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RELATIONS BETWEEN POLARIZATION PARAMETERS IN THE PROCESS $N+N \rightleftarrows d+\pi$

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1. The development of investigations with polarized beam and polarized targets has increased the interest in studying the polarization phenomena in the reactions

$$pp \to d + \pi^+ \tag{1}$$

and in the inverse process

$$\pi^+ + d \to pp. \tag{2}$$

Below a justification of the exact relation

$$L_{00} = 2\sqrt{3} < T_{22} >_{00} + \sqrt{2} < T_{20} >_{00} = 1 + 3A_{nn} \tag{3}$$

is presented which connects the average values of the tensors $<T_{20}>_{00}$ and $<T_{22}>_{00}$ describing the quadrupole polarization of deuterons in process (1) in the case of unpolarized beam and unpolarized target with quantity A_{nn}, which is the contribution to the differential cross section of process (1) having proton beam and target polarized perpendicularly to the reaction plane. The conclusions obtainable from (3) and existing measurements on A_{nn} are discussed. The generalization of relation (3) for the case of quadrupolarization of deuterons with polarized protons in process (1) is presented.

Relation (3) gives a new possibility in the investigation of deuteron polarization, to measure $<T_{22}>_{00}$ only, and $<T_{20}>_{00}$ can be calculated from (3) if A_{nn} is known. Process (2) can be used for making the experimental control of the polarization state of deuteron target easier. We notice that due to equality (3) the $<T_{20}>_{00}$, $<T_{22}>_{00}$ and A_{nn} quantities cannot vanish simultaneously at a given arbitrary energy and momentum transfer. Taking into account the T-invariance all the conclusions on the polarization parameters of process (1) can be transformed[1,2] to inverse process (2).

For a process similar to (1) with the production of a scalar particle an analogous relation can be given with changing the sign before A_{nn}

$$2\sqrt{3} < T_{22} >_{00} + \sqrt{2} < T_{20} >_{00} = 1 - 3A_{nn}. \tag{4}$$

2. For prooving relations (3)-(4) we have used the approach of L.Shechter paper/2/. The amplitude $M_p(\Theta)$ for process (1) is pseudoscalar, leading to the triplet state of nucleons in the final states and contains six scalar functions having definite symmetry under substitution Θ_π to $\pi-\Theta_\pi$ due to the Pauli principle (Θ_π is the pion angle in c.m. system). We have the notation for the deuteron angle Θ_Λ in the laboratory and Θ_d in c.m. system, where $\Theta_d=\pi-\Theta_\pi$, respectively. The value of Θ_Λ corresponding to $\Theta_\pi=\pi/2$ is denoted by Θ_1.

We introduce two orthonormal vector triad

$$\vec{\ell}=\frac{\vec{k}'+\vec{k}}{|\vec{k}'+\vec{k}|},\quad \vec{m}=\frac{\vec{k}'-\vec{k}}{|\vec{k}'-\vec{k}|};\quad \vec{n}=[\vec{\ell}\,\vec{m}]=[\vec{k}\vec{k}'\,]/|[\vec{k}\vec{k}'\,]| \tag{5}$$

in the c.m. system and

$$\vec{k},\vec{n},\vec{s}=[\vec{k}\,\vec{n}] \tag{6}$$

in the laboratory system of reaction (1).

Requiring the parity conservation at the reflection in the reaction plane we have the following condition for the amplitude M_p

$$(\vec{\sigma}_1\vec{n})(\vec{\sigma}_2\,\vec{n})M_p(\vec{\sigma}_1\vec{n})(\vec{\sigma}_2\vec{n})=-M_p. \tag{7}$$

If the beam and the target are polarized with $\vec{P}_1$ and $\vec{P}_2$, respectively, then the density matrix of the initial protons is equal to

$$\rho_0=\frac{1}{4}[1+(\vec{\sigma}_1\vec{P}_1)][1+(\vec{\sigma}_2\vec{P}_2)] \tag{8}$$

and the differential cross section has the form

$$\sigma_{\vec{P}_1\vec{P}_2}=\sigma_0[1+(\vec{P}_1\vec{n})A_1+(\vec{P}_2\vec{n})A_2+P_{1u}P_{2v}A_{uv}], \tag{9}$$

where A_1 and A_2 are the asymmetry of the cross section using normally polarized beam (target unpolarized) and normally polarized target (beam unpolarized), respectively

$$\sigma_0 A_{1,2}=1/4\,\mathrm{Sp}M(\vec{\sigma}_{1,2}\cdot\vec{n})M^+. \tag{10}$$

The definition of tensor A_{uv} is

$$\sigma_0 A_{uv}=1/4\,\mathrm{Sp}M\,\sigma_{1u}\sigma_{2v}M^+. \tag{11}$$

It has five non zero components: A_{nn}, $A_{\ell\ell}$, A_{mm}, $A_{\ell m}$, $A_{m\ell}$ in the c.m. and A_{nn}, A_{kk}, A_{ss}, A_{sk}, A_{ks} in the lab. system, respectively.

