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ON A SYSTEM OF RELATED MASSLESS DIPOLE GHOST FIELDS IN TWO SPACE-TIME DIMENSIONS

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1. INTRODUCTION

Dipole ghost fields have been originally introduced by Heisenberg[1] in order to justify the use of a double pole propagator in his nonlinear spinor field theory. Then the problem was dealt with by many authors among which one should especially mention Froissart[2] who investigated the problem in the framework of indefinite metrics field theory. Then it was realized that massless dipole ghosts are implicit in the manifest covariant formulation of Quantum Electrodynamics as a gauge theory[3]. In that connection one should, of course, mention the role of the dipole ghost for the construction of the solution of the Zwanziger model[4]. Dipole ghosts are also present in the consideration of the Schwinger model[5,6] (i.e., massless spinor electrodynamics in two space-time dimensions). The present paper considers a couple of massless dipole ghost fields (treated according to the Froissart model[2]) in a more complicated situation. The formulation of the problem is as follows. Consider the system of scalar and pseudoscalar fields $F(x)$ and $\tilde{F}(x)$, respectively, that obey the equation (duality condition):

$$\partial_\mu F(x) + \epsilon_{\mu\nu}\partial^\nu \tilde{F}(x) = 0, \tag{1.1}$$

where $\epsilon_{\mu\nu} = -\epsilon_{\nu\mu} = -\epsilon^{\mu\nu}$, $\epsilon_{01} = 1$ and the metrics is chosen to be $g_{\mu\mu} = (+,-)$. It is then obvious that these fields satisfy also

$$\Box F(x) = 0, \tag{1.2}$$

$$\Box \tilde{F}(x) = 0. \tag{1.3}$$

Now consider the scalar field $\Phi(x)$ and the pseudoscalar field $\tilde{\Phi}(x)$ that satisfy the following equations:

$$\Box\Phi(x) = \lambda_1 F(x), \tag{1.4}$$

$$\Box\tilde{\Phi}(x) = \lambda_2 \tilde{F}(x). \tag{1.5}$$

It is clear (in view of eqs. (1.2) and (1.3)) that $\Phi(x)$ and $\tilde{\Phi}(x)$ are in fact massless dipole ghosts, i.e., they satisfy

$$\Box^2\Phi(x) = 0, \tag{1.4a}$$

$$\Box^2\tilde{\Phi}(x)=0. \tag{1.5a}$$

It is now obvious that we have two Froissart systems of equations (eqs. (1.2)-(1.5)), which are however related through equation (1.1). Although such a system might seem somewhat artificial, it should be noted, that such a situation can take place in the case of a "non Schwinger" solution of the two dimensional massless spinor electrodynamics. Here "non Schwinger" means a solution that is covariant under both gauge and γ^5 -gauge transformations, unlike the standard case when γ^5-gauge invariance is lost in the quantization procedure.

In order to find out the explicit solution of the above-formulated problem some boundary conditions are needed. The existence of the relation (1.1) indicates that in general we should modify the conditions that are usually imposed in the Froissart model. However this does not concern Poincare invariance, which should be present in the solution. As for locality, it is eq. (1.1) that leads to certain modifications. It is in general known that scalar fields that are related with pseudoscalar through equations of the type (1.1) cannot be mutually local /7-9/. However, we can still ask that the rest of the commutation relations are local. The requirement that some commutators should be canonical will be commented on in the text since at this point we cannot give a brief formulation of this condition.

2. COMMUTATION RELATIONS

In this section we write down the proper commutation relations. To start with we begin with the observation that translational invariance implies that

$$[F(x),F(y)]=[F(x),\tilde{F}(y)]=[\tilde{F}(x),\tilde{F}(y)]=0. \tag{2.1}$$

That is readily seen from eqs. (1.2)-(1.5) and the consideration of the appropriate commutators of $F(x)$ (or $\tilde{F}(x)$) and $\Phi(x)$ (or $\tilde{\Phi}(x)$). Further on the same arguments when applied to the commutator of $\Phi(x)$ and $\tilde{\Phi}(x)$ lead to the fact that the only independent commutation functions are the following:

