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THE STRUCTURE FUNCTIONS
OF RELATIVISTIC SYSTEMS COMPOSED
OF TWO PARTICLES WITH $1 / 2$ SPINS

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## 1. INTRODUCTION

The problem of the description of deep-inelastic leptonhadron scattering is one of the most actual problems of the high energy physics. In ref./1/ this problem has been stidued on the basis of general principles of quantum field theory. This paper can be considered as a continuation of our paper ${ }^{2 /}$ and is devoted to the investigation of lepton-hadron scattering processes on the basis of the relativistic bound state wave functions (WF) of two spin $1 / 2$ quarks. The relativistic WF, we shall use, are solutions of a covariant three-dimensional equation derived in the framework of a single-time approach to a relativistic two-body problem ${ }^{13-5 /}$. This equation coincides in from with a covariant two-body equation that occurs in the framework of a covariant Hamiltonian formulation of quantum field theory ${ }^{/ 6 /}$. The covariant three-dimensional equation can appear to be more convenient than the four-dimensional Bethe-Salpeter equation because of the existing suitable methods for finding their solutions by passing to a relativistic configurational representation introduced in ref. 7 /.

A general formalism for describing deep-inelastic processes in the framework of the covariant single time equations was proposed earlier in ref. ${ }^{18 /}$ and developed in ref. ${ }^{19 /}$. The aim of the present paper is to develop an approach that would permit us to use the solutions of spin equations found earlier in the relativistic configurational representation.

The paper is organized as follows. The next section contains basic formulae of a covariant three-dimensional approach based on the single-time description of two-particle systems. In the third section the meson structure functions are expressed through the covariant single-time two-body WF.
2. THE COVARIANT THREE-DIMENSIONAL EQUATION FOR TWO SPIN PARTICLES

The Bethe-Salpeter wave function (WF) of the quark-antiquark system has the form

$$
\begin{equation*}
\Psi_{\mathrm{MK}}^{\alpha \beta}\left(\mathrm{x}_{1}, \mathrm{x}_{2}\right)=\langle 0| \mathrm{T}\left\{\psi_{1}^{\alpha}\left(\mathrm{x}_{1}\right) \bar{\psi}_{2}^{\beta}\left(\mathrm{x}_{2}\right)\right\}|\mathrm{M}, \overrightarrow{\mathrm{~K}}\rangle, \tag{2.1}
\end{equation*}
$$

Here $\psi_{1}^{a}\left(x_{1}\right)$ and $\bar{\psi}_{2}^{\beta}\left(x_{2}\right)$ are fermion operators (in the Heisenberg representation), $M$ and $\vec{K}$ are the mass and momentum of the bound state. A procedure of the transition from (2.1) to the covariantly defined single-time (quasi-potential) WF $\widetilde{\Psi}_{M K}^{a \beta}\left(p_{1}, p_{2}\right)^{15,10 /} \quad$ is as follows

$$
\tilde{\Psi}_{\mathrm{MK}}^{a \beta}\left(\mathrm{p}_{1}, \mathrm{p}_{2}\right)=\iint \mathrm{d}^{4} \mathrm{x}_{1} \mathrm{~d}^{4} \mathrm{x}_{2} \exp \left(\mathrm{ip}_{1} \mathrm{x}_{1}+\mathrm{ip}_{2} \mathrm{x}_{2}\right) \delta\left[\lambda \mathscr{P}_{1}\left(\mathrm{x}_{1}-\mathrm{x}_{2}\right)\right] \Psi_{\mathrm{MK}}^{\alpha \beta}\left(\mathrm{x}_{1}, \mathrm{x}_{2}\right),
$$

where $\lambda^{\mu} \mathscr{\mathscr { S }}^{\equiv} \mathscr{P}^{\mu} / \sqrt{\mathscr{P}^{2}}$ is a four-vector of the system velocity. The presence of the $\delta\left[\lambda \mathscr{P}\left(\mathrm{x}_{1}-\mathrm{x}_{2}\right)\right]$ function in (2.2) leads to the coincidence of the quark proper times $\tau_{1,2}=\lambda \mathfrak{P x}{ }_{1,2}$ with the proper time of the system $\tau=\lambda \mathcal{P} X$, where $X \xlongequal{1,2}\left(x_{1}+x_{2}\right) / 2^{/ 11 /}$. (Therefore the symbol of $T$-product in (2.2) can be dropped). Upon performing the covariant time equating in (2.2) the vectors $p_{1}$ and $p_{2}$ can be considered as belonging to the mass hyperboloid $\mathrm{p}_{1,2}^{02}-\overrightarrow{\mathrm{p}}_{1,2}^{2}=\mathrm{m}^{2 / 12 /}$. Taking into account translational invariance we can separate the motion of the mass centre of the system in the WF

$$
\begin{equation*}
\tilde{\Psi}_{\mathrm{MK}}^{a \beta}\left(\mathrm{p}_{1}, \mathrm{p}_{2}\right)=(2 \pi)^{4} \delta^{(4)}(\mathcal{P}-K) \tilde{\Psi}_{\mathrm{MK}}^{\alpha \beta}(\mathrm{p}), \tag{2.3}
\end{equation*}
$$

$$
\begin{equation*}
\widetilde{\Psi}_{M K}^{\alpha \beta}(p)=\int d^{4} x \exp (\mathrm{ppx}) \delta[\lambda \mathcal{P} \mathrm{x}]_{M K}^{\alpha \beta}(\mathrm{x}) . \tag{2.4}
\end{equation*}
$$

where $x=x_{1}-x_{2}, \mathscr{P}=p_{1}+p_{2}, p=\left(p_{1}-p_{2}\right) / 2$. The field operator transformation law permits us to represent the quark relative motion $W F$ (2.4) in a form ${ }^{15 / *}$

$$
\begin{align*}
& \tilde{\Psi}_{M K}^{a \beta}(\mathrm{p})=\mathrm{S}_{1}\left(\mathrm{~L}_{\mathrm{K}}\right) \mathrm{S}_{2}\left(\mathrm{~L}_{\mathrm{K}}\right) \int \mathrm{d}^{3} \mathrm{x}^{\prime} \exp \left(-\mathrm{i} \vec{\Delta}_{\mathrm{p}, \mathrm{~m} \mathrm{\lambda} \mathrm{~K}} \overrightarrow{\mathrm{x}}^{\prime}\right) \times \\
& \left.\times<0\left|\psi_{1}^{a}\left(0, \overrightarrow{\mathrm{x}}^{\prime} / 2\right) \bar{\psi}_{2}^{\beta}\left(0,-\overrightarrow{\mathrm{x}}^{\prime} / 2\right)\right| \mathrm{M}, \overrightarrow{0}\right\rangle, \tag{2.5}
\end{align*}
$$

where

$$
\mathrm{x}_{\mu}^{\prime}=\left(\mathrm{L}_{\left.\lambda_{\mathrm{K}}^{-1} \mathrm{x}\right)_{\mu}, \quad \lambda_{\mathrm{K}}^{\mu}=\mathrm{K}^{\mu} / \sqrt{\mathrm{K}^{2}}=\mathscr{P}^{\mu} / \sqrt{\mathcal{P}^{2}} \equiv \lambda^{\mu} \mathscr{P}, ~ ; ~, ~}\right.
$$

and $\mathrm{S}(\mathrm{L}) \quad$ is a matrix of a finite-dimensional representation of the Lorentz group. From (2.5) it follows that the WF of the relative motion depends on the tiree-dimensional vector only (see for details $/ 5,12 /$ ) : $\vec{\Psi}_{M K}^{a \beta}\left(\vec{\Lambda}_{\mathrm{p}, \mathrm{m} \lambda \mathscr{P}}\right) \equiv \widetilde{\Psi}_{\mathrm{MK}}^{a \beta}(\mathrm{p})$. The vector $\vec{\Delta}_{\mathrm{p}, \mathrm{m} \lambda \mathrm{g}}$ has a sence of a covariantly defined momentum of a quark in the c.m.s. /13/

$$
\begin{equation*}
\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda \mathscr{P}} \equiv \overrightarrow{\left(\mathrm{~L}_{\lambda \mathscr{P}}^{-1} \mathrm{p}\right)} \equiv\left(\overrightarrow{\mathrm{L}_{\lambda}-1} \frac{\mathrm{p}_{1}-\mathrm{p}_{2}}{2}\right)=\vec{\Delta}_{\mathrm{p}_{1}, \mathrm{n} \lambda \mathcal{P}}= \tag{2.6}
\end{equation*}
$$