The polarization of deuterons is described by the average values of spin-tensors T_{JM}, which according to ref.[3] are given by the following relations

$$T_{10} = \sqrt{\frac{3}{2}} S_z ; T_{11} = -\frac{\sqrt{3}}{2}(S_x + S_y i); \quad T_{22} = \frac{\sqrt{3}}{2}(S_x + iS_y)^2 \tag{12}$$
$$T_{21} = -\frac{\sqrt{3}}{2}[(S_x + iS_y)S_z + S_z(S_x + iS_y)]; \; T_{20} = \frac{1}{\sqrt{2}}(3S_z^2 - 2) \; ; T_{J,-M} = (-1)^M T_{JM}^+ .$$

We choose the axis oy along the normal to the reaction plane and the axises z and x in the reaction plane, whose exact definition would be given below.

If the beam and the target are unpolarized in the initial state than the polarization of the final state deuterons can be given in terms of the quantities $< T_{JM}>_{00}$

$$\sigma_0 <T_{JM}>_{00} = \frac{1}{4} SpMM^+ T_{JM} . \tag{13}$$

Taking into account that the spin operator of the deuteron can be presented as $\vec{S} = \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$, for the $<T_{JM}>_{00}$ average values using the (7) we obtain

$$<iT_{11}>_{00} = \frac{\sqrt{3}}{4} <\sigma_{1y} + \sigma_{2y}> \; ; \qquad <T_{10}>_{00} = 0$$

$$<T_{22}>_{00} = \frac{\sqrt{3}}{4} <\sigma_{1x}\sigma_{2x} - \sigma_{1y}\sigma_{2y}>$$

$$<T_{21}>_{00} = -\frac{\sqrt{3}}{4} <\sigma_{1z}\sigma_{2x} + \sigma_{1x}\sigma_{2z}> \tag{14}$$

$$<T_{20}>_{00} = \frac{1}{2\sqrt{2}} <3\sigma_{1z}\sigma_{2z} - 1> ,$$

where

$$\sigma_0 <\sigma_{1v}\sigma_{2u}> = 1/4\, SpMM^+ \sigma_{1v}\sigma_{2u} .$$

If one introduces the angle $\theta = 1/2\Theta_\pi - \Theta_\Lambda$ [2], then the unit vectors $\vec{i}$, $\vec{j}$ and $\vec{q}$ along the axes x,y and z respectively can be expressed through the vectors $\vec{\ell}$, $\vec{m}$ and $\vec{n}$ in the following way[2]

$$\vec{i} = \vec{\ell} \sin\theta + \vec{m} \cos\theta$$

$$\vec{j} = \vec{n}$$
$$\vec{q} = \vec{\ell}\cos\theta - \vec{m}\sin\theta . \qquad (15)$$

It is necessary to add the relativistic spin rotation effect/4/ to the result of ref./2/. Due to this additional rotation the unit vectors $\vec{i}$ and $\vec{q}$ turn by angle Ω around $\vec{n}$ axis.

The result of the rotation by angle Ω around the $\vec{n}$ axis for arbitrary vector $\vec{a}$ is given by expression/5/

$$R_{\vec{n}}(\Omega)\vec{a} = \vec{a}(\vec{n}\vec{a})(1-\cos\Omega) + \vec{a}\cos\Omega + [\vec{n}\vec{a}]\sin\Omega \qquad (16)$$

and

$$R_{\vec{n}}(\Omega)\vec{\ell} = \vec{\ell}\cos\Omega + \vec{m}\sin\Omega$$
$$R_{\vec{n}}(\Omega)\vec{m} = \vec{m}\cos\Omega - \vec{\ell}\sin\Omega \qquad (17)$$

then

$$\vec{i}_R = R_{\vec{n}}(\Omega)\vec{i} = \vec{\ell}\sin\theta' + \vec{m}\cos\theta'$$
$$\vec{q}_R = R_{\vec{n}}(\Omega)\vec{q} = \vec{\ell}\cos\theta' - \vec{m}\sin\theta' \qquad (18)$$

take place, where

$$\theta'(\Theta_d) = \theta(\Theta_d) - \Omega(\Theta_d).$$

The comparison of (18) and (15) shows, that relativistic effect of spin rotation is effectively reduced to a change of θ for θ' in the proper formulae of ref./2/.

For Ω angle, defined as a difference from π of sum of inner angles of the Wick triangle /4/ $\omega + \Theta + \pi - \Theta_d$, we have

$$\Omega = \Theta - \Theta_\Lambda - \omega_d . \qquad (19)$$

Therefore angle Ω is calculated as a difference between the Wick angle ω and its nonrelativistic limit. It is clear, that $\omega < \Theta - \Theta_\Lambda$ and $\Omega \geq 0$.

