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SPINORS IN SELF-DUAL YANG-MILLS FIELDS IN MINKOWSKI SPACE

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1. INTRODUCTION

At present, in order to get rid of the infrared divergences in gauge theories, one usually introduces a cut-off /1/ . In QED this operation makes physical sense if all inelastic processes with non-observable long-wave photons are taken into account. However, in QCD the cut-off is a formal mathemtical tool which is justified only by referring to confinement and a finite hadron size. On the other side, one of the central problems of the theory is to give a proof for confinement based on the behaviour of the gluon fields over large distances, that is on their long-wave singularities. Finally, there seems to arise a logical circle: the confinement is justified by infrared singularities and the removal of them by the confinement.

In our papers /2,3/ we have made an attempt to remove the infrared divergences in QCD without reference to confinement but by analogy with quantum liquid /4,5,6/ . From this point of view the infrared behaviour is described by macroscopic (global) excitations of the liquid as a whole, being accompanied by spontaneous vacuum symmetry breaking. This analogy for a non-Abelian gauge theory leads to vacuum fields, satisfying self-dual equations in the Minkowski space.

The main aim of this paper is the relativistic-covariant description of a spinor field within a self-dual vacuum.

To understand the points of departure, we describe briefly the results of papers /2,3/ .

The infrared problem in Yang-Mills theory with the action

$$S=-\frac{1}{4}\int d^4x\, F^a_{\mu\nu}F^{a\mu\nu}\ ;\qquad F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+g\varepsilon^{abc}A^b_\mu A^c_\nu \tag{1.1}$$

arises far before quantization, when independent dynamical variables are defined by solving the constraint equations, i.e., the classical equations for components A_0^a , which have no canonical momenta /7/

$$\frac{\delta S}{\delta A_0^a} = \left(\nabla_i^2 A_0\right)^a - \nabla_i^{ab} \partial_0 A_i^b = 0 \, ; \left(\nabla_i^{ab} = \partial_i \delta^{ab} - g \varepsilon^{abc} A_i^c\right). \qquad (1.2)$$

The infrared singularities are connected with zero eigenvalues of the operator (∇_i^2) . If these zeroes exist (this is the starting point) then the general solution of eq. (1.2) can be represented as a sum of solutions of the homogeneous and inhomogeneous equations

$$A_0^a = (\partial_0 C(t)) \Phi^a + \left[\frac{1}{\nabla_i^2}\right]^{ab}_{Reg} \left[\nabla_i \partial_0 A_i\right]^b \qquad (1.3)$$

with $\nabla_i^2 \Phi = 0$. The operator $[1/\nabla_i^2]_{Reg}$ is defined in a class of functions for which $[\nabla^2]_{Reg} \neq 0$. The coefficient $\partial_0 C(t)$ just represents a global dynamical variable. It has covariant properties under gauge transformations provided we connect it with the Pontryagin index

$$\nu[A] = \frac{g^2}{32\pi^2} \int d^4x \, F^a_{\mu\nu} \, {}^*F^{a\mu\nu} \quad ; \left({}^*F^a_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}\right)$$

(For solutions (1.3) it has the form of an integral over time for the derivative; $\nu = \int dt \, \partial_0 \underline{\nu}$; $C \to \underline{\nu}$) /2,3/ .

Substituting eq. (1.3) into eq. (1.1) we obtain the action and the Hamiltonian expressed in terms of the global variable $P = \delta S / \delta(\partial_0 \underline{\nu})$ and transverse electric (E) and magnetic (B) fields.

$$H_{tot} = \frac{1}{2} \int d^3x \left(E_D^2 + B_D^2\right) + \frac{1}{2}\left[P^2 \left(\frac{g^2}{8\pi^2}\right)^2 + 1\right] \frac{\left(\int d^3x \, \nabla\Phi B\right)^2}{\int d^3x (\nabla\Phi)^2} \qquad (1.4)$$

$$E_D = E - \nabla\Phi \frac{\int d^3x \, E \nabla\Phi}{\int d^3x (\nabla\Phi)^2} \, ; \, B_D = B - \nabla\Phi \frac{\int d^3x \, B \nabla\Phi}{\int d^3x (\nabla\Phi)^2} \qquad (1.5)$$

$$\nabla_i E_i = \nabla_i B_i = \nabla_i (\nabla_i \Phi) = 0 \, . \qquad (1.6)$$

The Hamiltonian depends upon field Φ via the characteristic functionals: $\int d^3x (\nabla\Phi)^2$; $\int d^3x \, E \nabla\Phi$; $\int d^3x \, B \nabla\Phi$; which due to Eqs. (1.6) are non-zero only for singular fields $B, E, \nabla\Phi$ (i.e., nondifferentiable at certain points of $R(3)$ or nonvanishing at spatial infinity in $R(3)$. Within this context the following alternative arises:

1) For regular fields the global dynamics vanishes and we obtain the usual Yang-Mills theory $H = \frac{1}{2}\int d^3x \left(E^2 + B^2\right)$ with an infrared unstable perturbation theory.