${ }^{*}{ }^{\mathrm{L}} \bar{\lambda}^{\mathrm{I}}$ is a pure Lorentz transformation to the rest fra-
me of the bound system, i.e., $\mathrm{L}_{\lambda}^{-1} \mathrm{~K}=(\mathrm{M}, \overrightarrow{0})\left(\lambda_{\mathscr{F}}=\lambda_{\mathrm{K}}\right)$. me of the bound system, i.e., $\mathrm{L}_{\lambda_{\mathscr{F}}}^{-1} \mathrm{~K}=(\mathrm{M}, \overrightarrow{0})\left(\lambda_{\mathscr{F}}=\lambda_{\mathrm{K}}\right)$.

$$
=-\vec{\Delta}_{\mathrm{p}_{2}}, \mathrm{~m} \mathrm{\lambda} \mathscr{\mathscr { P }}=\overrightarrow{\mathrm{p}}_{1}(-) \mathrm{m} \dot{\mathcal{P}}=\overrightarrow{\mathrm{p}}_{1}-\frac{\overrightarrow{\mathscr{P}}}{\mathrm{M}}\left(\mathrm{p}_{1}^{o}-\frac{\overrightarrow{\mathscr{P}}_{\mathrm{p}_{1}}}{\mathscr{\mathscr { P }}_{0}+\mathrm{M}}\right)
$$

and can be considered as a spatial component of the four-vector $\Delta_{\mathrm{p}, \mathrm{m} \lambda \mathcal{G}}^{\mu} \equiv\left(\mathrm{L}_{\lambda \mathscr{g}}^{-1} \mathrm{p}\right)^{\mu}$. This vector belongs to the same mass-shell hyperboloid $p_{1,2}^{0}-\vec{p}_{1,2}^{2}=m^{2}$ 1ike the vectors $\vec{p}_{1}$ and $\overrightarrow{\mathrm{p}}_{2}$. As is know, the upper sheet of such a hyperboloid serves as a model of the Lobachevsky momentum space. Under arbitrary Lorentz transformations $\mathrm{L}\left(\mathrm{p}^{\prime}=\mathrm{Lp}, \mathscr{P}^{\prime}=\mathrm{L} \mathcal{P}^{\prime}\right)$ the components of a four-vector $\Delta_{\mathrm{p}, \mathrm{m} \lambda \boldsymbol{\rho}}^{\mu}$ transforms as follows

$$
\begin{align*}
& \Delta_{\mathrm{p}, \mathrm{~m} \lambda \mathscr{P}}^{\circ}=\Delta_{\mathrm{p}, \mathrm{~m} \lambda \mathscr{P}}^{\circ} \equiv \lambda^{\mu} \mathrm{p}_{\mu}=\mathscr{P}_{\mathrm{p}}^{\mu}{ }_{\mu} M  \tag{2.7}\\
& \left(\vec{\Lambda}_{\mathrm{p}^{\prime}, \mathrm{m} \lambda \mathscr{P}}\right)_{\mathrm{i}}=\mathrm{R}_{\mathrm{ij}}: \mathrm{V}^{-1}\left(\mathrm{~L}^{-1}, \mathscr{P}\right\}\left(\vec{\Lambda}_{\mathrm{p}, \mathrm{~m} \lambda \mathscr{P}}\right)_{\mathrm{j}}
\end{align*}
$$

where the matrix $R_{i j}\left\{\mathrm{~V}^{-1}\left(\mathrm{~L}^{-1}, \mathscr{P}\right)\right\} \quad$ contains the Wigner rotation.

The two-time Green function of the quark-antiquark system will be defined in analogy with (2.1), (2.2) ${ }^{15,10,14 /}$

$$
\begin{equation*}
\because \iiint \int \mathrm{d}^{4} \mathrm{x}_{1} \mathrm{~d}^{4} \mathrm{x}_{2} \mathrm{~d}^{4} \mathrm{y}_{1} \mathrm{~d}^{4} \mathrm{y}_{2} \exp \left(i \mathrm{p}_{1} \mathrm{x}_{1}+i \mathrm{p}_{2} \mathrm{x}_{2}-i \mathrm{q}_{1} \mathrm{y}_{1}-i \mathrm{q}_{2} \mathrm{y}_{2}\right) \cdot x \tag{2.8}
\end{equation*}
$$

$$
\times \delta\left[\lambda \mathscr{P}\left(\mathrm{x}_{1}-\mathrm{x}_{2}\right)\right] \delta\left[\lambda_{\mathrm{Q}}\left(\mathrm{y}_{1}-\mathrm{y}_{2}\right)\right] \mathrm{C}^{a_{1} a_{2} a_{3} \alpha_{4}}\left(\mathrm{x}_{1}, \mathrm{x}_{2} ; \mathrm{y}_{1}, \mathrm{y}_{2}\right),
$$

where

$$
\begin{equation*}
\mathrm{G}^{a_{1} a_{2} a_{3} \alpha_{4}}\left(\mathrm{x}_{1}, \mathrm{x}_{2} ; \mathrm{y}_{1}, \mathrm{y}_{2}\right)=\langle 0| \mathrm{T}\left\{\psi_{1}^{a_{1}}\left(\mathrm{x}_{1}\right) \tilde{\psi}_{2}^{-a_{2}}\left(\mathrm{x}_{2}\right) \psi_{2}^{a_{3}}\left(\mathrm{y}_{2}\right) \tilde{\psi}_{1}^{a_{4}}\left(\mathrm{y}_{1}\right)\right\}|0\rangle \tag{2.9}
\end{equation*}
$$

With the help of the representation of the $T$-product through the $\theta$-function a spectral representation for $\widehat{G}$ can be obtained (see $/ 3,5 /$ ). Near the bound state pole with $\sqrt{9^{2}}=\mathrm{M}$ from the spectral representation we find the following expression for the Green function ${ }^{15 /}$ :

$$
\begin{align*}
& \tilde{\mathrm{G}}^{a_{1} a_{2} a_{3} a_{4}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} \rho ; \vec{\Delta}_{\mathrm{q}, \mathrm{~m} \lambda \mathscr{\rho}} ; \mathscr{\rho}^{2}\right) \simeq}  \tag{2.10}\\
& \simeq \frac{\mathrm{i}(2 \pi)^{3}}{2 \mathrm{M}} \vec{\Psi}_{\mathrm{MK}}^{a_{1} a_{2}}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda \rho}\right) \otimes \tilde{\Psi}_{\mathrm{MK}}^{a_{3} a_{4}\left(\vec{\Delta}_{\mathrm{q}, \mathrm{~m} \lambda}\right)} \\
& \sqrt{\mathscr{\rho}^{2}}-\mathrm{M}+\mathbf{i} \epsilon
\end{align*}
$$

Following to ${ }^{/ 5 /}$ we shall introduce projections of all quantities onto the subspace of the positive-energy states:

$$
\begin{aligned}
& \overrightarrow{\mathrm{G}}_{\sigma_{1} \sigma_{2} \sigma_{3} \sigma_{4}}^{(+)}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \overrightarrow{\mathrm{\Delta}}_{\mathrm{q}, \mathrm{~m} \lambda_{\mathscr{F}}} ; \mathscr{P}^{\mathcal{T}}=\frac{1}{16 \mathrm{~m}^{4}} \overrightarrow{\mathrm{u}}_{a_{1}}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda_{\mathscr{F}}} ; \sigma_{1}\right) \mathrm{v}_{a_{2}}\left(-\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \mathscr{P}_{2}^{(2.11)}\right) \times\right.
\end{aligned}
$$

Positive- and negative-energy bispinors $u$ and $v$ of free quarks with mass $m$ are normalized by the condition

$$
\overrightarrow{\mathrm{u}}_{\alpha}(\overrightarrow{\mathrm{p}}, \sigma) \mathrm{u}^{\alpha}\left(\overrightarrow{\mathrm{p}}, \sigma^{\prime}\right)=-\mathrm{v}_{a}(\overrightarrow{\mathrm{p}}, \sigma) \mathrm{v}^{a}\left(\overrightarrow{\mathrm{p}, \sigma^{\prime}}\right)=2 \mathrm{~m} \delta \sigma^{\prime} .
$$

For the Green function (2.12) in the case of free quarks we have

$$
\begin{align*}
& \tilde{\mathrm{G}}_{0 \sigma_{1} \sigma_{2} \sigma_{3} \sigma_{4}}^{(+)}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \overrightarrow{\mathcal{S}}_{\mathrm{q}, \mathrm{~m} \lambda} ; \mathscr{P}^{2}\right)=\mathrm{i}(2 \pi)^{3}\left(2 \mathrm{~m} \cdot 2 \Delta_{\mathrm{p}, \mathrm{~m} \lambda}^{\circ} \dot{\mathscr{F}}^{\left.2 \Delta_{\mathrm{q}, \mathrm{~m} \lambda} \mathscr{P}^{\circ}\right)^{-1} \times}\right. \\
& \times \delta\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \boldsymbol{\rho}_{\mathscr{P}}}-\vec{\Delta}_{\mathrm{q}, \mathrm{~m} \lambda_{\mathscr{P}}}\right) \delta_{\sigma_{1} \sigma_{4}} \delta_{\sigma_{2} \sigma_{3}}\left[\sqrt{\left.\mathscr{P}^{2}-2 \Delta_{\mathrm{p}, \mathrm{~m} \lambda}^{\circ} \mathscr{P}^{-\mathrm{i} \epsilon}\right]^{-1}}\right. \tag{2.13}
\end{align*}
$$