For process (1) the Wick angle for deuterons is given by expression

$$\sin\omega_d = \frac{M_d}{M}\left[\frac{1-4M^2/s}{(1+\Delta/s)^2 - 4M_d^2/s}\right]^{1/2}\sin\Theta_\Lambda , \qquad (20)$$

where $\Delta = M_d^2 - m_\pi^2$, $s = 2M^2 + 2ME$; M_d, M and m_π are masses of the deuteron, nucleon and pion respectively, E is equal to total energy of protons in the lab.system, and

$$\theta'(\Theta)=\omega_d(\Theta)+\frac{3}{2}\Theta_\pi-\pi \tag{21}$$

and

$$\theta'(\pi/2)=\omega_d(\pi/2)-\pi/4 . \tag{22}$$

The polarization tensor N_{uv}

$$\sigma_0 N_{uv}=1/4\,\mathrm{Sp}MM^+\sigma_{1u}\sigma_{2v} \tag{23}$$

like a tensor A_{uv}, has five components. The condition (7) for process (1) leads to the relation [2]

$$N_{nn}=-A_{nn} . \tag{24}$$

It is true for amplitude M_p, that [2]

$$\mathrm{Sp}MM^+(\vec{\sigma}_1\vec{\sigma}_2)=\mathrm{Sp}MM^+ . \tag{25}$$

From (18) and (13) it follows, that

$$N_{xx}+N_{zz}=N_{\ell\ell}+N_{mm}=1-N_{nn}=1+A_{nn} . \tag{26}$$

After taking into account (15) and (18) the left-hand part of formula (3) has the following form

$$L_{00}=1/2\{3(N_{xx}+N_{zz}-N_{nn})-1\}. \tag{27}$$

With the help of (24) and (26) we obtain formula (3).

3. A. Let us consider scalar particle production in the reaction similar to (1). The invariance under the reflection in the reaction plane

$$(\vec{\sigma}_1\vec{n})(\vec{\sigma}_2\vec{n})M_s(\vec{\sigma}_1\vec{n})(\vec{\sigma}_2\vec{n})=M_s \tag{28}$$

gives for scalar particle production case

$$N_{nn}=A_{nn} \tag{29}$$

and

$$N_{xx}+N_{zz}=1-A_{nn} . \tag{30}$$

Substituting (28)-(29) into equation (27) we obtain relation (4).

B. It is interesring, that the form of (3)-(4) relations does not change if we apply a magnetic field directed perpendicularly to the reaction plane. The effect of such a magnetic field gives a rotation of the second rank tensor of quadrupolarization in the reaction plane with respect to the direction of the motion. Nevertheless the quantity

$$L_{00} = \langle 3(S_x^2 + S_z^2 - S_y^2) - 2 \rangle_{00} \tag{31}$$

has cylindrical symmetry, so it is not changed by the rotation around the axis oy. Though, due to the effect of magnetic field, the components $\langle T_{2M} \rangle_{00}$ are transformed into other $\langle T_{2T} \rangle'_{00}$ components (see formulae of ref.[6]), the combination of $\langle T_{22} \rangle_{00}$ and $\langle T_{20} \rangle_{00}$ defining the L_{00} quantity is not changed by the presence of magnetic field

$$L'_{00} = 2\sqrt{3} \langle T_{22} \rangle'_{00} + \sqrt{2} \langle T_{20} \rangle'_{00} = 2\sqrt{3} \langle T_{22} \rangle_{00} + \sqrt{2} \langle T_{20} \rangle_{00} = L_{00} \,. \tag{32}$$

This is valid not only for the $pp \to d\pi^+$ reaction but for an arbitrary two body process.

C. Relation (3) is valid at arbitrary energy of collision and momentum transfer. At the angles $\Theta_\pi = 0$ and π the quantity $\langle T_{22} \rangle_{00}$ kinematically vanishes and relation (3) gives a direct connection between $\langle T_{20} \rangle_{00}$ and $A_{nn}(0)$

$$\sqrt{2} \langle T_{20} \rangle_{00} = 1 + 3A_{nn} \quad (\Theta = 0, \pi). \tag{33}$$

The quadrupole polarization in reaction (1) has not yet been measured*.