2) Spontaneous vacuum symmetry breaking, by shifting $A_i(x,t)$ with a singular nondynamical field $A_{i\,vac}(x)$

$$A_i = A_{i\,vac}(x) + A_{i\,Reg}(x,t)$$

is the condition for the existence of the global variable. Here A_{Reg} and operator $[1/\nabla^2]_{Reg}$ are defined within the class of functions, vanishing at the singularity points [x]. The Hamiltonian (1.4) is obtained just for this case. The characteristic functionals here play a role analogous to "constant" fields in the σ - model.

Starting from Eqs. (1.4),(1.5), one can show that the "quasiparticle" Hamiltonian $\sim \int d^3x \left(E_D^2 + B_D^2\right)$ is finite provided the singular vacuum fields satisfy the integrability conditions /5/

$$E(A_{vac}) \sim B(A_{vac}) \sim \nabla\Phi.$$

The vacuum energy $H_{vac} \sim \int d^3x (\nabla\Phi)^2$ may be infinite. However, this energy is subtracted from the total energy by redefinition of the asymptotic states with the help of the interaction representation: $H_{Reg} = H_{tot} - H_{vac}$. The obtained theory reminds one of a quantum liquid /4,5,6/. In the microscopic Bogolubov theory /6/ the criteria for an energetically favoured condensate A_{vac} are the existence of a Hermitean Hamiltonian and the infrared stability of the theory.

We should remember that due to the infinite vacuum energy the classical theory of the quantum liquid is an approximation, which has no physical meaning. (The cause of the classical infinity is explained by a more fundamental theory, which takes into account the size of a "liquid atom").

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x The same situation arises in the σ -model in a finite volume of $R(3)$: "constant" vacuum fields have a jump at the boundary of the volume, while "quasiparticles" fulfil the zero boundary condition.

For the existence of the global dynamics and the vacuum symmetry breaking, we have to pay with a loss of the usual relativistic covariance. However, for self-dual fields $A^a_\mu = (\Phi^a; A^a_{i\,vac})$

$$E = -\nabla\Phi = \pm iB \quad \Rightarrow \quad {}^*F^a_{\mu\nu}(A^{(\pm)}) = \pm i F_{\mu\nu}(A^{(\pm)}) \qquad (1.7)$$

the covariance group of the new vacuum contains the group of transformations (mixing internal and external indices), the algebra of which coincides with that of the Poincare group. The new relativistic group gives additional dynamics caused by an ambiguous choice of field components A^a_0.

This paper is organized as follows. In section 2 the classification for self-dual fields is given, in section 3 we describe spinor fields within a self-dual singular background in a Lorentz-invariant way; in section 4 the complete set of solutions of the corresponding Dirac equations are obtained, in section 5 the spectra of observables (i.e. conserved quantities) are calculated. In the Appendix the generalization of ADHM-construction /9/ to the Minkowski-space is given.

2. SELF-DUAL FIELDS IN MINKOWSKI SPACE

Recently, there is a considerable progress in the construction of Euclidean self-dual non-Abelian fields /9-11/. Some of these results, in particular the ansatz /10/, can be generalized to the Minkowski space

$$A^{a(\pm)}_\mu = \frac{1}{g}\Sigma^{a(\pm)}_{\mu\nu}\partial^\nu \ln\rho, \qquad (2.1)$$

where

$$\Sigma^{a(\pm)}_{0i} = \pm i\delta^a_i \quad ; \Sigma^{a(\pm)}_{ij} = \varepsilon^{aij} \quad ; \quad \Sigma^{a(\pm)}_{\mu\nu} = -\Sigma^{a(\pm)}_{\nu\mu}$$

and the function ρ has to satisfy the d'Alambert equation

$$\Box\rho = 0 \qquad ; \qquad (\partial_\mu\partial^\mu = \Box). \qquad (2.2)$$

The stationary vacuum fields correspond to the factorizable solution of Eq. (2.2): $\rho = \exp(ik_0t)\,\Phi(\vec{x})$, where $\Phi(\vec{x})$ is an arbitrary function, satisfying equation $(\partial_i^2 + k_0^2)\Phi = 0$. For the vacuum at rest there are no preferable directions and due to the spherical symmetry we obtain immediately a solution

$$\rho = e^{ik_0t}\,\frac{\sin k_0|x|}{|x|}, \qquad (2.3)$$

where k_0 has a real value and defines a scale similar to the photon momentum in the conform-invariant QED without charge ($k_0 = i\varkappa$ corresponds to finite energy monopole solutions /11/, which are not considered here). The solution (2.1), (2.3), up to a gauge transformation, coincides with the one, considered in ref. /12/.

The transformation group, preserving the scale k_0, coincides with the Poincare group with the usual Lorentz generators $L_{\mu\nu}$ replaced by

$$L'_{\mu\nu} = L_{\mu\nu} + \sum_{\mu\nu}^{a(\pm)} T^a, \tag{2.4}$$

where T^a is the colour generator.

By the Poincare group transformation, the field (2.1), (2.3) transforms into the vacuum moving with an arbitrary velocity $\vec{V}$ starting from an arbitrary point X

$$\rho(x) \to \rho_{V,X}(x) = \rho(x'+X) \; ; \qquad x'_\mu = \Lambda_\mu^\nu x^\nu . \tag{2.5}$$

As will be shown below, the theory of quasiparticles in such a vacuum reminds us of a relativistic generalization of the bag model /13,14/. Therefore, we shall call the vacuum (2.1), (2.3), (2.5) self-dual bag.