$$1/\lambda_1[\Phi(x),F(y)]=1/\lambda_2[\tilde{\Phi}(x),\tilde{F}(y)]=D(x-y), \tag{2.2}$$

$$1/\lambda_1[\Phi(x),\tilde{F}(y)]=1/\lambda_2[\tilde{\Phi}(x),F(y)]=\tilde{D}(x-y), \tag{2.3}$$

$$[\Phi(x),\Phi(y)] = H_1(x-y), \tag{2.4}$$

$$[\tilde{\Phi}(x),\tilde{\Phi}(y)] = H_2(x-y), \tag{2.5}$$

$$[\Phi(x),\tilde{\Phi}(y)] = \tilde{H}(x-y). \tag{2.6}$$

Having in mind the system of equations (1.1)-(1.5) for the fields, it is obvious that the functions $D(x)$, $\tilde{D}(x)$, $H_1(x)$, $H_2(x)$ and $\tilde{H}(x)$ satisfy the following system of equations:

$$\partial_\mu D(x) + \epsilon_{\mu\nu}\partial^\nu \tilde{D}(x) = 0, \tag{2.7}$$

$$\Box H_1(x) = \lambda_1^2 D(x), \tag{2.8}$$

$$\Box H_2(x) = \lambda_2^2 D(x), \tag{2.9}$$

$$\Box \tilde{H}(x) = \lambda_1\lambda_2 \tilde{D}(x). \tag{2.10}$$

A system of commutation functions that satisfy eq. (2.7) has been studied by many authors (see, f.i., papers /7-11/) mainly in connection with the massless Thirring model. It is well known that the condition $D(x)$ to be canonical, i.e.,

$$\partial_0 D(x)\big|_{x^0=0} = -\delta(x^1) \tag{2.11}$$

fixes the solution of (2.7) in the following form:

$$D(x) = -\frac{1}{2}\epsilon(x^0)\theta(x^2), \tag{2.12}$$

$$\tilde{D}(x) = -\frac{1}{2}\epsilon(x^1)\theta(-x^2). \tag{2.13}$$

Now consider eqs. (2.8)-(2.10). Having in mind that $D(x)$ and $\tilde{D}(x)$ are homogeneous functions of zero degree, it is obvious that one can write down the following explicit expressions for the general solutions:

$$H_1(x) = \frac{\lambda_1^2 x^2}{4} D(x) + c_1 D(x) \equiv \lambda_1^2 H_0(x) + c_1 D(x), \tag{2.14}$$

$$H_2(x) = \frac{\lambda_2^2 x^2}{4} D(x) + c_2 D(x) \equiv \lambda_2^2 H_0(x) + c_2 D(x), \tag{2.15}$$

$$\tilde{H}(x)=\frac{\lambda_1\lambda_2 x^2}{4}\tilde{D}(x)+c_3\tilde{D}(x)\equiv\lambda_1\lambda_2\tilde{H}_0(x)+c_3\tilde{D}(x). \qquad (2.16)$$

Here $H_0(x)$ and $\tilde{H}_0(x)$ are just particular solutions of (2.8) and (2.10), respectively, for $\lambda_1=\lambda_2=1$, while the terms determined by the constants c_i, $i=1,2,3$ which are just solutions of the homogeneous equations with definite parity* involve the possible arbitrariness of the solutions. It is obvious that $H_1(x)$ and $H_2(x)$ can be made canonical if we fix $c_1=c_2=1$ on the contrary to $\tilde{H}$ which can be fixed by some other conditions. In what follows, however, we keep the constants c_1, c_2 and c_3 arbitrary in order to be ready to meet some other requirements. Since solutions of eqs. (2.8) and (2.9) of the type (2.14) and (2.15) have been already written (see, f.i., papers/6,10/), the aim of the present section is to give a more detailed treatment of the infrared properties of the positive and negative frequency parts of these functions.