The measurement of $A_{nn}(0)$ has been carried out in SIN at proton energies of 515 and 575 MeV[10] and it has been extended to 494 and 536 MeV. In the investigated region ($0.8 \geq \cos^2\Theta_\pi \geq 0$ at 575 MeV) the value of A_{nn} is near to $-(0.7 \div 0.9)$ which gives an evidence of dominant transitions from singlet states of the initial NN system. The extrapolation of the $A_{nn}(\Theta_\pi)$ data to $\cos^2\Theta_\pi = 1$ (for backward deuterons in c.m. system) gives the value $A_{nn}(0) \approx -0.8$ In this case formula (33) gives $\sqrt{2} \langle T_{20} \rangle_{00} = -1.4$. The extension of measurements on A_{nn} and $\langle T_{20} \rangle_{00}$ to the region of small angle pion production is of interest for

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* In paper[7] deiteron polarization $\langle iT_{11} \rangle_{00}$ was determined at T_p = 670 MeV at three values of angle Θ_d. This data are in general agreement with the results of Niskanen[8]. In the work of Tripp[9] the polarization of deuterons $\langle iT_{11} \rangle_{00}$ was measured at 340 MeV.

reactions (1)-(2) which are under theoretical and experimental study for a long time [11,12,8,2,1]. Recent calculation using the triangle diagram approximation [13] gives a definite prediction [14] for $\langle T_{20}(\Theta_d=\pi)\rangle_{00}$. Among others the model gives a positive $\langle T_{20}(\Theta_d=\pi)\rangle_{00}$ below the threshold of the Δ production in the NN collisions and $\langle T_{20}(\Theta_d=\pi)\rangle_{00}$ turns into zero near the threshold having a negative value after it. The above results using simple extrapolation contradict to this prediction of the triangle model in sign and absolute values. The extension of measurements of A_{nn} for smaller angles O (in order to remove the uncertainties due to the extrapolation) and for other energies has a definite interest. These new experiments could help in the construction of a consistent model of processes (1)-(2) and would be important for the elaboration of a more general theoretical treatment for the reactions of similar types.

D. The existence of relation (3) expresses that the spin-spin effects in reactions (1)-(2) cannot turn into zero at arbitrary energies. This important statement must be taken into account at elaboration of a model. At high energies the cross section of process (1) is strongly decreased with energy and this fact gives a limitation for the effective use of relation (1). Taking into account that at high energies the main contribution to the cross section of (1) is given by the exchange of nucleonic reggeon with intercept $\alpha_N(0) \simeq -0.4$ we obtain

$$\frac{d\sigma}{dt}\Big|_{t'\simeq 0} \sim \left(\frac{s}{s_0}\right)^{2[\alpha_N(0)-1]} \approx (s/s_0)^{-2.8}. \tag{34}$$

The experimental cross section of process (1) was measured up to 21 GeV in works by Dekkers et al. [11], Anderson et al. [11], Heinz et al [11], Allaby et al. [11].

In the energy region where the observation of the cross section of process (1) is possible, this process may be used for measurement of the polarization of high energy proton beam, if the target is polarized. As the progress of the experimental technics allows the observation of smaller and smaller cross sections, the energy region of the investigation of polarization effects in process (1) and the use of that for the study of spin effects in other process can be extended.

E. The measurement of the contribution to the differential cross section, obtained with a target with known polarization parameters $\langle T_{22}\rangle_t$ and $\langle T_{20}\rangle_t$, of course, allows the determination of $A_{nn}(\Theta)$ using relation (3). As the results of work [15] for p-d scattering show, the measurement of $\langle T_{20}\rangle_{00}$ is possible at $\Theta_d=\pi$ that is in the kinematical region where the measurements of $A_{nn}(\Theta)$ in process (1) are absent.

The more complete experimental investigation (adding $A_{sk} = A_{xz}$ in the notation of work [10]) of polarization parameters in reaction (1)-(2) is of interest for obtaining additional information about the possible existence of two-proton resonances of resonance-like mechanism.

F. All the conclusions given for the process (1) are valid for the reaction

$$n+p \to d+\pi^{\circ} \tag{35}$$

also, as due to the isotopic invariance all the appropriate polarization parameters in the two processes are the same.The relation (3) is true for (35) and other similar processes with production of other pseudoscalar particles, for example for

$$n+p \to d+\eta^{\circ}.$$

Relation (4) waits for the discovery of scalar particle production in a process like (1).

G. We consider now the relation given by Pauli principle at $\Theta=\pi/2$, where the transition matrix has only three scalar functions [1,2] and has a form given in Appendix 1. This fact makes the direct reconstruction of the amplitude on the ground of experimental data easier and leads to new relations.

The scalar amplitude $E=|E|e^{i\phi_e}$ and the combinations of amplitudes $A+C=|a|e^{i\phi_a}$ and $A-C=|b|e^{i\phi_b}$ can be expressed by formulae

$$\begin{aligned}
&8|a|^2/\sigma_0 = 1-A_{nn}+A_{kk}+A_{ss}\\
&32|E|^2/\sigma_0 = 1-A_{nn}-A_{kk}-A_{ss}\\
&4|b|^2/\sigma_0 = 1+A_{nn}\\
&\mathrm{tg}(\phi_a-\phi_e)=A_1/A_{ks}=-A_1/A_{sk}=A_2/A_{sk}\,.
\end{aligned} \tag{36}$$