Obviously, the configuration

$$\rho_N = \sum_{i=1}^{N} \rho_{k_{0i}, V_i, X_i}(x) \tag{2.6}$$

corresponds to an N-bag vacuum.

The general solutions of the Euclidean self-duality equation are obtained by the A.D.H.M. construction /9/ (see also refs. /15,16/; a variant of the A.D.H.M. construction for the Minkowskian solutions (2.1), (2.3) is considered in the Appendix.

3. DESCRIPTION OF SPINOR FIELDS IN A SINGULAR SELF-DUAL VACUUM

We consider the spinor fields within an external field in such a way that the whole theory becomes invariant under the transformation of the Poincare group. Usually in the case of existence of zero modes Poincare-invariance is restored by applying the method of collective coordinates. Since in our case of singular fields these zero mode states are not normalizable, we do not refer to this method. We shall rather use an analogy with the two-component superfluid theory /5/. That is, the dynamics of the vacuum fields is considered to be independent of the quasiparticle excitations.

Let us consider the Lagrangian

$$\mathcal{L} = i\bar{\Psi}^i \gamma^\mu [\delta_i^j \partial_\mu + \sum_{\mu\nu}^{a(+)} \frac{(\tau^a)_i^j}{2i} \partial^\nu \ln\rho] \Psi_j + \partial_\mu \rho\, \partial^\mu \lambda \qquad (3.1)$$

which obeys asymptotic states of the spinor fields in the vacuum (2.1),(2.2). Here $\lambda(x)$ is a Lagrange factor. Variation with respect to $\lambda(x)$ gives the vacuum self-duality condition, i.e.,the "free" equation (2.2).

The Lagrangian (3.1) is invariant with respect to the Poincare group. In accordance with the Noether theorem the following quantities (observables) are conserved: energy-momentum, $P_\mu = (H, P_i)$; spin $\vec{S}$; charge Q

$$P_\nu = \int_{\Omega(\rho)} d^3 z^\mu T_{\mu\nu} \; ; \; S^i = \varepsilon^{ijk} \int_{\Omega(\rho)} d^3 z^\mu M_{\mu(jk)} \; ; \; Q = \int_{\Omega(\rho)} d^3 z^\mu J_\mu \, . \qquad (3.2)$$

Here z_μ is a space-like surface chosen in such a way that the z_μ -projection of the bag world lines describe a finite region inside z_μ . $\Omega(\rho)$ is the integration region which is defined by the zeroes of $\rho(x)$. (For example, in the case of vacuum (2.1),(2.3), we have $Q = \int_{|x| < \pi k_0^{-1}} d^3x\, J_0(x)$). J_μ , $T_{\mu,\nu}$, $M_z(\mu,\nu)$ are the current vector, energy-momentum and angular momentum tensors.

$$T_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi)} \partial_\nu \Psi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \rho)} \partial_\nu \rho + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \lambda)} \partial_\nu \lambda - g_{\mu\nu} \mathcal{L}$$

$$M_{z(\mu\nu)} = x_\nu T_{\mu z} - x_\mu T_{\nu z} + i\bar{\Psi}^i \gamma_z \left(\delta_i^j \frac{\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu}{4} + \sum_{\mu\nu}^{a(+)} \left(\frac{\tau^a}{2i}\right)_i^j\right) \Psi_j \qquad (3.3)$$

$$J_\mu = \bar{\Psi}^i \gamma_\mu \Psi_i \, .$$

The quantities (3.3) are defined by the solutions of the equations of motion

$$\gamma^\mu \nabla_\mu \Psi = 0 \quad ; \quad \bar{\Psi} \gamma^\mu \overleftarrow{\nabla}_\mu = 0 \qquad (3.4)$$

$$\Box \lambda = -\frac{1}{\rho} \partial_\nu \left(i \bar{\Psi} \sum_{\nu\mu}^{a(+)} \frac{\tau^a}{2i} \gamma^\mu \Psi\right) \quad ; \quad \Box \rho = 0 \, . \qquad (3.5)$$

Here $\nabla_\mu = (\partial_\mu + \sum_{\mu\nu}^{a(+)} \frac{\tau^a}{2i} \partial^\nu \ln\rho)$, the spinors have to satisfy zero boundary conditions at the points of the vacuum singularities

$$\bar{\Psi}\big|_{\rho=0} = \Psi\big|_{\rho=0} = 0 \, . \qquad (3.6)$$

It is easy to prove the covariance of the observables. Within this context the observables, calculated for a moving bag

$$\rho_v(x) = \rho(x') \ ; \ x'_\mu = \Lambda_\mu^\nu x_\nu ; \left(t' = \frac{t - v x_1}{\sqrt{1-v^2}} \ ; \ x'_1 = \frac{x_1 - vt}{\sqrt{1-v^2}} \ ; \ x'_{2,3} = x_{2,3} \right) \quad (3.7)$$

are connected with the observables of the bag at rest (x') by the Lorentz transformations.