Let us introduce the quasipotential operator $\overrightarrow{\mathrm{V}}^{1 / 3 /}$

$$
\begin{equation*}
-\mathrm{i}\left(2 \Delta_{\mathrm{p}, \mathrm{~m} \lambda}^{\circ} \mathscr{P} 2 \Delta_{\mathrm{q}, \mathrm{~m} \lambda}^{\circ} \mathscr{\rho}\right) \overrightarrow{\mathrm{V}}=\left[\tilde{\mathrm{G}}_{0}^{(+)}\right]^{-1}-\left[\overrightarrow{\mathrm{G}}^{(+)}\right]^{-1} . \tag{2.14}
\end{equation*}
$$

With the help of (2.11)-(2.14) it is easy to obtain for the WF

$$
\begin{equation*}
\phi_{\sigma_{1} \sigma_{2}}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda \mathscr{P}}\right)=2 \Delta_{\mathrm{p}, \mathrm{~m} \lambda \mathcal{P}}^{\circ} \tilde{\Psi}_{\sigma_{1} \sigma_{2}}^{(+)}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda \mathscr{P}}\right) \tag{2.15}
\end{equation*}
$$

the equation *

$$
\begin{aligned}
& 2 \Delta_{\mathrm{p}, \mathrm{~m} \lambda}^{\circ} \mathscr{\rho}\left[\sqrt{\mathscr{P}^{2}}-2 \Delta_{\mathrm{p}, \mathrm{~m} \lambda}^{\circ} \mathscr{P}^{] \phi_{\sigma_{1}} \sigma_{2}}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda \mathscr{P}}\right)=\right. \\
& \left.=(2 \pi)^{-3} \int \mathrm{~d}^{3} \vec{\Delta}_{\mathrm{q}, \mathrm{~m} \lambda} / 2 \mathscr{P}_{\mathrm{q}, \mathrm{~m} \lambda} \mathscr{P} \mathrm{~V}_{\sigma_{1} \sigma_{2} \sigma_{3} \sigma_{4}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} \mathscr{P}\right.} ; \vec{\Delta}_{\mathrm{q}, \mathrm{~m} \lambda} ; \mathscr{P}^{2}\right) \phi_{\sigma_{3} \sigma_{4}}\left(\vec{\Delta}_{\mathrm{q}, \mathrm{~m} \lambda}\right.
\end{aligned}
$$

The equation (2.16) coincides in its structure with the equation obtained earlier in the c.m.s. on the basis of the covariant Hamiltonian formulation of quantum field theory by Kadyshevsky and Mateev ${ }^{16 /}$ and that one, obtained in ref. /16,17/

[^0]in the framework of a single-time formulation. A normalization condition for the $W F \phi_{\sigma_{1}} \sigma_{2}$ for the case of the quasipotential $\overrightarrow{\mathrm{V}}$, independent of the system energy, takes the form ${ }^{14 /}$
 The properties of eq. (2.16) are described in reviews ${ }^{\text {/7,17/, }}$, where the problem of construction of a quasipotential from the matrix elements of the relativistic scattering amplitude was considered. The graphical representation of equation (2.16) is depicted on fig. 1 .

The WF of a system with a definite total spin $s$ and its projection $\sigma$ can be constructued as follows

$$
\begin{equation*}
\phi_{\mathrm{s}, \sigma}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda}\right)=\sum_{\sigma_{1}, \sigma_{2}= \pm 1 / 2}<\frac{1}{2} \frac{1}{2} ; \sigma_{1} \sigma_{2} \mid s \sigma>\phi_{\sigma_{1} \sigma_{2}}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \mathrm{\lambda}}\right) . \tag{2.18}
\end{equation*}
$$

Here $\left\langle\frac{1}{2} \frac{1}{2} ; \sigma_{1} \sigma_{2} \mid s \sigma\right\rangle$ are the Clebsh-Gordan coefficients. With the help of (2.18) we pass to the following equation ${ }^{15 /}$

$$
\begin{aligned}
& 2 \Delta_{\mathrm{p}, \mathrm{~m} \lambda \mathcal{F}}^{\circ}\left[\mathrm{M}-2 \Delta_{\mathrm{p}, \mathrm{~m} \lambda \rho}^{\circ}\right] \phi_{\mathrm{s}, \sigma}\left(\vec{\Lambda}_{\mathrm{p}, \mathrm{~m} \lambda \rho \rho}\right)=
\end{aligned}
$$

where

$$
\begin{align*}
& x<\frac{1}{2} \frac{1}{2} ; \sigma_{1}^{\prime} \sigma_{2}^{\prime} \mid \mathrm{s}^{\prime} \sigma^{\prime}>\cdot \mathrm{V}_{\sigma_{1} \sigma_{2} \sigma_{1}^{\prime} \sigma_{2}^{\prime}}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ;{\overrightarrow{\Delta_{\mathrm{k}}}, \mathrm{~m} \lambda \mathscr{P}} ; \mathscr{P}^{2}\right) \text {. } \tag{2.20}
\end{align*}
$$

For the pseudoscalar meson we find from (2.19)

$$
\begin{equation*}
\sqrt{2 \phi_{0,0}}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda \mathscr{P}}\right)=\phi_{1 / 2,-1 / 2}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda \mathscr{P}}\right)-\phi_{-1 / 2,1 / 2}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda \mathscr{}}\right) \text {. } \tag{2.21}
\end{equation*}
$$

To illustrate the connection of the formalism used here with the technique of the WF expansion over the $\gamma$ matrices we shall consider here that part of the WF $\phi_{\sigma_{1}} \sigma_{2}$ whose structure is defined by the $\gamma_{5}$ matrix:


Fig. 1. The graphical representation of equation (2.16).

$$
\phi_{\sigma_{1} \sigma_{2}}^{5}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda \mathscr{P}}\right)=\overline{\mathrm{u}}\left(\vec{\Delta}_{\left.\mathrm{p}, \mathrm{~m} \lambda_{\mathscr{P}} ; \sigma_{1}\right) \gamma_{5} \mathrm{v}\left(-\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \sigma_{\mathscr{F}}\right) \Phi\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda \mathscr{F}}\right),(2.22)}\right.
$$ where $\quad \vec{\Delta}_{\mathrm{p}, \mathrm{m} \lambda} \mathcal{P}=\vec{\Delta}_{\mathrm{p}_{1}, \mathrm{~m} \lambda} \mathscr{P}=-\vec{\Delta}_{\mathrm{p}_{2}, \mathrm{~m} \lambda \mathscr{P}}$ and $\Phi\left(\vec{\Delta}_{\mathrm{p}, \mathrm{m} \lambda \mathcal{P}}\right)$, is a scalar function. The direct calculations give:

$$
\begin{equation*}
\overline{\mathrm{u}}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda \mathcal{P}} ; \pm 1 / 2\right) y_{5} \mathrm{v}\left(-\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda \mathscr{P}} ; \mp 1 / 2\right)= \pm 2 \Delta_{\mathrm{p}, \mathrm{~m} \lambda}^{\circ} \mathcal{P} . \tag{2.23}
\end{equation*}
$$

The substitution of (2.24) into (2.22) leads to the relation

$$
\begin{equation*}
\phi_{0,0}^{5}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda \mathscr{P}}\right)=4 \Delta_{\mathrm{p}, \mathrm{~m} \lambda}^{\circ}\left({\sqrt{2})^{-1} \Phi\left(\vec{\Delta}_{p, \mathrm{~m} \lambda \mathscr{P}}\right) . . . .}\right. \tag{2.24}
\end{equation*}
$$