For the quantities describing the polarization of the final state deuterons, as defined in (12) and (14) when relativistic rotation of the spin was taken into account, we have for $\Theta=\pi/2$

$$\langle iT_{11}\rangle_{00} = 0$$

$$\sqrt{2}\,\langle T_{20}\rangle_{00} = 1 - \frac{3}{4}(1 - A_{nn})[1 - \sin 2\theta'(\pi/2)]$$

$$\langle T_{21}\rangle_{00} = -\frac{\sqrt{3}}{4}(1 - A_{nn})\cos 2\theta'(\pi/2) \tag{37}$$

$$\langle T_{22}\rangle_{00} = \frac{\sqrt{3}}{2}\{A_{nn} + \frac{1}{4}(1 - A_{nn})[1 - \sin 2\theta'(\pi/2)]\},$$

where $\theta'(\pi/2)$ is given in (22). The expression (37) follows from formulae (7) of ref. [2] with substitution of θ' for θ and when the relation

$$2N_{\ell m} = A_{nn} - 1$$

at $\Theta = \pi/2$ has been taken into account.

In some works the axis oz was directed along the path of the incident proton. At $T_p = 575$ MeV $2\Theta_1$ is equal to 23°. On neglecting the finite value of Θ_1 the quantity $\langle T_{21}(\pi/2)\rangle_{00}$ turns to zero. As can be seen from (37), the quantity $\langle T_{21}(\pi/2)\rangle_{00}$ is negative.

At 575 MeV, by taking into account the results of work [10] we obtain

$$2|b|^2/\sigma_0 = 0.05 \qquad 2|a|^2/\sigma_0 = 0.25 \qquad 8|E|^2/\sigma_0 = 0.7 \qquad \phi_a - \phi_e = -28°.$$

For the quadropolarization of deuterons at angle $\Theta = \pi/2$ with $A_{nn} = -0.9$ we have

$$\langle T_{21}\rangle_{00} = -0.41 : \langle T_{20}\rangle_{00} = 0.57 : \langle T_{22}\rangle_{00} = -0.72 .$$

When relativistic spin rotation effect is neglected $\langle T_{20}(\pi/2)\rangle_{00} = 1/\sqrt{2}$ as was mentioned in ref. [8] and $\langle T_{22}(\pi/2)\rangle_{00} = -0.78$ (when $A_{nn} = -0.9$). Relativistic spin rotation effect is maximal for $\langle T_{21}(\pi/2)\rangle_{00}$ and is relatively small for $\langle T_{20}\rangle$ and $\langle T_{22}\rangle$ at 575 MeV. These contributions will be more and more important on increasing the energy of particles and the accuracy of experiments.

4. For the proof of generalization of relation (3) for the case with polarized initial protons the following property of the transition matrix M_p is taken into account.

A. As in the final state the NN system is formed in triplet state, M_p can be written as

$$M_p = \hat{T} M_{0p} \, . \tag{38}$$

where $\hat{T}=1/4(3+\vec{\sigma}_1 \cdot \vec{\sigma}_2)$ is the triplet projection operator.

B. As it is true that

$$\hat{T}\sigma_{1v}\sigma_{2u}\hat{T} = \hat{T}\sigma_{1u}\sigma_{2v}\hat{T} \tag{39}$$

for arbitrary operator A we have

$$\begin{aligned} \mathrm{Sp} M A M^{+} \sigma_{1v}\sigma_{2u} &= \mathrm{Sp} M_{0p} A M^{+}_{0p} \hat{T}\sigma_{1v}\sigma_{2u}\hat{T} = \\ &= \mathrm{Sp} M A M^{+} \sigma_{2v}\sigma_{1u} . \end{aligned} \tag{40}$$

This means that

$$\mathrm{Sp} M A M^{+} \sigma_{1\ell}\sigma_{2m} = \mathrm{Sp} M A M^{+} \sigma_{1m}\sigma_{2\ell}$$

and

$$\mathrm{Sp} M A M^{+} \sigma_{1\ell}\sigma_{2n} = \mathrm{Sp} M A M^{+} \sigma_{1n}\sigma_{2\ell} .$$

C. Taking into account (38)-(40)

$$\mathrm{Sp} M A M^{+} (\vec{\sigma}_1 \vec{\sigma}_2) = \mathrm{Sp} M A M^{+} \tag{41}$$

and

$$\mathrm{Sp} M A M^{+} (\sigma_{1\ell}\sigma_{2\ell} + \sigma_{1m}\sigma_{2m}) = \mathrm{Sp} M A M^{+} (1 - \sigma_{1n}\sigma_{2n}). \tag{42}$$

The conditions A-C follow from the results of work /2/. With the help of (12) and (14), expression for the operator L

$$L = 2\sqrt{3}\, T_{22} + \sqrt{2}\, T_{20} = 3(S_x^2 + S_z^2 - S_y^2) - 2 + 3i(S_x S_y + S_y S_x) \tag{43}$$

can be written in the following way

$$\begin{aligned} L = \frac{3}{2}(\sigma_{1\ell}\sigma_{2\ell} + \sigma_{1m}\sigma_{2m} - \sigma_{1n}\sigma_{2n}) - 1/2 + \frac{i3}{4}[(\sigma_{1n}\sigma_{2\ell} + \\ + \sigma_{1\ell}\sigma_{2n})\sin\theta' + (\sigma_{1n}\sigma_{2m} + \sigma_{1m}\sigma_{2n})\cos\theta'] \end{aligned} \tag{44}$$

if we take into account the relations (15) and (18).