In particular, we have: $P_\mu(v \neq 0) = \Lambda_\mu^\nu P_\nu(v=0)$; $Q(v \neq 0) = Q(v=0)$. Let us prove, for example, the last relation, taking into account that for the bag (3.7) the surface element $d\sigma_\mu$ is chosen in the following form:

$$d^3\sigma_\mu = (d^3x \ ; \ dx_2\, dx_3\, dt \ ; \ 0 \ ; \ 0) \quad (3.8)$$

$$Q(v \neq 0) = \int\limits_{\Omega(\rho_v)} d^3x \, J_0(x/\rho_v) + \int\limits_{\Omega(\rho_v)} dx_2\, dx_3\, dt \, J_1(x/\rho_v) .$$

Let us pass to the new integration variable $(x \to x')$ (3.7) and apply the current transformation property

$$J_\mu(x/\rho_v(x)) = \Lambda_\mu^\alpha J_\alpha(x'/\rho(x')) .$$

Then we obtain the expression for $Q(v \neq 0)$

$$Q(v \neq 0) = \int\limits_{\Omega(\rho(x'))} d^3x' \left\{ \frac{1}{\sqrt{1-v^2}} \left[\frac{J_0(x') - vJ_1(x')}{\sqrt{1-v^2}} \right] + \frac{v}{\sqrt{1-v^2}} \left[\frac{J_1(x') - vJ_0(x')}{\sqrt{1-v^2}} \right] \right\} =$$

$$= \int\limits_{\Omega(\rho(x'))} d^3x' \, J_0(x') \equiv Q(v=0) \quad ; \qquad \left(J(x') = J(x'/\rho(x')) \right).$$

Thus, it is sufficient to calculate the spectrum of observables for the bag at rest.

We suppose that the spinor excitations describe the asymptotic hadron states. The hadron mass spectrum and spin are defined by Eqs. (3.2),(3.3),(3.4) and (3.5). The charge (3.8) may be used for the wave function normalization. The solution of Eq. (3.5) for the unphysical field $\lambda(x)$ is chosen in the form $\lambda(x) \sim (\bar{\psi}\psi)$. Therefore, all physical observables are equal to zero provided the spinor excitation is absent. In this sense the vacuum (2.1),(2.2) is Lorentz invariant.

4. THE SOLUTIONS OF DIRAC EQUATIONS FOR ARBITRARY $\rho(x)$

In order to solve Eq. (3.4), we shall use the two-component spinors

$$\bar{\psi} = (\psi_R^+ , \psi_L^+) \quad ; \qquad \psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad ;$$

$$\gamma_\mu = \begin{pmatrix} 0 & , \; \alpha_\mu^{(+)} \\ \alpha_\mu^{(-)} & , \; 0 \end{pmatrix} \; ; \; \left(\alpha_\mu^{(\pm)}\right)_B^A = \left(\delta_B^A \, ; \mp (\tau^k)_B^A\right) \tag{4.1}$$

$$\frac{1}{4}\left(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu\right) = \begin{pmatrix} \Sigma_{\mu\nu}^{a(-)} & , \; 0 \\ 0 & , \; \Sigma_{\mu\nu}^{a(+)} \end{pmatrix} \frac{\tau^a}{2i} \, .$$

We shall first consider the left spinors in the field $A_\mu^{(+)}$

$$\Delta\mathcal{L}_{(L)} = i\left(\psi_L^+\right)^{iA}\left(\partial_\mu + \Sigma_{\mu\nu}^{a(+)} \frac{\tau^a}{2i} \partial^\nu \ln\rho\right)_i^j \left(\alpha^{(+)\mu}\right)_A^C \left(\psi_L\right)_{jC} \, . \tag{4.2}$$

In the Euclidean space, to solve the Dirac equations, one applies the ADHM construction /9/ . However, in the Minkowski space this method yields not all the solutions of Eqs.(3.4) (see Appendix). We shall use here a more simple method, remembering the infrared factorization in QED.

Let us make two transformations of Eq. (4.2)

1) Using the identiyy

$$\Sigma_{\mu\nu}^{a(+)}\left(\frac{\tau^a}{2i}\right)_j^i \partial^\nu \ln\rho \left(\alpha^{(+)\mu}\right)_B^A = -\partial_\mu \left[\frac{\Lambda}{2}\ln\rho\right]_{jC}^{iA} \left(\alpha^{(+)\mu}\right)_B^C \, , \tag{4.3}$$

where $\Lambda_{jC}^{iA} = (\tau^a)_j^i \, (\tau^a)_C^A$, we remove from the Lagrangian the interaction term with the derivative by transformation:

$$\left(\psi_L^+\right)^{iA} = \left(\Phi_L^+\right)^{jB}\left[\mathcal{U}^{-1}(\rho)\right]_{jB}^{iA} \; ; \; \left(\psi_L\right)_{iA} = \left[\mathcal{U}(\rho)\right]_{iA}^{jB}\left(\Phi_L\right)_{jB} \, , \tag{4.4}$$

where Φ_L^+ and Φ_L are new spinors, and the matrix $\mathcal{U}(\rho)$ is of the form

$$\mathcal{U}(\rho) = \exp\left\{\frac{\Lambda}{2}\ln\rho\right\} = I \, \frac{3\rho^{1/2} + \rho^{-3/2}}{4} + \Lambda \frac{\rho^{1/2} - \rho^{-3/2}}{4} \tag{4.5}$$

(In order to find the explicit form of the matrix $\exp(\Lambda t) = IB + \Lambda D$ one needs to solve the differential equation: $\frac{d}{dt}\left[IB(t) + \Lambda D(t)\right] = \Lambda\left[IB(t) + \Lambda D(t)\right]$ taking into account the relation $\Lambda \cdot \Lambda = 2I - \Lambda$.