3. The Structure functions of a composite meson

Deep-inelastic scattering of a lepton on a hadron with the momentum $\overrightarrow{\mathcal{P}}$, mass $M$, spins and polarization $\sigma$ is described by the structure function $F_{i}$ introduced by the following standard decomposition of the current product

$$
\begin{align*}
& \mathrm{W}_{\mu \nu}(\mathscr{P}, \mathrm{q})=(8 \pi)^{-1} \sum_{\mathrm{X}, a}(2 \pi)^{4} \delta^{(4)}\left(\mathscr{P}+\mathrm{q}-\mathscr{P}_{\mathrm{X}}\right)\langle\overrightarrow{\mathscr{P}}, \mathrm{M}, \mathrm{~s}, \sigma| \mathrm{J}_{\mu}(0)|\mathrm{X}, a\rangle \times \\
& \left.\times<\mathrm{X}, a\left|\mathrm{~J}_{\nu}(0)\right| \overrightarrow{\mathscr{P}}, \mathrm{M}, \mathrm{~s}, \sigma\right\rangle=\left(-\mathrm{g}_{\mu \nu}+\frac{\mathrm{q}_{\mu} \mathrm{q}_{\nu}}{\mathrm{q}^{2}}\right) \frac{\mathrm{F}_{1}}{2 \mathrm{M}}+  \tag{3.1}\\
& +\left(\mathscr{P}_{\mu}-\frac{\mathscr{P}_{\mathrm{q}}}{\mathrm{q}^{2}} \mathrm{q}_{\mu}\right)\left(\mathscr{P}_{\nu}-\frac{\mathscr{P}_{\mathrm{q}}}{\left.\mathrm{q}^{2} \mathrm{q}_{\nu}\right)} \frac{\mathrm{F}_{2}}{\mathrm{M}(\mathscr{P} \mathrm{q})}-\frac{\mathrm{i} \epsilon \mu \nu a \beta}{2 \mathrm{M}\left(\mathscr{P}_{\mathrm{q}}\right)} \mathscr{P}^{a} \mathrm{q}^{\beta} \mathrm{F}_{3}\right.
\end{align*} .
$$

Here $q$ is the momentum transfer and $\mathscr{P}_{\mathrm{X}}$ is the total momentum of particles in a final state $|X, a\rangle$ with spin properties denoted by the symbola. We shall use the following standard variables:

$$
Q^{2}=-q^{2}, \quad \nu=\mathscr{P} q / M, \quad X \equiv 1 / \omega=Q^{2} / 2 M \nu, \quad W^{2}=\left(\mathscr{P}_{+} q\right)^{2} .
$$

To express a matrix element of the current operator $J_{\mu}(0)$ through the WF (2.11), let us consider the following Greenlike function $/ 8 /$.

$$
\begin{equation*}
\mathrm{R}_{\mu}^{\alpha \beta}\left(\mathrm{X}, a \mid \mathrm{y}_{1}, \mathrm{y}_{2}\right)=\langle\mathrm{X}, a| \mathrm{T}\left\{\mathrm{~J}_{\mu}(0) \psi_{2}^{a}\left(\mathrm{y}_{2}\right) \psi_{1}^{-\beta}\left(\mathrm{y}_{1}\right)\right\}|0\rangle \tag{3.2}
\end{equation*}
$$

and its single-time Fourier transform $(\lambda \equiv \lambda \mathscr{\rho})$

$$
\begin{align*}
& \ddot{\mathrm{R}}_{\mu}^{\alpha \beta}\left(\mathrm{X}, a \mid \vec{\Lambda}_{\mathrm{p}, \mathrm{~m} \mathrm{\lambda}} ; \mathscr{P}^{2}\right)=\iint \mathrm{d}^{4} \mathrm{y}_{1} \mathrm{~d}^{4} \mathrm{y}_{2} \exp \left(-\mathrm{ip} \mathrm{p}_{1}-\mathrm{ip}_{2} \mathrm{y}_{2}\right) \times \\
& \times \delta\left[\lambda\left(\mathrm{y}_{1}-\mathrm{y}_{2}\right)\right] \mathrm{R}_{\mu}^{\alpha \beta}\left(\mathrm{X}, a \mid \mathrm{y}_{1}, \mathrm{y}_{2}\right) . \tag{3.3}
\end{align*}
$$

Let us project the $\tilde{\mathrm{R}}_{\mu}^{a \beta}$ onto the subspace of the positiveenergy states:

$$
\begin{aligned}
& \tilde{\mathrm{R}}_{\mu \sigma_{1} \sigma_{2}}^{(+)}\left(\mathrm{X}, a \mid \vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \mathscr{P}^{2}\right)=\frac{-1}{4 \mathrm{~m}^{2}} \overrightarrow{\mathrm{R}}_{\mu}^{a \beta}\left(\mathrm{X}, \alpha \mid \vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \mathscr{P}^{2}\right) \times \\
& \times \overline{\mathrm{v}}_{\alpha}\left(-\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \sigma_{2}\right) \mathrm{u}_{\beta}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \sigma_{1}\right)
\end{aligned}
$$

and in analogy with (2.18) let pass to functions with a definite total spin and its projection:

$$
\begin{equation*}
\tilde{\mathrm{R}}_{\mu \mathrm{s} \sigma}^{(+)}\left(\mathrm{X}, a \mid \vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \mathscr{P}^{2}\right)=\sum_{\sigma_{1}, \sigma_{2}= \pm 1 / 2}^{\mathrm{L}}<\frac{1}{2} \frac{1}{2} ; \sigma_{1} \sigma_{2} \mid \mathrm{s} \sigma>\mathrm{R}_{\mu \sigma_{1} \sigma_{2}}^{(+)}\left(\mathrm{X}, a \mid{\overrightarrow{\Delta_{\mathrm{p}, \mathrm{~m} \lambda}}}^{\mathscr{P}^{2}}\right) \tag{3.4}
\end{equation*}
$$

where $\lambda \equiv \lambda \rho$. By using the representation of the $T$-product through the $\theta$ function a spectral respresentation can be $\overline{\mathscr{P}^{2}}=\mathrm{M}$
obtained that gives near the bound state pole with the following expression (see $/ 8 /$ ):

$$
\begin{align*}
& \text { following expression } \\
& \overrightarrow{\mathrm{R}}_{\mu \mathrm{s} \sigma}^{(+)}\left(\mathrm{X}, a \mid \vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \mathscr{P}\right) \sim \frac{\mathrm{i}(2 \pi)^{3}}{2 \mathrm{M}} \frac{\langle\mathrm{X}, a| \mathrm{J}_{\mu}(0)|\overrightarrow{\mathscr{P}}, \mathrm{M}, \mathrm{~s}, \sigma\rangle}{\sqrt{\mathscr{} 2}-\mathrm{M}+\mathrm{i} \epsilon} \vec{\Psi}_{\mathrm{s} \sigma}^{(+)}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda}\right),  \tag{3.5}\\
& \tilde{\Psi}_{\mathrm{s} \sigma}^{(+)}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda}\right)={ }_{\sigma_{1}, \sigma_{2}= \pm 1 / 2}^{\mathrm{L}}<\frac{1}{2} \frac{1}{2} ; \sigma_{1} \sigma_{2} \mid \mathrm{s} \sigma>\tilde{\Psi}_{\sigma_{1}}^{(+)}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda}\right)
\end{align*}
$$

Following ref. ${ }^{18 /}$ it is convenient to introduce a generalized vertex function $\tilde{\Gamma}_{\mu}^{(+)}$through the relation

$$
\tilde{\mathrm{R}}_{\mu}^{(+)}=\tilde{\Gamma}_{\mu}^{(+)} \tilde{\mathrm{G}}^{(+)},
$$

where $\mathrm{G}^{(+)}$is the two-time covariant Green function (2.12). Comparing the expressions (3.5), (3.6) and taking into account (2.10), (2.12) near the bound state pole we find

The vertex function $\vec{\Gamma}_{\mu}^{(+)}$can be calculated by expanding it in the coupling constant. In the lowest order ${ }^{/ 8 /}$

$$
\begin{equation*}
\vec{\Gamma}_{0 \mu}^{(+)}=\tilde{\mathrm{R}}_{0 \mu}^{(+)}\left[\tilde{\mathrm{G}}_{0}^{(+)}\right]^{-1} \tag{3.8}
\end{equation*}
$$

From the spectral representation for $\tilde{R}_{\mu}^{(+)}$we find near the bound state pole with meson quantum numbers

$$
\begin{align*}
& \tilde{R}_{0 \mu \mathrm{~s} \sigma}^{(+)}\left(\mathrm{X}, a \mid \vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \mathscr{P}^{2}\right)=\frac{\mathrm{i}(2 \pi)^{3}}{\left(2 \Delta_{\mathrm{p}, \mathrm{~m} \lambda}^{\circ}\right)^{2}} \times \\
& \quad\langle\mathrm{X}, a| J_{\mu}(0)\left|\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ;-\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \mathrm{s}, \sigma\right\rangle  \tag{3.9}\\
& \sqrt{\mathscr{\mathscr { P }}^{2}}-2 \Delta_{\mathrm{p}, \mathrm{~m} \lambda^{+i} \epsilon}^{\circ}
\end{align*}
$$

where we have passed to the total spin s and its projection $\sigma$

$$
\begin{equation*}
\left|\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ;-\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \mathrm{s}, \sigma\right\rangle=\sum_{\sigma_{1}, \sigma_{2}= \pm 1 / 2}\left\langle\frac{1}{2} \frac{1}{2} ; \sigma_{1} \sigma_{2}\right| \mathrm{s} \sigma>\left|\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \sigma_{1} ;-\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \sigma_{2}\right\rangle . \tag{3.10}
\end{equation*}
$$

