parameter  $\lambda$  is imaginary,  $\lambda = if$  and expression (33) coincides with formula (11). And vice versa, at small energies, when the mixing is large, expression (33) can be expanded by the small parameter  $f/\Delta q$  and for the imaginary part of amplitude  $F$  we obtain

$$\text{Im}F(u) = 1 - e^{-P_q fu} - \frac{P_q(1-P_q)f^2}{(\Delta q)^2} e^{-P_q fu} \times \\ \times [1 - e^{-(2P_q - 1)fu} \cos(\Delta q u)]. \quad (35)$$

In limit  $f/\Delta q \rightarrow 0$  only the first two terms, corresponding to the Glauber-Sitenko approximation, are retained in this expression. This can be expected as the complete mixing takes place at distances of internuclear order. The last term of (35) is of interest too. In the investigated two-channel case it coincides with formula of Kondratyuk and Karmanov for the first inelastic correction /19/. Indeed, if, following the approach of work /19/, we assume that the quark and all the products of the quark's diffraction dissociation have the same interaction amplitude with nucleon, then  $P_q = 1/2$ . The factor  $\exp(-P_q fT)$  describes the absorption of particle in the nucleus. The factor  $P_q(1-P_q)f^2$  in the two channel approximation is equal to  $\sigma_{diff}$ . The factor  $[1 - \cos(\Delta q u)]/(\Delta q)^2$  of expression (35) coincides with the factor  $1/2 |F(\Delta q)|^2$  in

the formula of work<sup>/19/</sup>. In the given case the longitudinal form factor of nucleus is equal to  $F(\Delta q) = \int^0_1 dz \exp(i\Delta q z)$ .

It can be also seen from this comparison that the mixing of different components of hadron during transmission through a nucleus is equivalent to the influence of the nuclear form factor. This apparently leads to a decreasing energy dependence of the total hadron-nucleus cross section.

It is worth while noting that if one considers the mixing process on a probability level<sup>/20/</sup> one finds that corrections to asymptotic expression (11) decrease with energy as  $1/E$ . Our above analysis showed that the terms of the order of  $1/E$  in the amplitude have no imaginary part and only terms of the order of  $O(1/E^2)$  give a contribution to the total cross section.

The pass to the multi-channel problem<sup>/21/</sup> makes important alterations in the obtained results. The energy dependence of the mixing parameter  $\Delta q = \mu^2/E$  is due to the fact that we have retained only two states with fixed masses. In the real case when the energy is increased the nuclear form factor provides opportunity for the production of higher and higher masses in the intermediate state. In the parton model this means that although the hadron energy is increased there are always such components in the passive state for which the gap in the rapidity scale, unoccupied by partons, does not increase with energy. The existence of such a component leads to the continuous "pump over" of norm from the active state to the passive one with the increase of energy. It was shown in work<sup>/21/</sup> that this fact leads to relation between the section of diffraction dissociation into high mass and the logarithmic derivative of  $P_q$  over the rapidity:

$$s \frac{d\sigma_{diff}}{dq_{\perp}^2 dM^2} = \frac{s}{M^2} \frac{d\sigma_{el} P_q^{-2}(s)}{dq_{\perp}^2} \frac{dP_q(M^2)}{d \ln(M^2/s_0)} \quad (36)$$

The derivative  $dP_q/d \ln(s/s_0)$  is negative and turns to zero at  $s \rightarrow \infty$  in the pomeron theory with intercept  $\alpha_p(0) > 1$ . In the energy region available now the derivative is small and has a small energy dependence. This gives the explanation of the smallness of the triple pomeron constant and of the approximate Feynman scaling in the diffraction cross section, observable experimentally. The scaling means that the diffractive production of states with a given longitudinal momentum transfer  $\Delta q$  does not depend on energy. In such a case the corrections on mixing would not depend on energy. But as the energy is increased the scaling must be drastically broken due to expression (36) and formula for  $P_q(s)$  which has been found in work<sup>/22/</sup> in the parton cascade model

$$P(s) = P_\infty \left[ 1 - (1 - P_\infty) \left( \frac{s}{s_0} \right)^{1 - \alpha_p(0)} \right]^{-1} \quad (37)$$

It can be seen from (37) that the derivative of  $P_q$  in (36) decreases as power of the energy if energy is large enough  $\ln(s/s_0) \gg (\alpha_p(0) - 1)$ .