2) We symmetrize the Lorentz and colour indices in Φ^+, Φ by

$$(\sigma_2)_{AA'}(\Phi_L^+)^{iA'} = \frac{1}{\sqrt{2}}\left(A_0^+ + A_k^+\sigma_k\right)_A^i \tag{4.6}$$

$$(\Phi_L)_{iA'}(\sigma_2)^{A'A} = \frac{1}{\sqrt{2}}\left(A_0 + A_k\sigma_k\right)_i^A .$$

Finally, we obtain the Lagrangian (4.2) in terms of the fields A^+, A

$$\Delta\mathcal{L}_{(L)} = i\{A_0^+[\partial_0 A_0 - \partial_k(A_k\rho^2)] + A_k^+[\partial_0 A_k - \partial_k(A_0\rho^{-2}) + i\varepsilon^{kab}\partial_a A_b]\} . \tag{4.7}$$

The Dirac equations corresponding to (4.7) are of the form

$$\left.\begin{aligned}&\partial_0 A_0 - \partial_k(A_k\rho^2) = 0\\ &\partial_0 A_k - \partial_k(A_0\rho^{-2}) + i\varepsilon^{kab}\partial_a A_b = 0\end{aligned}\right\} \tag{4.8}$$

$$\left.\begin{aligned}&\partial_0 A_0^+ - \rho^{-2}\partial_k A_k^+ = 0\\ &\partial_0 A_k^+ - \rho^2\partial_k A_0^+ - i\varepsilon^{kab}\partial_a A_b^+ = 0\end{aligned}\right\} \tag{4.9}$$

and are easily solved, if the vector fields A_k, A_k^+ are represented as the sum of a gradient and a transverse field

$$A_k = \partial_k\chi + V_k \quad ; \quad A_k^+ = \partial_k\chi^+ + V_k^+ \tag{4.10}$$

$$\partial_k V_k^+ = \partial_k V_k = 0 .$$

Equation (4.8) takes the form

$$1)\quad \partial_0 A_0 - \partial_k(\rho^2\partial_k\chi) = 2\rho(\partial_k\rho)V_k \tag{4.8^I}$$

$$2)\quad \partial_0\partial_k\chi - \partial_k(A_0\rho^{-2}) = -G_{(+)kb}V_b .$$

Here we have introduced the notation

$$G_{(\pm)kb} = \left(\delta_{kb} - \frac{\partial_k\partial_b}{\partial^2}\right)\partial_0 \pm i\varepsilon^{kab}\partial_a . \tag{4.11}$$

From the second Eq. (4.8)^I we may pick up the transverse and longitudinal parts acting in them with the operators ∂_k and $G_{(-)ak}$, respectively. We obtain the relation between χ and A_0

$$\partial_k^2(\partial_0\chi) = \partial_k^2(A_0\rho^{-2}) \tag{4.12}$$

and the equation for the vector field

$$\Box V_k = 0 \quad ; \quad G_{(+)kb} V_b = 0 . \tag{4.13}$$

The set of equations (4.12), (4.8)[1] is solved by substitution

$$A_0 = \rho^2 \partial_0 \frac{\varphi_L}{\rho}$$

which leads to an equation for the scalar field φ_L with a source term

$$\Box \varphi_L = 2(\partial_k \rho) V_k .$$

Finally, we have the following solution for Eq.s (4.8)

$$\begin{aligned} A_0 &= \rho^2 \partial_0 \left[\frac{\varphi_L^0}{\rho} + \frac{1}{\rho} \frac{1}{\Box} 2(\partial_k \rho) V_k \right] \quad ; \quad \Box \varphi_L^0 = 0 \\ A_k &= \partial_k \left[\frac{\varphi_L^0}{\rho} + \frac{1}{\rho} \frac{1}{\Box} 2(\partial_\ell \rho) V_\ell \right] + V_k \quad ; \quad G_{(+)kb} V_b = 0 . \end{aligned} \tag{4.14}$$

In the same way, one may find the solutions of Eq. (4.9)

$$\begin{aligned} A_0^+ &= \frac{\partial_0 \varphi_L^+}{\rho} \\ A_k^+ &= \frac{\partial_k}{\partial^2} \left[\rho^2 \partial_0 \left(\frac{\partial_0 \varphi_L^+}{\rho} \right) \right] + \frac{1}{\Box} G_{(+)kb} j_b^+ + V_k^+ , \end{aligned} \tag{4.15}$$

where

$$j_b^+ = j_b(\varphi_L^+) = \frac{2\partial_\ell}{\partial^2} \left[\partial_b (\partial_0 \varphi_L^+) \partial_\ell \rho - \partial_\ell (\partial_0 \varphi_L^+) \partial_b \rho \right] . \tag{4.16}$$