Taking into account (40) and (42) the expression

$$SpMAM^+L = \frac{3}{2} SpMAM^+(\sigma_{1\ell}\sigma_{2\ell} + \sigma_{1m}\sigma_{2m} - \sigma_{1n}\sigma_{2n}) -$$

$$- \frac{1}{2} SpMAM^+ + \frac{i3}{4} SpMAM^+[(\sigma_{1n}\sigma_{2\ell} + \sigma_{1\ell}\sigma_{2n})\sin\theta' + (\sigma_{1n}\sigma_{2m} + \sigma_{1m}\sigma_{2n})\cos\theta']$$

can be written as follows

$$SpMAM^+L = SpMAM^+(1 - 3\sigma_{1n}\sigma_{2n}) + \frac{i3}{2} SpMAM^+\sigma_{1n}(\sigma_{2\ell}\sin\theta' + \sigma_{2m}\cos\theta'). \tag{45}$$

If $A=1$ one obtains formulae (3) immediately.

If $A=\sigma_{1n}$ having the notation for the left side of (45) as $\sigma_0 L_{n0}$ and using (7) we obtain

$$L_{n0} = A_1 + 3A_2. \tag{46}$$

Similarly, introducing the

$$\sigma_0 L_{0n} = 1/4\, SpMM^+\sigma_{2n}$$

and

$$\sigma_0 L_{nn} = 1/4\, SpMM^+\sigma_{1n}\sigma_{2n}$$

we obtain from (45) that

$$L_{0n} = A_2 + 3A_1 \tag{47}$$

and

$$L_{nn} = A_{nn} + 3. \tag{48}$$

Notice that due to the Pauli principle at $\Theta=\pi/2$ $A_1 + A_2 = 0$ and

$$L_{n0}(\pi/2) = -L_{0n}(\pi/2) = 2A_2(\pi/2) = -2A_1(\pi/2). \tag{49}$$

If we accept the density matrix (8) for the initial transversely polarized protons ($\vec{P}_1 = P_1\vec{n}$ and $\vec{P}_2 = P_2\vec{n}$) as the operator A then

$$\sigma_{P_{1n}P_{2n}}\langle L_{nn}\rangle = \sigma_0[L_{00} + P_{1n}L_{n0} + P_{2n}L_{0n} + P_{1n}P_{2n}L_{nn}]. \tag{50}$$

We introduce the cross sections in the pure spin states

$$
\begin{aligned}
\sigma(\uparrow\uparrow) &= \sigma_0(1+A_1+A_2+A_{nn}) \\
\sigma(\downarrow\downarrow) &= \sigma_0(1-A_1-A_2+A_{nn}) \\
\sigma(\uparrow\downarrow) &= \sigma_0(1+A_1-A_2-A_{nn}) \\
\sigma(\downarrow\uparrow) &= \sigma_0(1-A_1+A_2-A_{nn})
\end{aligned}
\tag{51}
$$

and we have the notations $\langle L_{n,n}\rangle$, $\langle L_{-n,-n}\rangle$, $\langle L_{n,-n}\rangle$ and $\langle L_{-n,n}\rangle$ for the values of the operator L in these states. From (50)-(51) we have

$$
\begin{aligned}
\langle L_{n,n}\rangle &= \langle L_{-n,-n}\rangle = 4 \\
\langle L_{n,-n}\rangle &= \langle L_{-n,n}\rangle = -2 .
\end{aligned}
\tag{52}
$$

In the experiments where the beam and the target are polarized in the reaction plane the $L_{\ell\ell}$, L_{mm}, $L_{\ell m}$ and $L_{m\ell}$ can be obtained. Formula (45) yields

$$
\begin{aligned}
L_{\ell\ell} &= A_{\ell\ell} - 3A_{mm}; \quad L_{\ell m} = A_{\ell m} + 3A_{m\ell} \\
L_{mm} &= A_{mm} - 3A_{\ell\ell}; \quad L_{m\ell} = A_{m\ell} + 3A_{\ell m} .
\end{aligned}
\tag{53}
$$