With the help of (2.13), where we perform an anologous addition of spins, we find

$$
\tilde{\Gamma}_{0 \mu \mathrm{~s} \sigma}^{(+)}\left(\mathrm{X}, a \mid \vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \mathscr{P}^{2}\right)=2 \Delta_{\mathrm{p}, \mathrm{~m} \lambda}^{\circ}<\mathrm{X}, a\left|\mathrm{~J}_{\mu}(0)\right| \vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ;-\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \mathrm{s}, \sigma>\text {. (3.11) }
$$

Here the matrix element of the current operator describes a transition of a quark and antiquark state into a final hadron state $|\mathrm{X}, \alpha\rangle$. Substituting (3.11) into (3.7) we find the formula

$$
\begin{align*}
& \langle\mathrm{X}, a| \mathrm{J}_{\mu}(0)|\overrightarrow{\mathscr{P}}, \mathrm{M}, \mathrm{~s}, \sigma\rangle= \\
& \left.=\int \mathrm{d}^{3} \vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} / 2 \Delta_{\mathrm{p}, \mathrm{~m} \lambda}^{\circ}<\mathrm{X}, a\left|\mathrm{~J}_{\mu}(0)\right| \vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ;-\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} ; \mathrm{s}, \sigma\right\rangle \phi_{\mathrm{s}, \sigma}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda}\right) \tag{3.12}
\end{align*}
$$

that gives with the he1p of (2.21) and (3.1)

$$
\begin{aligned}
& \mathrm{W}_{\mu \nu}(\mathcal{P}, \mathrm{q})=(8 \pi)^{-1} \underset{\mathrm{X}, a}{ }(2 \pi)^{4} \delta^{(4)}\left(\mathscr{F}+\mathrm{q}-\mathcal{F}_{\mathrm{X}}\right) \iint \mathrm{d}^{3} \vec{\Delta}_{\mathrm{p}, \mathrm{~m} \lambda} / 2 \Delta_{\mathrm{p}, \mathrm{~m} \lambda}^{\circ} \times
\end{aligned}
$$

$$
\begin{align*}
& x<\mathrm{X}, a\left|J_{\nu}(0)\right|{\overrightarrow{\Delta_{\mathrm{p}}^{\prime}}{ }^{\prime}, \mathrm{m} \lambda} ;-\vec{\Delta}_{\mathrm{p}}^{\prime}, \mathrm{m} \lambda ; \mathrm{s}, \sigma^{\prime}>\phi_{\mathrm{s}, \sigma^{\prime}}\left(\overrightarrow{\mathrm{U}}_{\mathrm{p}}{ }^{\prime}, \mathrm{m} \lambda\right) . \tag{3.13}
\end{align*}
$$

Let us use (3.13) for the calculation of the structure functions for electromagnetic scattering on a pseudoscalar.meson processes. In what follows we shall estrizt our consideration to the final stace being a free quark-antiquark stata:
$\mathrm{W}_{\mu \nu}(\mathscr{P}, \mathrm{q})=(8 r)^{-1}{\underset{r_{1}}{1}, r_{2}}_{\Sigma}^{\int} \iiint \mathrm{d}^{3} \vec{\Delta}_{\mathrm{p}, \mathrm{m} \lambda} / \varepsilon \Delta_{\mathrm{p}, \mathrm{m} \lambda}^{\circ} \cdot \mathrm{d}^{3} \vec{\Delta}_{\mathrm{p}}^{\prime}, \mathrm{m} \lambda / 2 \Delta_{\mathrm{p}}^{\circ}, \mathrm{m} \lambda \times$
$\times \mathrm{d}^{4}{ }^{4}{ }_{1} \theta\left(\mathrm{k}{ }_{1}^{\circ}\right) \delta\left(\mathrm{k}_{1}^{2}-\mathrm{m}^{2}\right) \cdot \mathrm{d}^{4} \mathrm{k}_{2} \theta\left(\mathrm{k}_{2}^{\circ}\right) \delta\left(\mathrm{k}_{2}^{2}-\mathrm{m}^{2}\right) \cdot(2 \pi)^{4} \delta^{(4)}\left(\mathcal{P}+\mathrm{a}-\mathrm{k}_{1}-\mathrm{k}_{2}\right) \times$
$\left.\times \vec{\phi}_{0,0}\left(\vec{\Delta}_{\mathrm{p}, \mathrm{m} \lambda}\right)<\vec{\Delta}_{\mathrm{p}, \mathrm{m} \lambda} ;-\vec{\Delta}_{\mathrm{p}, \mathrm{m} \lambda} ; 0,0 \mid J_{\mu} ; 0\right) \mid \vec{\Delta}_{\mathrm{k}_{1}, \mathrm{~m} \lambda} ; r_{1} ; \vec{\Delta}_{\mathrm{k}_{2}, \mathrm{~m} \lambda}: r_{2}>x$


a)

6)

Fig. 2. The diagrams that give a contribution into a generalized vertex function $\tilde{\Gamma}_{\mu}^{(+)}$in the mmpulse approximation.

In the impulse approximation (see fig. 2) the current operator between two-quark states can be represented in the form

$$
\begin{align*}
& \left\langle\vec{\Delta}_{\mathrm{k}_{1}, \mathrm{~m} \lambda} ; r_{1} ; \vec{\Delta}_{\mathrm{k}_{2}, \mathrm{~m} \lambda} ; r_{2}\right| \mathrm{J}_{\mu}(0) \mid \vec{\Delta}_{\mathrm{p}_{1}, \mathrm{~m} \lambda} ; \sigma_{1} ; \vec{\Delta}_{\mathrm{p}_{2}, \mathrm{~m} \lambda} ; \sigma_{2}>= \\
& =\mathrm{Q}_{1}<\vec{\Delta}_{\mathrm{k}_{1}, \mathrm{~m} \lambda} ; r_{1}\left|\mathrm{~J} \mathrm{~J}_{\mu}(0)\right| \vec{\Delta}_{\mathrm{p}_{1}, \mathrm{~m} \lambda} ; \sigma_{1}>\cdot 2 \Delta_{\mathrm{k}_{2}, \mathrm{~m} \lambda}^{\circ} \delta\left(\vec{\Delta}_{\mathrm{k}_{2}, \mathrm{~m} \lambda}-\vec{\Delta}_{\mathrm{p}_{2}, \mathrm{~m} \lambda}\right) \delta_{r_{2}} \sigma_{2}+ \\
& \left.+\mathrm{Q}_{2}<\vec{\Delta}_{\mathrm{k}_{2}, \mathrm{~m} \lambda} ; r_{2}\left|\mathrm{~J}_{\mu}(0)\right| \vec{\Delta}_{\mathrm{p}_{2}, \mathrm{~m} \lambda} ; \sigma_{2}\right\rangle \cdot 2 \Delta_{\mathrm{k}_{1}, \mathrm{~m} \lambda}^{\circ} \delta\left(\vec{\Delta}_{\mathrm{k}_{1}, \mathrm{~m} \lambda} \vec{\Delta}_{\mathrm{p}_{1}, \mathrm{~m} \lambda}\right) \delta_{r_{1} \sigma_{1}}, \\
& \left\langle\vec{\Delta}_{\mathrm{k}_{\mathrm{i}}, \mathrm{~m} \lambda} ; r_{\mathrm{i}}\right| \mathrm{J}_{\mu}(0) \mid \vec{\Delta}_{\mathrm{p}_{\mathrm{i}}, \mathrm{~m} \lambda} ; \sigma_{\mathrm{i}}>=\mathrm{u}\left(\vec{\Delta}_{\mathrm{k}_{\mathrm{i}}, \mathrm{~m} \lambda} ; r_{\mathrm{i}}\right) \gamma_{\mu} \mathrm{u}\left(\vec{\Delta}_{\mathrm{p}_{\mathrm{i}}, \mathrm{~m} \lambda} ; \sigma_{\mathrm{i}}\right) \tag{3.15}
\end{align*}
$$

Here $Q_{1}\left(Q_{2}\right)$ is the quark (antiquark) charge (in the units of e). After substituting (3.15), (3.16) into (3.14) we find

$$
\begin{align*}
& \mathrm{W}_{\mu \nu}(\mathscr{P}, \mathrm{q})=(2 \pi)^{3} / 4 \sum_{\text {spin }} \iint \mathrm{d}^{3}{\overrightarrow{\Delta_{\mathrm{k}}^{1}}}, \mathrm{~m} \lambda \tag{3.17}
\end{align*} / 2 \Delta_{\mathrm{k}_{1}, \mathrm{~m} \lambda}^{\circ} \cdot \mathrm{d} 3_{\Delta_{\mathrm{k}_{2}, \mathrm{~m} \lambda} / 2 \Delta_{\mathrm{k}_{2}, \mathrm{~m} \lambda}^{\circ} \quad \times}^{\times 0^{(4)}\left(\dot{\mathscr{P}}+\mathrm{Q}_{1}-\mathrm{k}_{2}\right) \cdot\left(\mathrm{Q}_{1}^{2} \mathrm{~h}_{1 \mu \nu}+\mathrm{Q}_{1} \mathrm{Q}_{2} \mathrm{~h}_{2 \mu \nu}+\mathrm{Q}_{2} \mathrm{Q}_{1} \mathrm{~h}_{3 \mu \nu}+\mathrm{Q}_{2}^{2} \mathrm{~h}_{4 \mu \nu}\right) .}
$$