At energies available at accelerators the correction to the parton states mixing does not become extinct, their account can lead to the decrease of the total hadron-nucleus cross sections. For example at the calculation of the first inelastic correction the substitution  $d\sigma_{diff}/dM^2 \sim M^{-2}$  leads to a negative contribution increasing logarithmically with energy. The effect of mixing for all the inelastic corrections can be simply found by the eigenstate method. Indeed, ac-

Table 2

A	$P_q = 0.5$	$P_q = 0.55$	$P_q = 0.6$
$^{12}\text{C}$	0.03	0.02	0
$^{27}\text{Al}$	0.02	0.01	-0.01
$^{64}\text{Cu}$	0.01	0	-0.01
$^{207}\text{Pb}$	0	-0.01	-0.02

cording to relation (36) it is enough to introduce an energy dependence for  $P_q$  in formula (11) as for example in (37). It can be seen from (11) that the simultaneous decrease of  $P_q$  with energy and the increase of  $\sigma_{tot}^{qN}$  lead to a decrease with energy of the partial amplitude  $f_{qA}^{qN}(b)$  for large  $T(b)$  and to an increase of  $f_{qA}(b)$  for small  $T(b)$ . In Table 2 the values of the logarithmic derivative  $d \ln \sigma_{NA}^{tot} / d \ln(s/s_0)$  at 200 GeV are given via the value of  $P_q$ . For the calculation we have used expression (37).

Table 2 shows that the energy dependence of the hadron-nucleus total cross section is very sensitive to the value of  $P_q$ . The data<sup>/12/</sup> show a possible decrease of the n-<sup>207</sup>Pb and n-<sup>238</sup>U cross sections with energy. Higher accuracy of data is needed.

The other manifestation of energy dependence of  $\sigma_{tot}^{qN}$  and  $P_q$  is an additional shrinkage of the diffraction cone for nuclei in comparison with the nucleon. This effect has been first observed experimentally in the elastic p-d scattering<sup>/23/</sup> and has been explained<sup>/24/</sup> by the increase of the inelastic

corrections with energy. One can see however that already in the Glauber-Sitenko approximation additional shrinkage of the diffraction cone on nuclei exists due to the increase of the hadron-nucleon total cross section. The energy dependence of the inelastic corrections due to the decrease of  $P_q$  with energy enhances the effect as can be seen from (11). As it has been already noted, for sufficiently heavy nuclei the partial amplitude  $f_{qA}(\vec{b})$  decreases with energy in the center of the nucleus and increases with energy in the peripheral region. This leads to a particularly rapid increase of the interaction radius, i.e., large additional shrinkage of the diffraction cone. The results are presented in Fig. 3 for  $-t_{\min}(E)$ .

The results of the calculation of  $\alpha'_{\text{eff}}(A)$  the effective slope of the Pomeron trajectory describing the shrinkage of

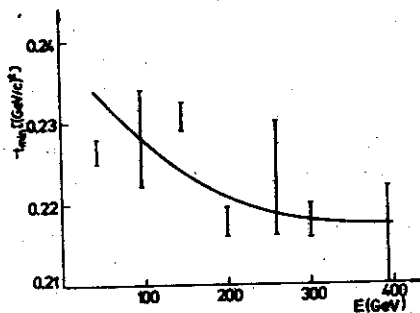


Fig.3. The motion of position of the first diffraction minimum for the  $p$ - $^4\text{He}$  elastic scattering cross section with the incident energy. The experimental values are estimated from the results of work <sup>/16/</sup>. The theoretical curve is calculated in the quark-parton model.

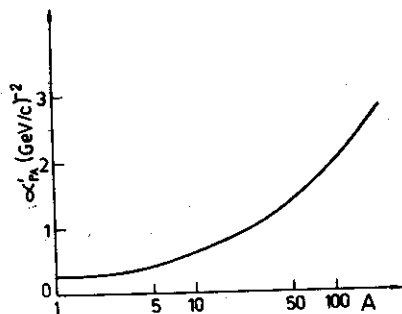


Fig.4 The dependence of the effective slope of the Pomeron trajectory on the mass number  $A$  according to the quark-parton model.

the diffraction cone are shown in fig. 4. We have used the formula

$$\alpha'_{\text{eff}}(A) = \frac{d}{d \ln(s/s_0)} \left[ \frac{1}{2\sigma_{hA}^{\text{tot}}} \int d^2\vec{b} \cdot b^{-2} f^{hA}(\vec{b}) \right], \quad (38)$$

Where the amplitude  $f_{LS}^A(\vec{b})$  has been calculated using the formulae of the 4th section which takes into account the energy



dependence of  $\sigma_{tot}^{qN}$ ,  $P_q$  and  $R_{qN}^2$ . It is seen that  $a_{eff}(A)$  rapidly increases with  $A$ . In this connection we expect the higher value of  $a_{eff}$  for NN scattering than for  $\pi N$  and  $KN$ .