The parameters $L_{\ell\ell}$, L_{mm}, $L_{m\ell}$ and $L_{\ell m}$ are expressed in terms of the corresponding quantities in laboratory system as follows

$$
\begin{aligned}
L_{\ell\ell} &= (A_{kk}-A_{ss})\cos\Theta + (A_{ks}+A_{sk})\sin\Theta - (A_{ss}+A_{kk}) \\
L_{mm} &= -(A_{kk}-A_{ss})\cos\Theta - (A_{ks}+A_{sk})\sin\Theta - (A_{ss}+A_{kk}) \\
L_{\ell m} &= (2\cos\Theta-1)A_{ks} + (2\cos\Theta+1)A_{sk} - 2(A_{kk}-A_{ss})\sin\Theta \\
L_{m\ell} &= (2\cos\Theta+1)A_{ks} + (2\cos\Theta-1)A_{sk} - 2(A_{kk}-A_{ss})\sin\Theta .
\end{aligned}
\tag{54}
$$

The expressions for $L_{\ell 0}$, L_{m0}, $L_{0\ell}$, L_{0m}, $L_{n\ell}$, $L_{\ell n}$, L_{mn} and L_{nm} are given in Appendix 2.

In analogue with (46)-(48), expressions for production of scalar particles in the reaction of type (1) can be obtained straightforwardly. In this case

$$L_{n0} = A_1 - 3A_2 \qquad L_{0n} = A_2 - 3A_1$$
$$L_{nn} = A_{nn} - 3. \tag{55}$$

Besides this using (4) and (51)

$$\langle L_{n,n} \rangle = \langle L_{-n,-n} \rangle = -2$$
$$\langle L_{n,-n} \rangle = \langle L_{-n,n} \rangle = 4. \tag{56}$$

APPENDIX 1. THE WILKIN RELATION

Recently Wilkin has established [16] a useful relation among the A_{kk}, A_{nn} and A_{ss} components of the tensor A_{uv} for the process (1) in forward direction. This relation can be written as follows

$$A_{kk} \mp A_{ss} \mp A_{nn} = 1 \tag{A.1}$$

in our notation, where the upper (lower) indices correspond to the production of pseudoscalar (scalar) particle. The extrapolation of data from [10] to $\Theta_\pi = 0$ has given for the first time the possibility of the direct determination of negative parity of positive pion.

For the demonstration of relation (A1) we begin with general structure of amplitude of the reaction of type (1) with forward pseudoscalar (scalar) particle in the final state.

In the case of pseudoscalar particle we start from the general expression

$$M_p = \hat{T} M_{0p} = \hat{T}[A'(\sigma_{1k} + \sigma_{2k}) + B'(\sigma_{1k} - \sigma_{2k}) + C'(\vec{k}[\vec{\sigma}_1 \vec{\sigma}_2])]. \tag{A.2}$$

As

$$\hat{T}(\sigma_{1k} + \sigma_{2k}) = \sigma_{1k} + \sigma_{2k} \tag{A.3a}$$

and

$$\hat{T}[\sigma_{1k} - \sigma_{2k} + i(\vec{k}[\vec{\sigma}_1 \vec{\sigma}_2])] = \sigma_{1k} - \sigma_{2k} + i(\vec{k}[\vec{\sigma}_1 \vec{\sigma}_2]) \tag{A.3b}$$

we obtain the final result

$$M_p = \hat{T}\{\alpha_p(\sigma_{1k} + \sigma_{2k}) + \beta_p[\sigma_{1k} - \sigma_{2k} + i(\vec{k}[\vec{\sigma}_1 \vec{\sigma}_2])\}, \tag{A.4}$$

where on taking into account (A.3) the $\hat{T}$ factor can be omitted. So $M_p(0)$ contains two independent scalar functions.

For the production of scalar particle the transition matrix is written in the form

$$M_s = \hat{T} M_{0s} = \hat{T}[A_s + B_s \sigma_{1k}\sigma_{2k} + C_s(\vec{\sigma}_1 \vec{\sigma}_2)] \tag{A.5}$$

and using the relations

$$(\vec{\sigma}_1 \vec{\sigma}_2)^2 = 3 - 2(\vec{\sigma}_1 \vec{\sigma}_2), \quad \hat{T}(\vec{\sigma}_1 \vec{\sigma}_2)\hat{T} = \hat{T}$$

and

$$\hat{T}(1 + \sigma_{1k}\sigma_{2k}) = 1 + \sigma_{1k}\sigma_{2k}$$

(A.5) can be written equivalently as follows

$$M_s = \hat{T} M_{0s} = \hat{T}\{A_s + C_s + B_s \sigma_{1k}\sigma_{2k}\} = \hat{T}(\alpha + \beta \sigma_{1k}\sigma_{2k}) \tag{A.6}$$

which contains two independent scalar amplitude also.