The fields φ_L^+ and V_k^+ satisfy the equations

$$\begin{aligned} &\Box \varphi_L^+ = 0 \quad ; \quad \varphi_L^+ \big|_{\rho = 0} = 0 \\ &G_{(-)kb} V_b^+ = 0 \; ; \; (\Box V_\ell^+ = 0) \; ; \quad V_\ell^+ \big|_{\rho=0} = 0 . \end{aligned} \tag{4.17}$$

Analogously, the equations for the right spinors are solved. The solutions coincide up to the sign with the expressions (4.14), (4.15) with A_L^+, A_L replaced by A_R , A_R^+ respectively,

$$\begin{aligned} (A_0)_R &= \frac{\partial_0 \varphi_R}{\rho} \quad ; \quad \Box \varphi_R = 0 \quad ; \quad G_{(-)kb} (V_b)_R = 0 \\ (A_k)_R &= -\left[\frac{\partial_k}{\partial^2} \rho^2 \partial_0 \left(\frac{\partial_0 \varphi_R}{\rho} \right) \right] + \frac{1}{\Box} G_{(+)kb} j_b(\varphi_R) + (V_k)_R \end{aligned} \tag{4.18}$$

$$(A_0^+)_R = \rho^2 \partial_0 \left[\frac{\varphi_R^+}{\rho} + \frac{1}{\rho\Box} 2(\partial_k\rho)(V_k^+)_R \right] \; ; \Box\varphi_R^+ = 0$$

$$(A_k^+)_R = \partial_k \left[\frac{\varphi_R^+}{\rho} + \frac{2}{\rho\Box}(\partial_\ell\rho)(V_\ell^+)_R \right] + (V_k^+)_R \; ; \quad G_{(+)kb}(V_b^+)_R = 0 \, , \tag{4.19}$$

where A_R, A_R^+ are related with the spinors ψ_R^+, ψ_R by

$$(\psi_R)_i^{\ A} = (\psi_R)_{iA'}(\sigma_2)^{A'A} = \frac{1}{\sqrt{2}}\left[A_0\rho^{3/2} + \rho^{-1/2}(A_k\sigma_k) \right]_{i(R)}^{\ A} \tag{4.20}$$

$$(\psi_R^+)_A^{\ i} = (\psi_R^+)^{iA'}(\sigma_2)_{A'A} = \frac{1}{\sqrt{2}}\left[A_0^+\rho^{-3/2} + \rho^{1/2}(A_k^+\sigma_k) \right]_{A(R)}^{\ i} \, .$$

Once we know the solutions of the Dirac equations (3.4), we obtain immediately the solution of eq. (3.5) for the Lagrange factor. Therefore, the set of equations (3.4), (3.5) is completely integrable.

As it will be shown in the Appendix, using the ADHM construction one may obtain the scalar variant $(V_k = V_k^+ = 0)$ of the solutions for ψ_L and ψ_R^+ . The analog to the ADHM solutions for ψ_L^+, ψ_R (4.15), (4.18) do not exist (the corresponding Euclidean solutions are equal to zero).

5. CALCULATION OF THE CONSERVED QUANTITIES

Let us calculate the observables (3.2) for the bag at rest (2.1), (2.3) with the scale $k_0 = 1$. The vector fields in effect coincide with electromagnetic waves in a cavity with an ideally conductible surface provided both the electric and magnetic oscillation have the same energy. But for the spherical bag this condition is not fulfilled, therefore we conclude $V_k = V_k^+ = 0$ /17/ .

We expand the scalar excitations $\varphi_{L(R)}$, $\varphi_{L(R)}^+$ into a complete set of spherical functions $\Phi_{n,\ell,m}$; $\Phi_{n,\ell,m}^*$:

$$\Phi_{n\ell m}(\vec{x} = |x|(\cos\varphi\sin\theta, \sin\varphi\sin\theta, \cos\theta)) \sim e^{im\varphi} P_\ell^m(\cos\theta)\, j_\ell(|x|\varkappa_{n\ell})$$

$$\varphi_{L(R)} = \sum_{n,\ell,m} \left(\Phi_{n,\ell,m}^*(\vec{x})\, e^{-i\varkappa_{n,\ell}t} a_{n,\ell,m}^{(-)} + \Phi_{n,\ell,m}(\vec{x})\, e^{i\varkappa_{n,\ell}t} b_{n,\ell,m}^{(+)} \right)$$

$$\varphi_{L(R)}^+ = \sum_{n,\ell,m} \left(\Phi_{n,\ell,m}^*(\vec{x})\, e^{-i\varkappa_{n,\ell}t} b_{n,\ell,m}^{(-)} + \Phi_{n,\ell,m}(\vec{x})\, e^{i\varkappa_{n,\ell}t} a_{n,\ell,m}^{(+)} \right) , \tag{5.1}$$

where $a^{(\pm)}$, $b^{(\pm)}$ are the expansion coefficients and $\varkappa_{n,\ell}$ are numbers to be determined from the boundary condition

$$j_\ell(\varkappa_{n,\ell}\,r)=0\,. \tag{5.2}$$

For example, the basic state corresponds to the value $\varkappa_{n=1,\,\ell=0}=1$. The wave functions are normalized such that the probability to find a "quasiparticle" in the bag is equal to unity

$$Q=\int\limits_{\Omega(\rho)} d^3x\, J_0=\int\limits_{\Omega(\rho)} d^3x\left(A_0^+A_0+A_k^+A_k\right)_{(L)}+(L\to R)=\sum_{n,\ell,m}\left(a^{(+)}a^{(-)}+b^{(-)}b^{(+)}\right).$$