## 7. THE REAL PART OF THE HADRON-NUCLEUS ELASTIC SCATTERING AMPLITUDE

Although the quark-nucleon amplitude  $f$  was introduced as a pure imaginary one the quark-nucleus amplitude (33) has a real part. This real part has the following properties<sup>/18/</sup>: it is negative, it has a maximum value which position depends on the atomic number and the mixing parameter  $\mu^2$ . At  $\mu^2 = -1$  (GeV/c)<sup>2</sup> the maximum (minimum) has the position at a few tens of GeV. At higher energies the ratio  $Re f^{hA}/Im f^{hA}$  decreases as  $1/E$ .

It is clear that all of these results could be obtained also in the basis of physical states if one sums all the graphs. In such a method the origin of real part is particularly clear. Because inelastic intermediate state is produced with another mass, and, as a collary with another wave number, there is a phase shift between the incoming and outgoing waves which results in a real part of the scattering amplitude.

In the multichannel case the energy behaviour of the real part is quite different. The expression for the real part of scattering amplitude at angle  $0^\circ$ , corresponding to the first order inelastic correction can be written as follows;

$$\frac{Re f_{hA}}{Im f_{hA}} = - \frac{4\pi}{\sigma_{hA}^{tot}} \int d^2 b dM^2 \frac{d^2 \sigma_{diff}^{hN}}{dq_{\perp}^2 dM^2} \Big|_{q_{\perp}^2=0} e^{-\frac{1}{2} \sigma_{hN}^{tot} T(\vec{b})} \times \int_{-\infty}^{+\infty} d\ell_1 d\ell_2 \rho(\vec{b}, \ell_1) \rho(\vec{b}, \ell_2) \sin\left(\frac{\mu^2}{2E} |\ell_1 - \ell_2|\right). \quad (39)$$

This formula can be obtained on the basis of the same assumptions as formula of Kondratyuk and Karmanov<sup>/19/</sup> for the total cross section correction.

It can be seen from (39) that if the  $M^2$ -dependence of the inelastic diffraction cross section has the form  $d\sigma_{diff}/dM^2 \sim M^{-2}$  then the ratio  $Re f^{hA}/Im f^{hA}$  does not depend on energy. As it has been noted in the previous section the scaling behaviour of  $\sigma_{diff}$  has an approximate character and with increase of the energy and  $M^2$  the cross section has a strong decrease which leads to the decrease of the real part of the amplitude with energy. Nevertheless in the wide energy region now available at accelerators the triple Pomeron term in  $\sigma_{diff}$  makes a

negative, logarithmically increasing contribution to the total cross section and makes a constant negative contribution to Ref.<sup>NA</sup>

Formula (39) can be written taking into account the quark structure of hadron. In addition, the inelastic shadowing significantly "enlights" the nucleus. Because the real part has a small value it can be taken into account only in the first order of approximation. On the basis of these assumptions new expression can be obtained, for example, for the NA scattering:

$$\frac{\text{Ref}_{NA}}{\text{Im}f_{NA}} = -\frac{12\pi}{\sigma_{NA}^{\text{tot}}} \int d^2\vec{b} dM^2 \frac{d^2\sigma_{\text{diff}}^{qN}}{dq_{\perp}^2 dM^2} \Big|_{q_{\perp}^2=0} [1 - P_q + \quad (40)$$

$$+ P_q e^{-\sigma_{qN}^{\text{tot}} T(\vec{b})/2 P_q}]^3 \times \int_{-\infty}^{+\infty} dl_1 dl_2 \rho(\vec{b}, l_1) \rho(\vec{b}, l_2) \times$$

$$\times \sin\left(\frac{\mu^2}{2E} |\vec{l}_1 - \vec{l}_2|\right).$$

We have calculated Ref<sup>NA</sup>/Imf<sup>NA</sup> using formula (39) and (40). The diffraction dissociation cross section has been taken in the following form:

If  $M^2 < 5 \text{ GeV}^2$ , then /12/

$$\frac{d^2\sigma_{NN}}{dq_{\perp}^2 dM^2} \Big|_{q_{\perp}^2=0} = 26,47z - 35,97z^2 + 18,47z^3 - 4,14z^4 + 0,34z^5 \quad (41)$$

where  $z = M^2 - (M_N + m_{\pi})^2$ .