Using the general expression (A.4) and (A.6) in forward direction we obtain for the production of pseudoscalar pion

$$\begin{aligned} &\sigma_0 = 2|\alpha_p|^2 + 4|\beta_p|^2 \\ &\sigma_0 A_{kk} = 2|\alpha_p|^2 - 4|\beta_p|^2 \\ &\sigma_0 A_{nn} = \sigma_0 A_{ss} = -4|\beta_p|^2 \end{aligned} \tag{A.7}$$

and for the production of scalar particle

$$\begin{aligned} &\sigma_0 = \frac{1}{2}(|\alpha|^2 + |\beta|^2) + \frac{1}{4}|\alpha+\beta|^2 \\ &\sigma_0 A_{nn} = \sigma_0 A_{ss} = \frac{1}{4}|\alpha-\beta|^2 \\ &\sigma_0(1 + A_{kk}) = |\alpha+\beta|^2 . \end{aligned} \tag{A.8}$$

Taking into account, that at $\Theta_\pi = 0$ $A_{ss} = A_{nn}$, for the spinless particle for both parities we obtain (A.1) immediately.

It can be seen from (A.7) and (A.8) that at $\Theta_\pi = 0$ A_{nn} is negative (or zero) for pseudoscalar particle and is positive (or zero) for scalar particles. Therefore the fact, that $A_{nn}(0)$ is negative in process (1) already demonstrated the negative parity of positive pion. This conclusion, based on the extrapolation of one experimental value, less sensitive to the extrapolarion procedure to $\Theta_\pi = 0$.

As it was mentioned in /16/ relation (A.1) is valid for $\Theta_\pi = \pi/2$ in the case of pseuodscalar particle production. We present here the demonstration.

Due to the Pauli principle $M_p(\pi/2)$ contains three scalar amplitudes and can be written in the following form

$$M_p(\pi/2)/\sqrt{2} = A(\sigma_{1k}+\sigma_{2k}) + iC(\sigma_{1n}\sigma_{2k'}+\sigma_{1k'}\sigma_{2n}) + \\ + E[\sigma_{1k'}-\sigma_{2k'}+i(\sigma_{1n}\sigma_{2k}-\sigma_{1k}\sigma_{2n})]. \quad (A.9)$$

As at $\Theta=\pi/2$ $\vec{s}=\vec{k}$, $A_{ss}=A_{k'k'}$. We can write

$$M_p(\pi/2)(\sigma_{1k}\sigma_{2k}-\sigma_{1k'}\sigma_{2k'}-\sigma_{1n}\sigma_{2n}) = M_p(\pi/2). \quad (A.10)$$

In this case

$$A_{kk}-A_{k'k'}-A_{nn} = A_{kk}-A_{ss}-A_{nn} = 1$$

and we obtain (1) for the production of pseudoscalar particle.

The existence of the relation (A.1) at $\Theta_\pi=0,\pi/2$, the equality $A_{nn}=A_{ss}$ at $\Theta_\pi=0$ and the relation (3) are useful for evaluation of the experimental data and for the phenomenological analysis.

APPENDIX 2

If we have for $A=\sigma_{1\ell}$, σ_{1m}, $\sigma_{2\ell}$, σ_{2m}, $\sigma_{1n}\sigma_{2\ell}$, $\sigma_{1\ell}\sigma_{2n}$, $\sigma_{1n}\sigma_{2m}$, and $\sigma_{1m}\sigma_{2n}$ respectively, then the first two terms in (45) vanish and for nonzero elements we have

$$\sigma_0 L_{\ell 0} = \frac{i3}{8} \mathrm{Sp} M \sigma_{1m}\sigma_{2n} M^+ \sigma_{2(q)_R}$$

$$\sigma_0 L_{m0} = -\frac{i3}{8} \mathrm{Sp} M \sigma_{1\ell}\sigma_{2n} M^+ \sigma_{2(q)_R}$$

$$\sigma_0 L_{0\ell} = \frac{i3}{8} \mathrm{Sp} M \sigma_{1n}\sigma_{2m} M^+ \sigma_{2(q)_R}$$

$$\sigma_0 L_{0m} = -\frac{i3}{8} \mathrm{Sp} M \sigma_{1n}\sigma_{2\ell} M^+ \sigma_{2(q)_R}$$

$$\sigma_0 L_{n\ell} = \frac{i3}{8} \mathrm{Sp} M \sigma_{2m} M^+ \sigma_{2(q)_R}$$

$$\sigma_0 L_{n\ell} = \frac{i3}{8} \mathrm{Sp} M \sigma_{2\ell} M^+ \sigma_{2(q)_R}$$

$$\sigma_0 L_{nm} = \frac{i3}{8} \mathrm{Sp} M \sigma_{1m} M^+ \sigma_{2(q)_R}$$

$$\sigma_0 L_{mn} = \frac{i3}{8} \mathrm{Sp} M \sigma_{1\ell} M^+_{2(q)_R}$$

The unit vector $\vec{q}$ along the path of the deuteron and its rotation value $\vec{q}_R$ are given by last expressions in (15) and (18).

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