In what follows it is convenient to use the cone variables. So we introduce the notation

$$x_\pm=x_0\pm x_1 \qquad x_\pm=x^\mp \qquad d^2x=-\frac{1}{2}dx_+dx_-$$
$$\partial_\pm=\partial_0\pm\partial_1=\partial^\mp \qquad x^2=x_+x_-. \qquad (2.17)$$

The infrared problem for the functions $D(x)$ and $\tilde{D}(x)$ has been dealt with in many papers (see, f.i., refs./7-11/); we would just write here the solution in terms of the cone variables without any further comment. So we have for

$$D^\pm(x)=\pm\frac{1}{4\pi}\int_0^\infty\frac{dp}{p}\{e^{\mp ipx_+}+e^{\mp ipx_-}-2\theta(\kappa-p)\}$$
$$=\mp\frac{1}{4\pi}\ln(-\mu^2x^2\pm i0x^o), \qquad (2.18)$$

where $\mu=e^\gamma\kappa$, $\gamma=-\Gamma'(1)$ is a parameter with a mass dimension, that is needed to make the argument of the logarithm dimensionless. For the functions $\tilde{D}^\pm(x)$ we have

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* We remind that $H_1(x)$ and $H_2(x)$ are scalars, while $\tilde{H}(x)$ is a pseudoscalar under space-time inversions, and it is exactly this condition that renders the arbitrariness of the solutions in the form written above.

$$\tilde{D}^{\pm}(x) = \mp \frac{1}{4\pi} \int_0^\infty \frac{dp}{p} \{ e^{\mp ipx_+} - e^{\mp ipx_-} \} = \pm \frac{1}{4\pi} \ln \frac{x_+ \mp i0}{x_- \mp i0}. \quad (2.19)$$

The main problem is how to define the frequency parts of $H_0(x)$ and $\tilde{H}_0(x)$, and to formulate therefore a consistent infrared regularization procedure. For the purpose one can use of course the standard procedure. Since we are mainly interested in the infrared regularization problem, we can proceed in a more straightforward way. Namely, we shall define $H_0^{\pm}(x)$ and $\tilde{H}_0^{\pm}(x)$ as the solutions of the following equations:

$$\Box H_0^{\pm}(x) = 4\partial_+ \partial_- H_0^{\pm}(x_+, x_-) = D^{\pm}(x_+, x_-), \quad (2.20)$$

$$H_0^+(x) + H_0^-(x) = iH_0(x),$$

$$\Box \tilde{H}_0^{\pm}(x) = 4\partial_+ \partial_- \tilde{H}_0^{\pm}(x_+, x_-), \quad (2.21)$$

$$\tilde{H}_0^+(x) + \tilde{H}_0^-(x) = i\tilde{H}_0(x),$$

where in the r.h.s. of these equations we consider $D^{\pm}(x)$ and $\tilde{D}^{\pm}(x)$ in the form of one-dimensional Fourier integrals (eqs. (2.18) and (2.19), respectively). This gives an insight of the needed regularization. Indeed, since in the cone variables the D'Alembertian is factorized, it is evident that $H_0^{\pm}(x)$ and $\tilde{H}_0^{\pm}(x)$ are determined by $D^{\pm}(x)$ and $\tilde{D}^{\pm}(x)$, respectively, up to an additive linear combinations of x_+ and x_-. The latter are needed in order to regularize the leading singularity (the first order pole) of the integrands. Thus we are forced to write down the following expressions for $H_0^{\pm}(x)$ and $\tilde{H}_0^{\pm}(x)$:

$$H_0^{\pm}(x) = \frac{i}{16\pi} \int_0^\infty \frac{dp}{p^2} \{ x_- [e^{\mp ipx_+} - 1 \pm ipx_+ \theta(\kappa - p)] + x_+ [e^{\mp ipx_-} - 1 \pm ipx_- \theta(\kappa - p)] \} = \frac{x^2}{4} (D^{\pm}(x) \pm \frac{2}{\pi}), \quad (2.22)$$

$$\tilde{H}_0^{\pm}(x) = \frac{i}{16\pi} \int_0^\infty \frac{dp}{p^2} \{ x_- [e^{\mp ipx_+} - 1] - x_+ [e^{\mp ipx_-} - 1] \} = \frac{x^2}{4} \tilde{D}^{\pm}(x). \quad (2.23)$$