If  $M^2 > 5 \text{ GeV}^2$  then /25/

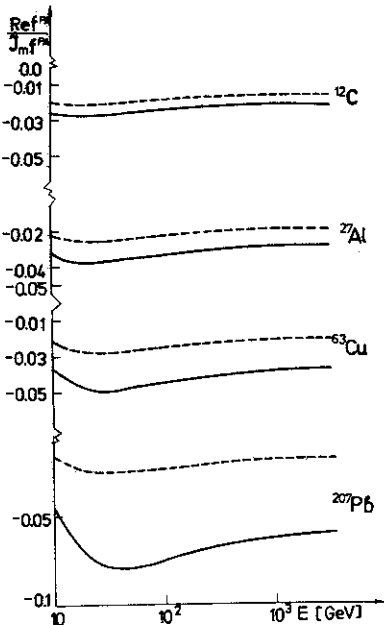


Fig.5. The energy dependence of the forward Ref<sup>NA</sup>/Imf<sup>NA</sup> ratio for the <sup>12</sup>C, <sup>27</sup>Al, <sup>63</sup>Cu and <sup>207</sup>Pb nuclei. The solid lines are calculated in the quark-parton model (formula 40), the dotted lines correspond to the calculations which neglect the nucleon structure (expression 39).

$$\frac{d^2 \sigma_{NN}}{d\vec{q}_\perp^2 dM^2} \Big|_{\vec{q}_\perp^2=0} = G_{PPP}(0)/M^2 + G_{PPR}(0)/M^3, \quad (42)$$

where  $G_{3P}(0) = 3.4 \text{ mb/GeV}^2$ ,  $G_{PPR}(0) = 3.4 \text{ mb/GeV}$ .

In formula (40) expressions (41) and (42) for  $\sigma_{\text{diff}}^{NN}$  have to be multiplied by the factor  $\sigma_{\text{tot}}^{qN} / \sigma_{\text{tot}}^{NN}$ . The results of calculations are presented in Figure 5. It can be seen that the real part corresponding to formula (40) has higher absolute values than the calculated one using (39). This can be explained by a significant enlightening effect caused by the inelastic corrections. As can be expected, the importance of enlightening strongly increases with the increase of atomic number, see Fig. 5.

It worthwhile noting that to the real part of the scattering amplitude calculated above one should add a part connected with the increase of the cross section with energy and the contribution of the secondary Reggeons.

## 8. SECONDARY REGGEONS

The problem of nuclear renormalization of secondary reggeons in this approach has been studied in ref. /26/ in connection with  $K_s$ -regeneration on nuclei. The regeneration amplitude on nuclei can be well described by formula /28/:

$$f_{LS}^A = (f_{LS}^N)_\omega \int d^2 b T(\vec{b}) (1 - P_s + P_s e^{-\sigma_{qN}^{\text{tot}} T(\vec{b})}) e^{-\sigma_{qN}^{\text{tot}} T(\vec{b})/2} \quad (43)$$

Here /27/

$$(f_{LS}^N)_\omega = \frac{\beta_\omega^{KN}}{2\pi \cos(\pi\alpha_\omega/2)} (2m_N p_L)^{\alpha_\omega - 1}, \quad (44)$$

$\beta_\omega^{KN} = 10.46 \text{ mb/GeV}^2$  - residue of the  $\omega$ -reggeon,  $\alpha_\omega = 0.44$  - intercept of the  $\omega$ -trajectory,  $p_L$  - momentum of  $K$ -meson.

Formula (43) takes into account that the  $\omega$  exchange is possible only for the u-quark, but the s-quark is coupled only to the low lying  $\phi$ -trajectory and thus s-quark is a spectator. The change of phase of  $f_{LS}^A$  in comparison with  $f_{LS}^N$  is small and can be neglected. Note that the energy dependence of expression (44) is quite sensitive to the logarithmic derivative of  $P_q$  with respect to the rapidity which is determined in /26/ using experimental data and is found to be equal to -0.08 in a good agreement with the results of section 5 of the present work.