The values $h_{i \mu \nu}$ are products of the quark currents of (3.16) type and the wave functions. For example,

$$
\begin{aligned}
& \mathrm{h}_{1}^{\mu \nu}=\sum_{\sigma_{1}, \sigma_{2}= \pm 1 / 2}\left\langle\frac{1}{2} \frac{1}{2} ; \sigma_{1} \sigma_{2} \mid 0,0\right\rangle \overrightarrow{\mathrm{u}}\left(-\vec{\Delta}_{\mathrm{k}_{2}, \mathrm{~m} \lambda} ; \sigma_{1}\right) y^{\mu} \mu_{\mathrm{u}}\left({\overrightarrow{\Lambda_{\mathrm{k}_{1}}, \mathrm{~m} \lambda}} ; \tau_{1}\right) \times{ }_{(3.18)} \\
& \times \overrightarrow{\mathrm{u}}\left(\overrightarrow{\Delta_{\mathrm{k}_{1}, \mathrm{~m} \lambda}}{\left.; r_{1}\right)}^{\nu}{ }^{\nu} \mathrm{u}\left(-{\overrightarrow{\Delta_{\mathrm{k}}^{2}}}, \mathrm{~m} \lambda ; \sigma_{1}\right) \cdot\left|\phi_{0,0}\left(-\vec{\Delta}_{\mathrm{k}_{2}, \mathrm{~m} \lambda}\right)\right|^{2} .\right.
\end{aligned}
$$

The procedure of averaging over the quark spins in (3.17) leads to a typical form of the impulse approximation Lorentz structure (see, for example, /18/)

$$
\begin{align*}
& \tilde{\mathrm{h}}_{1}^{\mu \nu} \equiv \sum_{\mathrm{spin}} \mathrm{~h}_{1}^{\mu \nu} \equiv\left|\phi_{0,0}\left(-\vec{\Delta}_{\mathrm{k}_{2}, \mathrm{~m} \lambda}\right)\right|^{2} \cdot \operatorname{Sp}\left\{{\overrightarrow{\mathrm{u}}\left(-\vec{\Delta}_{\mathrm{k}_{2}}, \mathrm{~m} \lambda\right.}^{\vec{m}^{\prime}} \gamma^{\mu} \times \cdot\right. \\
& \times \mathrm{u}\left(\vec{\Delta}_{\mathrm{k}_{1}, \mathrm{~m} \lambda}\right){\left.\overrightarrow{\mathrm{u}}\left(\vec{\Delta}_{\mathrm{k}_{1}, \mathrm{~m} \lambda}\right) \gamma^{\nu} \mathrm{u}\left(-\vec{\Delta}_{\mathrm{k}_{2}, \mathrm{~m} \lambda}\right)\right\}=}_{=\left|\phi_{0,0}\left(-\vec{\Delta}_{\mathrm{k}_{2}, \mathrm{~m} \lambda}\right)\right|^{2}\left\{\tilde{\Delta}_{\mathrm{k}_{2}, \mathrm{~m} \lambda}^{\mu} \Delta_{\mathrm{k}_{1}, \mathrm{~m} \lambda}^{\nu}+\vec{\Delta}_{\mathrm{k}_{2}, \mathrm{~m} \lambda}^{\nu} \Delta_{\mathrm{k}_{1}, \mathrm{~m} \lambda}^{\mu}-\right.}^{\left.-\mathrm{g}^{\mu \nu} \tilde{\Delta}_{\mathrm{k}_{2}, \mathrm{~m} \lambda}^{a}\left(\Delta_{\mathrm{k}_{1}, \mathrm{~m} \lambda}\right)_{a}\right\},}  \tag{3.19}\\
& \tilde{\Delta}_{\mathrm{k}_{2}, \mathrm{~m} \lambda^{\equiv}}^{\mu}\left(\Delta_{\mathrm{k}_{2}, \mathrm{~m} \lambda}^{\circ} ;-\vec{\Delta}_{\mathrm{k}_{2}, \mathrm{~m} \lambda}\right) .
\end{align*}
$$

An analogous expression for $\tilde{h}_{4 \mu \nu}$ can be obtained from expression (3.19) by changing $u(p, \sigma) \rightarrow v\left(p_{3} \sigma\right)$. One integration in (3.17) with the volume element $d^{3} k$ can be performed with the help of a $\delta\left(\mathscr{I}_{+} q-\mathrm{k}_{1}-\mathrm{k}_{2}\right)$ function. As a result, we obtain with account of (3.19) and the $W F$ reality property (i.e., $\bar{\phi}_{\mathrm{s}, \sigma}(\overrightarrow{\mathrm{p}})=$ $\left.=\phi_{\mathrm{s}, \sigma}(\overrightarrow{\mathrm{p}})\right)$ :

$$
\begin{align*}
& \mathrm{W}_{\mu \nu}(\mathscr{P}, \mathrm{q})=\frac{(2 \pi)^{3}}{4} \int \frac{\mathrm{~d}^{3} \vec{\Delta}_{\mathrm{k}, \mathrm{~m} \lambda}}{2 \Delta_{\mathrm{k}, \mathrm{~m} \lambda}^{\circ}}-\frac{1}{2 \Delta_{\mathrm{k}, \mathrm{~m} \lambda}^{\circ}} \delta\left(\mathrm{M}+\nu-\Delta_{\mathrm{k}, \mathrm{~m} \lambda}^{\circ}-\Delta_{\mathrm{k}}^{\circ}, \mathrm{m} \lambda^{\circ}\right) \times \\
& \times\left[\left(Q_{1}^{2}+Q_{2}^{2}\right) \cdot \phi_{0,0}^{2}\left(\overrightarrow{\Delta_{k, m}}\right) \cdot A_{\mu \nu}+2 Q_{1} Q_{2} \phi_{0,0}\left(\vec{\Delta}_{\mathrm{k}, \mathrm{~m} \lambda}\right) \phi_{0,0}\left(\overrightarrow{\Delta_{\mathrm{k}}}, \mathrm{~m} \lambda\right) \mathrm{B}_{\mu \nu}\right] . \tag{3.20}
\end{align*}
$$

Here $\vec{\Delta}_{k^{\prime}, m \lambda}=\vec{q}^{\prime}-\vec{\Delta}_{k, m \lambda}, \vec{q}^{\prime} \equiv\left(\vec{L}_{\lambda^{1}} \mathrm{q}\right), \Delta_{k^{\prime}, m \lambda}^{o}=\sqrt{m^{2}+\vec{\Delta}_{k}^{2}, m \lambda}$ and $A_{\mu \nu} \equiv A_{\mu \nu}\left(\vec{\Delta}_{k, m \lambda} ; \vec{\Delta}_{\mathrm{k}} ; \mathrm{m} \lambda\right)$ is constructed from $\overrightarrow{\mathrm{h}}_{1 \mu \nu}$ and $\tilde{\mathrm{h}}_{4 \mu \nu}$ and $\mathrm{B}_{\mu \nu} \equiv \mathrm{B}_{\mu \nu}\left({\overrightarrow{\Delta_{k}}}_{\mathrm{k}, \mathrm{m} \lambda} ;{\overrightarrow{\Delta_{\mathrm{k}}} ; \mathrm{m} \lambda}^{-}\right.$from $\tilde{\mathrm{h}}_{2 \mu \nu}$ and $\tilde{\mathrm{h}}_{3 \mu \nu}$. Now let us note that the integration in (3.20) is performed in fact with the volume element of the Lobachevsky momentum space $\mathrm{d}^{3}{\overrightarrow{\Delta_{k}}, \mathrm{~m} \mathrm{\lambda}} /\left(\Delta_{\mathrm{k}, \mathrm{m} \mathrm{\lambda}} \cdot \mathrm{~m}^{-1}\right)$ and the quark momenta belong to the upper sheet of the same hyperboloid as the vectors $\vec{k}$ and $\overrightarrow{\Delta_{k}}, m \lambda$.