This is equivalent to the condition

$$4\varkappa_{n,\ell}\int\limits_{\Omega(\rho)} d^3x\;\Phi^*_{n,\ell,m}\,\Phi_{n,\ell,m}=1\,. \tag{5.3}$$

Furthermore one obtains the following expressions for the energy (Hamiltonian), momentum P_i, and the spin projection (3.2)

$$P_i=\int\limits_{\Omega(\rho)} d^3x\, T_{0i}=\int\limits_{\Omega(\rho)} d^3x\left\{i\left[A_0^+\partial_i A_0+A_k^+\partial_i A_k\right]_{(L)}+(L\to R)+\partial_0\lambda\,\partial_i\rho+\partial_0\rho\,\partial_i\lambda\right\}\equiv 0 \tag{5.4}$$

$$H=\int\limits_{\Omega(\rho)} d^3x\left\{i\left[A_0^+\partial_0 A_0+A_k^+\partial_0 A_k\right]_{(L)}+(L\to R)+\partial_0\lambda\,\partial_0\rho+\partial_i\rho\,\partial_i\lambda\right\}=$$
$$=\sum_{n,\ell,m}\left(a^{(+)}a^{(-)}-b^{(-)}b^{(+)}\right)_{n,\ell,m}\frac{(\varkappa_{n,\ell}+1)(\varkappa_{n,\ell}-1)}{\varkappa_{n,\ell}} \tag{5.5}$$

$$S_z=\int d^3x\;\varepsilon^{3ij}\left[x^j T^{0i}-i\left(A_i^+A_j\right)_{(L)}-i\left(A_i^+A_j\right)_{(R)}\right]=\sum\left(a^{(+)}a^{(-)}-b^{(-)}b^{(+)}\right)m$$

$$m=\frac{1}{4\varkappa_{n,\ell}}\int\limits_{\Omega(\rho)} d^3x\;\Phi^*_{n\ell m}\left(i\,\varepsilon^{3ij}x^j\partial_i\right)\Phi_{n\ell m}\,. \tag{5.6}$$

The calculated observables are relativistic covariant (see section 2) and define the possible mass and spin spectra for the spherical bag.

For considering the N-bag vacuum, we should take into account that the asymptotic states of noninteracting hadrons are described, i.e.,the case when the distance between the bags is larger than their size. In this case the system is factorized and we may consider each bag separately, passing to the system of axes moving with the bag.

The effective spin of the "quasiparticle" in this model is equal to an integer number, as the sum of "colourspin" $\left(\frac{1}{2}\otimes\frac{1}{2}=0+1\right)$ and orbital momentum is conserved (see (3.3)). For the model of colourvector quarks $\left(\frac{1}{2}\otimes 1=\frac{1}{2}+\frac{3}{2}\right)$ the "quasiparticles" will have a normal halfinteger spin; therefore the problem of integer "quark" spin here is not important.

From Eq. (5.5) we see, that there are Goldstone modes which are necessary for the description of meson states. The Hamiltonian (5.5) is positive definite if we suppose, as usual, that Fermi statistics holds for the coefficients $a^{(\pm)}$, $b^{(\pm)}$

$$-b^{(-)}b^{(+)} = b^{(+)}b^{(-)}$$

and if we introduce a normal ordering for $a^{(\pm)}$, $b^{(\pm)}$ in quantizing the spinor fields. The quantization of the considered system will not lead to qualitatively new results, as the self-dual vacuum fields should be considered as c-number (or in more details as coherent states /2,3/).

Consequently, a picture arises which is similar to the hadron bag model /13,14/ with the relativistic covariant description of the hadron asymptotic states. The difference between the bag model /14/ and the one described here consists in zero boundary conditions for the quark fields. These is no problem in physical interpreting of "empty" bags as all their observables are equal to zero.

Concluding, let us remark that we have used nonobservable fields to abolish infrared divergences in Yang-Mills theory. The physical principle of the removal of the infrared divergences by nonobservable fields is used also in QED and, probably will work as well for other quantum systems with infinite number of degrees of freedom. Here, this principle leads to a possibility of constructive description of confinement.

Conclusion

In order to remove the infrared divergences in a non-Abelian theory, we follow the analogy with the theory for quantum liquid. In the quantum liquid case the infrared dynamics leads to spontaneous vacuum symmetry breaking. In the non-Abelian theory the vacuum fields satisfy the self-duality equation in the Minkowski space.

The main results of this paper consist in the complete solution of the Dirac equations with self-dual background fields (depending on a certain arbitrary function) and the calculation of the energy spectrum of the "vacuum + spinors" system.

These results lead to the following physical reasoning:

1) The physical vacuum is represented by an infinite number of gluon self-dual bags moving with various velocities in various directions. Physical observables of such a vacuum (energy, momentum, etc.) are equal to zero. In this sense the vacuum is Lorentz-invariant.