Since these expressions satisfy eqs. (2.20) and (2.21), it is obvious that their regularization is compatible with that of the functions $D^{\pm}(x)$ and $\tilde{D}^{\pm}(x)$. There is, however one particular point that needs to be discussed. It concerns the regularization of the leading singularities (the terms $x_{\pm}/p^2$). At first glance it seems that one should subtract the terms $x_{\pm}/p^2$ multiplied by $\theta(\kappa-p)$ as for the logarithmic terms. It is not difficult to see, however, that in such a case Lorentz invariance would be badly afflicted. Indeed, under Lorentz rotations at an angle χ such counter terms would generate additive terms of the kind $1/\chi(1-e^{\mp\chi})$. It is evident that this cannot be a group transformation law. Therefore, Lorentz covariance determines the regularization of the leading singularities uniquely.

At the end we must remind the Lorentz-transformation properties of the functions $D^{\pm}(x)$ and $\tilde{D}^{\pm}(x)$. As it can be seen from eqs. (2.18) and (2.19), they transform as follows:

$$D^{\pm}(\Lambda_{\chi}x)=D^{\pm}(x) \qquad \Lambda_{\chi}=\begin{pmatrix} \mathrm{ch}\chi & \mathrm{sh}\chi \\ \mathrm{sh}\chi & \mathrm{ch}\chi \end{pmatrix}. \tag{2.24}$$

$$\tilde{D}^{\pm}(\Lambda_{\chi}x)=\tilde{D}^{\pm}(x)\pm\frac{\chi}{2\pi}$$

Then the corresponding transformations of $H_0^{\pm}(x)$ and $\tilde{H}_0^{\pm}(x)$

$$H_0^{\pm}(\Lambda_{\chi}x)=H_0^{\pm}(x),$$

$$\tilde{H}_0^{\pm}(\Lambda_{\chi}x)=\tilde{H}_0^{\pm}(x)\pm\frac{\chi}{8\pi}x^2 \tag{2.25}$$

can be easily obtained in view of eqs. (2.22) and (2.23). It is quite important to note at this point that the equations (2.7)-(2.10) are invariant with respect to the transformations (2.24) and (2.25).

3. OPERATOR SOLUTION

In this section we give an explicit operator solution for the fields $F^{\pm}(x)$, $\tilde{F}^{\pm}(x)$, $\Phi^{\pm}(x)$ and $\tilde{\Phi}^{\pm}(x)$ satisfying the commutation relations obtained in the previous section. We consider first the fields $F^{\pm}(x)$ and $\tilde{F}^{\pm}(x)$. It has already been mentioned that they were intensively discussed in many papers /7-11/. Here we just write down the explicit solution in terms of the cone variables

$$F^{\pm}(x)=\sqrt{\frac{2}{\pi}}\int_0^{\infty}\frac{dp}{p}\{e^{\mp ipx_+}A^{\pm}(p)+e^{\mp ipx_-}B^{\pm}(p)-\theta(\kappa-p)[A^{\pm}(0)+B^{\pm}(0)]\}, \quad (3.1)$$

$$\tilde{F}^{\pm}(x)=\sqrt{\frac{2}{\pi}}\int_0^{\infty}\frac{dp}{p}\{e^{\mp ipx_+}A^{\pm}(p)-e^{\mp ipx_-}B^{\pm}(p)-\theta(\kappa-p)[A^{\pm}(0)-B^{\pm}(0)]\}. \quad (3.2)$$

It whould be noted that the above form of the infrared regularization is fixed (following the method of paper [11]) by two conditions. First of all it must be compatible with the regularization of the functions $D^{\pm}(x)$ and $\tilde{D}^{\pm}(x)$. And second the existence of both charges

$$Q^{\pm}\equiv\int_{-\infty}^{\infty}dx^1\partial_0 F^{\pm}(x)=\mp i[A^{\pm}(0)+B^{\pm}(0)], \quad (3.3)$$

$$\tilde{Q}^{\pm}\equiv\int_{-\infty}^{\infty}dx^1\partial_0\tilde{F}^{\pm}(x)=\mp i\,[A^{\pm}(0)-B^{\pm}(0)] \quad (3.4)$$

is required.