In a parton model the quark momenta belong to the massshell hyperboloid also. This is a consequence of the choice of a special reference frame where the hadron momenta $|\overrightarrow{\mathcal{P}}| \rightarrow \infty$, thus the quarks can be considered as the real particles. In contrast to the usual projection of the quark momentum $\vec{k}$ on the hadron momentum $\overrightarrow{\mathscr{P}}$, we shall use here an invariant projection of the covariantly defined quark momenta $\vec{\Delta}_{\mathrm{k}, \mathrm{m} \lambda}$ in the c.m.s. on the vector $\overrightarrow{\mathrm{q}}^{\prime} \equiv\left(\mathrm{L} \vec{\lambda}^{-1} \mathrm{q}\right)$, i.e. the quantity $\Delta_{\|}=$ $=\left(\vec{\Delta}_{k, m \lambda} \cdot \vec{q}^{\prime}\right) /\left|\vec{q}^{\prime}\right|^{/ 2 /}$. The $q_{0}^{\prime}$ and $|\vec{q}|$ are invariants:

$$
\mathrm{q}_{0}^{\prime} \equiv\left(\mathrm{L}_{\lambda}^{-1} \mathrm{q}\right)_{0}=\lambda^{\mu} \mathbf{q}_{\mu}=\mathscr{P} \mathbf{q} / \mathrm{M}=\nu,\left|\overrightarrow{\mathrm{q}}^{\prime}\right|=\sqrt{\mathrm{q}_{0}^{\prime 2}-\mathrm{q}^{\prime \mu} \mathrm{q}_{\mu}^{\prime}}=\sqrt{\nu \nu^{2} \mathrm{Q}^{2}} \cdot(3.21)
$$

Therefore the projection $\Delta_{\|}$and the modulus of the vector $\vec{\Delta}_{\perp}$ are invariant also:

$$
\begin{align*}
& \Delta_{\|} \equiv\left(\vec{\Delta}_{\mathrm{k}, \mathrm{~m} \lambda} \overrightarrow{\mathrm{q}^{\prime}}\right) /\left|\overrightarrow{\mathrm{q}}^{\prime}\right|=\left(\Delta_{\mathrm{k}, \mathrm{~m} \lambda}^{\circ} \mathrm{q}^{\circ \circ}-\Delta_{\mathrm{k}, \mathrm{~m} \lambda}^{\mu} \mathrm{q}_{\mu}^{\prime}\right) / \sqrt{\nu}{ }^{2}+\mathrm{Q}^{2} \\
& \mid \overrightarrow{\Delta_{\perp}} \tag{3.22}
\end{align*}
$$

what follows from the invariance of $q^{\circ}, \Delta_{k, m \lambda}{ }^{\circ}$ and $\left|\vec{q}^{\prime}\right|$ (see (2.7)). In terms of the $\Delta_{k, m \lambda}^{\circ}$ and $\Delta_{\|}$the scalar products ( $\mathscr{P}_{\mathrm{k}}$ ) and (qk) which appear in the $\delta\left(\mathrm{M}+\nu-\Delta_{\mathrm{k}, \mathrm{m} \lambda}^{\mathrm{o}}\right.$ -- $\Delta_{k}^{\circ}{ }^{\prime}, m \lambda$ ) function can be represented in the form
$\rho_{\mathrm{k}}=M \Delta_{\mathrm{k}, \mathrm{m} \lambda}^{\circ} ; \quad \mathrm{qk}=\mathrm{q}_{0}^{\circ} \Delta_{\mathrm{k}, \mathrm{m} \lambda}^{\circ}-\left|\overrightarrow{\mathrm{q}^{\prime}}\right| \Delta_{\|}=\nu \Delta_{\mathrm{k}, \mathrm{m} \lambda}^{\circ}-\sqrt{\nu 2_{+} \mathrm{Q}^{2}} \Delta_{\mathrm{i}}$.
After integration over the spherical angles we find from (3.18) the following expressions for the structure functions

$$
\begin{align*}
& F_{1}=M\left[V_{1}+\left(1+\nu^{2} / Q^{2}\right)^{-1} V_{2}\right] \\
& F_{2}=\nu / 2\left(1+\nu^{2} / Q^{2}\right)^{-1}\left[V_{1}+3\left(1+\nu^{2 /} / Q^{2}\right)^{-1} \cdot V_{2}\right],
\end{align*}
$$

where

$$
\begin{align*}
& \mathrm{V}_{1} \equiv-\mathrm{g}^{\mu \nu} \mathrm{W}_{\mu \nu}, \quad \mathrm{V}_{2} \equiv \mathrm{M}^{-2} \mathscr{\rho}^{\mu} \mathscr{P}^{\nu} \mathrm{W}_{\mu \nu} \\
& \mathrm{V}_{\mathrm{i}}=\frac{\eta_{+}}{(2 \pi)^{2} \sqrt{\nu}{ }^{2}+\mathrm{Q}^{2}} \eta_{-} \mathrm{d} \eta\left[\left(\mathrm{Q}_{1}^{2}+\mathrm{Q}_{2}^{2}\right) \phi_{0,0}^{2}(\eta) \mathrm{A}_{\mathrm{i}}+\right. \\
& \left.+2 \mathrm{Q}_{1} \mathrm{Q}_{2} \phi_{0,0}\left(\frac{\mathrm{M}+\nu}{\mathrm{m}}-\eta\right) \phi_{0,0}(\eta) \mathrm{B}_{\mathrm{i}}\right]
\end{align*}
$$

Here $\eta=\mathrm{m}^{-1} \Delta_{\mathrm{k}, \mathrm{m} \lambda}^{0} \quad$ and $\mathrm{A}_{\mathrm{i}}=\mathrm{A}_{\mathrm{i}}\left(\eta, \nu, \mathrm{W}^{2}\right), \quad \mathrm{B}_{\mathrm{i}}=\mathrm{B}_{\mathrm{i}}\left(\eta, \nu, \mathrm{W}^{2}\right)$ are obtained from the $A_{\mu \nu}$ and $\mathrm{B}_{\mu \nu}$ in (3.18) after integration and multiplication by the factors $\mathrm{g}^{\mu \nu}$ and $\mathrm{M}^{-2 \mathscr{P} \mu \mathscr{P} \nu}$ (see (3.21). It is easy to calculate that

$$
\begin{align*}
& \mathrm{A}_{1}=-\eta^{2}+\frac{\mathrm{M}+\nu}{\mathrm{m}} \eta-\frac{\mathrm{W}^{2}}{4 \mathrm{~m}^{2}}-\frac{1}{2}, \quad \mathrm{~A}_{2}=\frac{\mathrm{W}^{2}}{8 \mathrm{~m}^{2}} ;  \tag{3.23}\\
& \mathrm{B}_{1}=\eta^{2}-\frac{\mathrm{M}+\nu}{\mathrm{m}} \eta+\frac{1}{2},  \tag{3.24}\\
& \mathrm{~B}_{2}=\frac{1}{2}\left(\eta^{2}-\frac{\mathrm{M}+\nu}{\mathrm{m}} \eta+\frac{\mathrm{W}^{2}}{4 \mathrm{~m}^{2}}\right) .
\end{align*}
$$

The limits of the integration in eq. (3.22) are

$$
\begin{equation*}
\eta_{ \pm}=\frac{\nu+M \pm \epsilon \sqrt{\nu^{2}+Q^{2}}}{2 \mathrm{~m}}, \tag{3.25}
\end{equation*}
$$

where $\epsilon=\sqrt{1-4 m^{2} / W^{2}}$.
If the WF $\phi_{0,0}(\eta)$ decreases when $\eta \rightarrow \infty$ (the necessary condition of the convergence of the normalization integral for WF), one can neglect the interference terms containing the factors $B_{i}$, in the expressions for $\cdot V_{i}$, that is to say the interactions of the virtual photon with the quark and antiquark in the incoherent way which is a usual assumption of the parton model.