The real part of the  $\pi$ -A scattering amplitude contains the  $f$  and  $\rho$  pole contributions. In the  $p$ -A scattering the  $f$  and  $\omega$  poles have a dominant role which contributions are equal due to the approximate exchange degeneracy. So

$$\text{Ref}_R^{\text{NA}} \cong 2 \text{Ref}_\omega^{\text{NN}} \int d^2b T(\vec{b}) \exp(-\sigma_{qN}^{\text{tot}} T(\vec{b})/2P_q) \times \quad (45)$$

$$[1 - P_q + P_q \exp(-\sigma_{qN}^{\text{tot}} T(\vec{b})/2P_q)]^2$$

where

$$\text{Ref}_\omega^{\text{NN}} = \frac{\beta_\omega^{\text{NN}} \sin\left(\frac{\pi\alpha_\omega}{2}\right)}{2\pi \cos^2\left(\frac{\pi\alpha_\omega}{2}\right)} (2m_N p_L)^{\alpha_\omega - 1} \quad (46)$$

## 9. CONCLUSIONS

There has been proposed in refs <sup>5,6/</sup> a method of effective accounting of all inelastic screening corrections - the eigen state method. In the same refs. the concrete realization of the method was proposed within the quark-parton model. The two component approximation seems to be very convenient for calculations. This approximation leads to simple formula having two free parameters: the weight of the active component of the constituent quark and the cross section of the quark-nucleon interaction which can be easily established by the total hadron-nucleus cross section data. Other quantities, the differential cross section of the hadron-nucleus elastic scattering, the real part of the scattering amplitude, the triple pomeron constant, the  $K_s$  meson regeneration amplitude on nuclei are calculated without free parameters in a good agreement with the experimental data. So one can say that a quite convenient calculational scheme is proposed and realized which allows one to take into account all the inelastic screening corrections in the diffractive type hadron-nucleus processes.

It is interesting that the results (27) of the data analysis within the two component approximation give additional support for this approximation. Indeed the total cross section of two active constituent quark interaction in the c.m. system has the form:

$$(\sigma_{\text{tot act}}^{\text{qq}}) = \frac{\sigma_{\text{tot}}^{\text{qq}}}{P_q^2} \quad (47)$$

If we put here  $\sigma_{\text{tot}}^{\text{qq}} = 6$  mb and  $P_q = 0.5$  then we obtain  $(\sigma_{\text{tot act}}^{\text{qq}})_{\text{act}} = 24$  mb, which is very large cross section. The minimal radius of quark interaction can be found from here if formula (47) is equated with  $2\pi R_{\text{qq}}^2$  which gives  $R_{\text{qq}}^2 \cong 10$  (GeV/c)<sup>-2</sup>. This quantity is larger than the well-known value of squared Regge radius  $R^2 = 4\alpha_p' \ln s \cong 6$  (GeV/c)<sup>-2</sup> (at  $s = 400$  GeV<sup>2</sup>). Thus we have

the conclusion that the quarks in the active state are black which provides the basis of the two component approximation.

We note that there exists another way of realization of the eigen state method using the ideas of the quantum chromodynamics (QCD), as proposed in work<sup>/28/</sup>. As in the QCD only the coloured charge can interact then the colour singlet hadrons can interact only due to intrahadronic colour distribution in the impact parameter plane. Thus the cross section of hadrons interaction depends on their size<sup>/29/</sup>. Thus the hadron state with a defined transversal distance between quarks can be considered as an eigenstate of interaction. In this method the total hadron-nucleus cross section<sup>/28/</sup> and the  $K_S$ -meson regeneration amplitude on nuclei<sup>/30/</sup> have been calculated without free parameters in a good agreement with experiments.