2) Asymptotic states of the colour particles are "quasiparticle" excitations of the "vacuum". The "quasiparticle" wave function is given within the finite volume of one of the bags. The conserved values (observables) of the system ("vacuum" + particles) are Lorentz-covariant and the Hamiltonian defines the hadron mass spectrum.

In this way the resulting physical picture yields a bag-model--like description of hadron asymptotic states.

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APPENDIX

In order to determine the self-dual fields (2.1),(2.3) let us apply an analogy of the ADHM construction /9/.

In Euclidean space it consists in the following.

1) A linear matrix expression

$$\Delta^{AB}_{i\ell} = A_{i\ell}\, x^{AB} + B^{AB}_{i\ell} \quad ; \quad \ell = 0, 1, \dots, N \; ; \; i = 1, \dots, N$$

for quanternions $x^{AB} = \delta^{AB} x_0 + i \vec{\sigma}^{AB} \vec{x}$ is considered.

2) A solution of the algebraic equation

$$\sum_\ell M_\ell^{AC} (\Delta^+_{\ell i})_{CB} = 0 \quad ; \quad \sum_\ell M_\ell^{AC} (M_\ell^+)^B_C = I^{AB}$$

is found under the assumption that the matrix R^{-1} exits, where

$$(R_{\ell k})^{AC} = \sum_i (\Delta_{\ell i}\, \Delta^+_{ik})^{AC} . \qquad (A.1)$$

3) Then the solution of duality equation has the form

$$\hat{A}^{AB}_\mu = \left(A^a_\mu \frac{\sigma^a}{2i}\right)^{AB} = \sum_\ell (M_\ell)^{AC}\, \partial_\mu\, (M^+_\ell)^B_C \qquad (A.2)$$

and the Euclidean solution of Dirac equation

$$(\partial_\mu + \hat{A}_\mu)^{AC} (\psi_i)_C^B (\alpha^{(-)\mu}_{Euc})_B^D = 0 \quad ; \quad \alpha^{(-)\mu}_{Euc} = (I, i\vec{\sigma})$$

are represented in the form

$$(\psi_i)^B_{c(k)} = \sum_\ell (M_\ell^+)^{BA} (R^{-1}_{\ell k})_{AC} \, . \tag{A.3}$$

Our construction is based on integral equations for analytic functions.

We define the matrices

$$x_\tau^{(\pm)} = t - \tau \pm \vec{\sigma}\vec{x}$$

and introduce the functions of $x_\tau^{(\pm)}$

$$M_\tau^{(\mp)} = \frac{1}{\sqrt{\underline{\rho}}} \; \frac{e^{-i\tau k_0/2}}{x^{(\pm)} - i\varepsilon} \tag{A.4}$$

which are solutions of the equations

$$\int_{-\infty}^{\infty} d\tau \, M_\tau^{(+)} M_\tau^{(-)} = 1 \quad ; \quad \int_{-\infty}^{\infty} d\tau \, M_\tau^{(\mp)} x_\tau^{(\pm)} = 0 \tag{A.5}$$

$$\underline{\rho} = \int_{-\infty}^{\infty} d\tau \, \frac{e^{-ik_0\tau}}{x_\tau^2 + i\varepsilon\tau} = -2\pi\rho = -2\pi \, e^{-ik_0 t} \, \frac{\sin k_0 |\vec{x}|}{|\vec{x}|} \, . \tag{A.6}$$

In (A.5) the function $\exp(-i\frac{\tau}{2}k_0)$ is an analytic function at the lower half-plane of variable τ, and $(i\varepsilon)$ gives the rules of geting round the poles in integrating in (A.5),(A.6) over real axis. (Here we bare in mind that the integration over the arc of large radius does not give contribution in the lower halfplane).

We can easily convince ourselves that the solution of Eq.s (2.1), (2.3) takes the form

$$A_\mu^{(\pm)} = \int_{-\infty}^{\infty} d\tau \; M_\tau^{(\mp)} \, \partial_\mu \, M_\tau^{(\pm)} \, . \tag{A.7}$$

An analog of the matrix R_{ik}^{-1} (A.I) is the expression

$$R^{-1}_{\tau_1\tau_2} = \delta(\tau_1 - \tau_2)\frac{1}{x_{\tau_1}^2 + 2i\varepsilon\tau_1} - \frac{\exp\left(-i\,\frac{\tau_1+\tau_2}{2}\,k_0\right)}{\underline{\rho}\,(x_{\tau_1}^2 + i\varepsilon\tau_1)(x_{\tau_2}^2 + i\varepsilon\tau_2)} \, . \tag{A.8}$$

The solution of the Dirac equation with the zero boundary conditions takes the form of (A.3)

$$\Psi_{L(n)} = \int_{-\infty}^{\infty} d\tau_2 \, e^{in\tau_2 k_0} \int_{-\infty}^{\infty} d\tau_1 \, M_{\tau_1}^{(+)} (R^{-1})_{\tau_1 \tau_2} =$$

$$= -\frac{1}{2} \rho^{1/2} \left(\alpha^{(+)\mu} \partial_\mu \right) \left[\frac{\rho(nx)}{\rho(x)} \right]$$

which coincides with (4.4), (4.6), (4.14) for the vacuum (2.1), (2.3).

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