As it was done in ref. ${ }^{/ 2 /}$ we introduce $W F \quad \phi(y)$ :

$$
\begin{equation*}
\phi_{0,0}(\eta)=\frac{4 \pi}{\mathrm{~m} \sin \mathrm{hy}} \phi(y), \tag{3.26}
\end{equation*}
$$

depending on the quark rapidity $y=\ln \left(\eta+\sqrt{\left.\eta^{2}-1\right)}\right.$ and being related with WF in coordinate space by the following transformation:

$$
\begin{equation*}
\phi(y)=\int_{0}^{\infty} \mathrm{dr} \cdot \mathrm{r} \phi_{0,0}(\mathrm{r}) \sin \mathrm{rmy} \tag{3.27}
\end{equation*}
$$

Hence omitting the interference terms we get the following expressions for the structure functions $\mathrm{F}_{\mathrm{i}}$ :

$$
\begin{align*}
& \mathrm{F}_{1}\left(\zeta, W^{2}\right)=\frac{16 \mathrm{~m}^{2} \zeta\left(Q^{2}+Q_{2}^{2}\right)}{W^{2}-M^{2} \zeta^{2}} \int_{\left|\ln \frac{(1+\epsilon) M \zeta}{2 m}\right|}^{\ln \frac{(1+\epsilon) W^{2}}{2 m M \zeta}} \operatorname{dy} \operatorname{sinhy}|\phi(y)|^{2} \times \\
& \times\left[\frac{W^{2}+\zeta^{2} M^{2}}{2 m M \zeta} \operatorname{coshy}-\sinh ^{2} y-\frac{W^{2}}{4 m^{2}}-\frac{3}{2}+\right. \\
& \left.+\frac{M^{2} \zeta(1-\zeta) W^{2}\left(W^{2}-M^{2} \zeta\right)}{2 m^{2}\left(W^{2}-M^{2} \zeta^{2}\right)^{2}}\right] \tag{3.28}
\end{align*}
$$

$$
\begin{align*}
& \mathrm{F}\left(\zeta, W^{2}\right)=\frac{16 \mathrm{~m}^{2} \zeta\left(Q \frac{Q}{1}+Q_{2}^{2}\right)}{\left(W^{2}-M^{2} \zeta^{2}\right)^{3}}\left[W^{2}-M^{2} \zeta(2-\zeta)\right](1-\zeta)\left(W^{2}-M^{2} \zeta\right) \times \\
& \ln \frac{(1+\epsilon) W^{2}}{2 \mathrm{mM} \zeta} \\
& \times \quad \int \quad \frac{d y}{\sinh y}|\phi(y)|^{2} \cdot\left[\frac{W^{2}+\zeta^{2} M^{2}}{2 m M \zeta} \cosh y-\sinh ^{2} y-\right.  \tag{3.29}\\
& \left.\ln \frac{(1+\epsilon) M \zeta}{2 \mathrm{~m}} \right\rvert\,
\end{align*}
$$

where we have used a scaling variable

$$
\zeta=\frac{\nu+M-\sqrt{v^{2}+Q^{2}}}{M}
$$

$$
\left.-\frac{W^{2}}{4 m^{2}}-\frac{3}{2}+\frac{3 M^{2} \zeta(1-\zeta) W^{2}\left(W^{2}-M^{2} \zeta\right)}{2 m^{2}\left(W^{2}-M^{2} \zeta^{2}\right)}\right]
$$

which was described in detail in ref. ${ }^{/ 2,20 /}$. In our scheme it is more natural to consider the structure functions in terms of $W^{2}$ and $\zeta$ rather than Bjorken variables $Q^{2}$ and $x$.

In the deep inelastic region when $W^{2} \rightarrow \infty$ and $\zeta$ is fixed, the leading term of the structure functions expansion over inverse powers of $W^{2}$ has the following form

$$
\begin{aligned}
& \mathrm{F}_{2}\left(\zeta, W^{2}\right)=(1-\zeta) \cdot \mathrm{F}_{1}\left(\zeta, W^{2}\right)= \\
& \approx 8 M\left(Q_{1}^{2}+Q_{2}^{2}\right)(1-\zeta) \cdot \int_{\ln \frac{W^{2}}{\mathrm{mM} \zeta}}^{\left|\ln \frac{M}{m}\right|} \mathrm{dy} \frac{\cosh y-\zeta M / 2 \mathrm{~m}}{\sinh y}|\phi(y)|^{2}- \\
& -\frac{16 \mathrm{mM}^{2}\left(Q_{1}^{2}+Q_{2}^{2}\right) \zeta(1-\zeta)}{W^{2}} \ln \frac{W^{2}}{m M \zeta} \\
& \left.\ln \frac{M \zeta}{m} \right\rvert\,
\end{aligned}
$$

The first of these two equations is a relationship of the Cal -lan-Gross type in terms of the variables $W^{2}$ and $\zeta$.

## 4. CONCLUSIONS

In the present work we have expressed the structure functions of the hadron composed of two quarks through the covariant wave function of its relative motion. The quarks are here fermions with the spin $1 / 2$. The wave function obeys equation ( 2.20 ) which was earlier investigated in detail in ref. ${ }^{/ 15,17 / \text {, where its solutions were considered as well. }}$

Our further publications will be devoted to applications of the covariant two-particle wave functions, which are the solutions of eq. (2.20) with the quasipotential constructed of the matrix elements of the quark interaction amplitude in QCD ${ }^{\prime 21 /}$ for the structure function calculations. Here the investigation of the Bjorken's scaling violation as well as the comparison of the expressions obtained in this work for the structure functions with experimental data are of interest.

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## REFERENCES

1. Bjorken J.D. Phys.Rev., 1969, 179, p. 1547. Bogolubov N.N., Vladimirov V.S., Tavkhelidze A.N. Sov.J.Theor.Math. Phys., 1972, 12, p. 305. Vladimirov V.S., Zav'yalov B.I. Sov.J. Theor.Math. Phys., 1979, 40, p. 155.
2. Kapshay V.N. et a1. CERN, TH.3022, Geneva, 1981.
3. Logunov A.A., Tavkhelidze A.N. Nuovo Cim., 1963, 29, p. 380.
4. Kadyshevsky V.G., Tavkhelidze A.N. "Problems in Theoretical Physics", Essays dedicated to N.N. Bogolubov on the Occasion of his Sixtieth Birthday, Moscow, 1969, p. 261. Logunov A.A. et al. Sov.J.Theor.Math. Phys., 1971, 6, p. 157. Garsevanishvili V.R. et al. Sov.J.Theor.Math. Phys., 1975, 23, p. 310.
5. Faustov R.N. Ann.Phys., 1973, 78, p. 176.
6. Kadyshevsky V.G. Sov. JETP, 1964, 46, p. 654; 1964, 46, p. 872; Nucl.Phys., 1968, B6, p. 125. Kadyshevsky V.G., Mateev M.D. Nuovo Cim., 1968, 55A, p. 275.
7. Kadyshevsky V.G., Mir-Kasimov R.M., Skachkov N.B. Nuovo Cim., 1968, 55A, p. 233;Sov.J.Particles and Nuclei, 1972, 2, p. 635.
8. Faustov R.N. Proc. V Intern. Symposium on Many Particle Hadrodynamics, Eisenach and Leipzig, June 4-10, 1974, p. 769.
9. Savrin V.I. Sov.J.Theor.Math.Phys., 1976, 29, p. 347; 1979, 39 , p. 48.

Kvinikhidze A.N. et al. Sov.J.Particles and Nuclei, 1977, 8, p. 478. Krasnikov N.V., Chetyrkin K.G. Preprint INR, P-0036, Moscow, 1976. Atakishiev N.M., Mir-Kasimov R.M., Nagiev Sh.M. JINR, P2-80-635, Dubna, 1980.
10. Matveev V.A., Muradyan R.M., Tavkhelidze A.N. JINR, E23498, Dubna, 1967.
11. Shirokov Yu.M. Sov.JETP, 1951, 21, p. 748.
12. Skachkov N.B., Solovtsov I.L. Sov.J.Theor.Math. Phys., 1979, 41, p. 205; Sov.J.Nucl.Phys., 1979, 30, p. 1079.
13. Shirokov Yu.M. Sov. JETP, 1958, 35, p. 1005; Chgou GuanChgao, Shirokov M.I. Sov.JETP, 1958, 34, p. 1230; Macfarlane A.J, Rev.Mod.Phys., 1962, 34, p. 41.
14. Bogolubov N.N., Matveev V.A. Tavkhelidze A.N. Topics in Elementary Particle Theory. Proc. Intern. Seminar, Viena, Bulgaria. JINR, P2-4050, Dubna, 1968, p. 269; Matveev V.A. JINR, P2-3847, Dubna, 1968; Faustov R.N., Khelashvili A.A. JINR, P2-4345, Dubna, 1969.
15. Skachkov N.B. JINR, E2-81-78, Dubna, 1981; JINR, E2-7733, Dubna, 1973.
16. Logunov A.A. et al. Sov. J.Theor.Math.Phys., 1971, 6, p. 157; Kvinikhidze A.N., Stoyanov D.Ts. JINR, E2-5746, Dubna, 1971.
17. Skachkov N.B., Solovtsov I.L. Sov.J.Particles and Nuclei 1978, 9, p. 5.
18. Feynman R.P. Photon-Hadron Interactions. Benjamin W.A. Reading, Mass., 1972.
19. Georgi H., Politzer H.D. Phys.Rev., 1976, D14, p. 1829. Barbieri R. et a1. Phys.Let., 1976, 64B, p. 171; Nucl. Phys., 1976, B117, p. 50.
20. Savrin V.I., Skachkov N.B. Preprint TH.3050-CERN, Geneva, 1981; to be published in Nuovo Cim.
21. Savrin V.I., Skachkov N.B. Lett. Nuovo Cim., 1980, 29, p. 363; CERN, TH. 2822, Geneva, 1980.


[^0]:    * See also/15/, where this covariant spin equation was obtained on the basis of a Hamiltonian formulation of quantum field theory.