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#### APPENDIX

The first term of the sum over  $\ell$  in expression (16) corresponds to the case where only one quark of the hadron is in active state. This can be written directly as

$$\begin{aligned}
 F_{\pi A}^{(1)}(\vec{q}) = & \int d^2 \vec{b}_\pi e^{i\vec{q}(\vec{b}_\pi - \vec{r}_A)} \left\{ 1 - \left[ 1 - \frac{1}{(2\pi)^2 P_q} \times \right. \right. \\
 & \times \int d^2 \vec{k} e^{i\vec{k}(\vec{b}_1 - \vec{r}_A)} f_{qN}(\vec{k}) \rho_A(\vec{k}) \left. \right\}^A \times \\
 & \times \rho_\pi(\vec{b}_1, \vec{b}_2) \delta^2\left(\frac{\vec{b}_1 + \vec{b}_2}{2} - \vec{b}_\pi\right) d^2 \vec{b}_1 d^2 \vec{b}_2 .
 \end{aligned} \tag{A.1}$$

We choose the pion density function in factorized form:

$$\rho_\pi(\vec{b}_1, \vec{b}_2) = \rho_\pi(\vec{b}_1 - \vec{b}_\pi) \rho_\pi(\vec{b}_2 - \vec{b}_\pi),$$

where

$$\rho_\pi(\vec{b}_i - \vec{b}_\pi) = (2\pi R_\pi^2)^{-1/2} \exp\left[-\frac{(\vec{b}_i - \vec{b}_\pi)^2}{4R_\pi^2}\right]. \tag{A.2}$$

Carrying out in (A.1) a substitution of integration variable and taking  $f_{qN}(\vec{k})$  in Gaussian form we obtain expression (18) which in  $q$ -representation has an additional factor  $\exp(q^2 R_\pi^2/8)$ .

The second factor of the sum over  $\ell$  corresponds to two active quarks. It has the form:

$$\begin{aligned}
 F_{\pi A}^{(2)}(\vec{q}) = & \int d^2 \vec{b}_\pi e^{i\vec{q}(\vec{b}_\pi - \vec{b}_A)} \left\{ 1 - \left[ 1 - \frac{1}{(2\pi)^2 P_q} \times \right. \right. \\
 & \times \int d^2 \vec{k} (e^{i\vec{k}(\vec{b}_1 - \vec{r}_A)} + e^{i\vec{k}(\vec{b}_2 - \vec{r}_A)}) f_{qN}(\vec{k}) \rho_A(\vec{k}) + \\
 & + \frac{1}{(2\pi)^4 P_q^3} \int d^2 \vec{k}_1 d^2 \vec{k}_2 e^{i\vec{k}_1(\vec{b}_1 - \vec{r}_A)} e^{i\vec{k}_2(\vec{b}_2 - \vec{r}_A)} f_{qN}(\vec{k}_1) \times \\
 & \left. \left. \times f_{qN}(\vec{k}_2) \rho_A(\vec{k}_1 + \vec{k}_2) \right\} \rho_\pi(\vec{b}_1, \vec{b}_2) \delta^2\left(\frac{\vec{b}_1 + \vec{b}_2}{2} - \vec{b}_\pi\right) d^2 \vec{b}_1 d^2 \vec{b}_2.
 \end{aligned} \tag{A.3}$$

The first integral in the square brackets can be transformed to the form:

$$\frac{2}{(2\pi)^2 P_q} \int d^2 \vec{k} e^{i\vec{k}\left(\frac{\vec{b}_1 + \vec{b}_2}{2} - \vec{r}_A\right)} f_{qN}(\vec{k}) \rho_A(\vec{k}) \cos\left[\frac{\vec{k}}{2}(\vec{b}_1 - \vec{b}_2)\right]. \tag{A4}$$

We put into the argument of the cosinus  $\langle (\vec{b}_1 - \vec{b}_2)^2 \rangle = 2R_\pi^2$ , and considering the smallness of the quantity  $R_\pi^2/R_A^2$  we obtain the factor with  $l_1$  in expression (19).

The second integral in (A3) can be transformed to the form:

$$\frac{f_{qN}^2(0)}{(2\pi)^3 R_q^2} e^{-\langle (\vec{b}_1 - \vec{b}_2)^2 \rangle / 2R_{qN}^2} \int d^2 \vec{k} e^{-\frac{\vec{k}^2 R_{qN}^2}{8}} \rho_A(\vec{k}) e^{\frac{i\vec{k}}{2}(\vec{b}_1 + \vec{b}_2)}. \tag{A5}$$

Changing  $\langle (\vec{b}_1 - \vec{b}_2)^2 \rangle$  to  $\langle (\vec{b}_1 - \vec{b}_2)^2 \rangle = 2R_\pi^2$  we obtain (19)-(20).

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