The above expressions for the charges make clear the character of the singularities in the integrals (3.1) and (3.2) and at the same time fix the regularization up to the multiplicative function $\theta(\kappa-p)$.

The main problem is the construction of the explicit solutions for the fields $\Phi^{\pm}(x)$ and $\tilde{\Phi}^{\pm}(x)$. We shall define these fields as solutions of the equations

$$\Box\Phi^{\pm}(x)=\lambda_1 F^{\pm}(x), \quad (3.5)$$

$$\Box\tilde{\Phi}^{\pm}(x)=\lambda_2\tilde{F}^{\pm}(x). \quad (3.6)$$

The general solution of these equations can be written down in the following form:

$$\Phi^{\pm}(x)=\Phi_0^{\pm}(x)+\Phi_1^{\pm}(x)+c_1/2\lambda_1 F^{\pm}(x), \quad (3.7)$$

$$\tilde{\Phi}^{\pm}(x)=\tilde{\Phi}_0^{\pm}(x)+\tilde{\Phi}_1^{\pm}(x)+c_2/2\lambda_2\tilde{F}^{\pm}(x), \quad (3.8)$$

where $\Phi_0^{\pm}(x)$ and $\tilde{\Phi}_0^{\pm}(x)$ are particular solutions of eqs. (3.5) and (3.6), while the solutions of the homogeneous equations $\Phi_1^{\pm}(x)$ and $\tilde{\Phi}_1^{\pm}(x)$ are necessary in order to satisfy the commutation relations of the previous section (the constants c_1 and c_2 are introduced by eqs. (2.14) and (2,15)).

Now we first write down the following explicit expressions for $\Phi_0^{\pm}(x)$ and $\tilde{\Phi}_0^{\pm}(x)$:

$$\Phi_0^{\pm}(x) = \pm \frac{i\lambda_1}{2\sqrt{2\pi}} \int_0^\infty \frac{dp}{p^2} \{ x_-[e^{\mp ipx_+} A^{\pm}(p) - A^{\pm}(0) - p\theta(\kappa - p)(A'^{\pm}(0) \mp i x_+ A^{\pm}(0))]$$

$$+ x_+[e^{\mp ipx_-} B^{\pm}(p) - B^{\pm}(0) - p\theta(\kappa - p)(B'^{\pm}(0) \mp i x_- B^{\pm}(0))]\}, \qquad (3.9)$$

$$\tilde{\Phi}_0^{\pm}(x) = \pm \frac{i\lambda_2}{2\sqrt{2\pi}} \int_0^\infty \frac{dp}{p^2} \{ x_-[e^{\mp ipx_+} A^{\pm}(p) - A^{\pm}(0) - p\theta(\kappa - p)(A'^{\pm}(0) \mp i x_+ A^{\pm}(0))] \qquad (3.10)$$

$$- x_+[e^{\mp ipx_-} B^{\pm}(p) - B^{\pm}(0) - p\theta(\kappa - p)(B'^{\pm}(0) \mp i x_- B^{\pm}(0))]\},$$

where $A'^{\pm}(0)$ and $B'^{\pm}(0)$ are the first derivatives of $A^{\pm}(p)$ and $B^{\pm}(p)$ at $p=0$. It is evident that expressions (3.9) and (3.10) are solutions of eqs. (3.5) and (3.6). However, some comment on regularization procedure is needed. It concerns the terms that do not survive under the action of the D'Alembertian (i.e., $x_- A^{\pm}(0)$, $x_+ B^{\pm}(0)$, $\theta(\kappa - p)px_- A'^{\pm}(0)$ and $\theta(\kappa - p)px_+ B'^{\pm}(0)$). The analysis of the singularities at $p=0$ makes clear that such terms are indeed needed. However, their exact form is fixed by the requirements for Lorentz covariance and the necessity to reproduce the commutators obtained in the previous section. Further on, it is clear from eqs. (2.1), (3.9) and (3.10) that $\Phi_0^{\pm}(x)$, $\tilde{\Phi}_0^{\pm}(x)$, $F^{\pm}(x)$ and $\tilde{F}^{\pm}(x)$ should commute trivially. This consideration makes clear the introduction of the subsidiary fields $\Phi_1^{\pm}(x)$ and $\tilde{\Phi}_1^{\pm}(x)$, which should make possible to obtain the correct commutators. For the latter fields, we write down the following expressions:

$$\Phi_1^{\pm}(x) = \lambda_1 \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{dp}{p} \{ [e^{\mp ipx_+} C^{\pm}(p) - C^{\pm}(0)] + [e^{\mp ipx_-} G^{\pm}(p) - G^{\pm}(0)]\}, \qquad (3.11)$$

$$\tilde{\Phi}_1^{\pm}(x) = \lambda_2 \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{dp}{p} \{ [e^{\mp ipx_+} C^{\pm}(p) - C^{\pm}(0)] - [e^{\mp ipx_-} G^{\pm}(p) - G^{\pm}(0)]\}. \qquad (3.12)$$

In these expressions we do not use the standard regularization ($\theta(\kappa - p)$ is missed). Going ahead for a while, we can say that the latter is motivated by the requirement that all commutators should be Lorentz invariant, which can be achieved by a special choice of the field transformation properties only. And this in turn results in the above choice of the regularization.

Now we face the problem of formulation of such commutation relations between $A^{\pm}(p)$, $B^{\pm}(p)$, $C^{\pm}(p)$ and $G^{\pm}(p)$ which imply the commutators obtained in the previous section. A simple generalization of the method employed in paper /11/ leads to the following system of commutators

$$\frac{1}{pq}[C^{\pm}(p),A^{\mp}(q)]=\frac{1}{pq}[G^{\pm}(p),B^{\mp}(q)]=\pm\frac{1}{8}\{\frac{\delta(p-q)}{p}+\delta(p)\delta(q)\int_0^{\kappa}\frac{d\ell}{\ell}\} \quad (3.13)$$

$$\frac{1}{q}[C^{\pm}(0),A^{\mp}(q)]=\frac{1}{q}[G^{\pm}(0),B^{\mp}(q)]=\frac{1}{q}[C^{\pm}(q),A^{\mp}(0)]= \quad (3.14)$$

$$=\frac{1}{q}[G^{\pm}(q),B^{\mp}(0)]=\pm\frac{1}{8}\delta(q)$$

$$\frac{1}{q}[C^{\pm}(q),A'^{\pm}(0)]=\frac{1}{q}[G^{\pm}(q),B'^{\pm}(0)]=\mp\frac{1}{8}\delta'(q). \quad (3.15)$$

All other commutators are vanishing.

It is easy to check that using formulae (3.13)-(3.15), one can obtain the correct commutators between the fields $F^{\pm}(x)$ $\tilde{F}^{\pm}(x)$, $\Phi^{\pm}(x)$ and $\tilde{\Phi}^{\pm}(x)$ provided that the constant in formula (2.16) is fixed to be

$$c_3=\frac{1}{2}(c_2\frac{\lambda_1}{\lambda_2}+c\frac{\lambda_2}{\lambda_1}). \quad (3.16)$$

So we have partially removed the arbitrarity determined by the constants c_1 , c_2 and c_3 .

At the end of this section it is necessary to discuss briefly translational invariance. From eqs. (3.9) and (3.10) it is evident that $\Phi_0^{\pm}(x)$ and $\tilde{\Phi}_0^{\pm}(x)$ are lacking manifest translational covariance. And it is known /8/ that in order to deal with the problem of Poincare invariance of a dipole ghost correctly, one must use the representation as a Fourier integral over the whole space-time rather than the one-dimensional Fourier representation. In such a case we use the following equalities:

$$A^{\pm}(p,q)=-\frac{1}{p^2}\delta'(q)A^{\pm}(p,q), \quad (3.17)$$

$$B^{\pm}(p,q)=-\frac{1}{q^2}\delta'(p)B^{\pm}(p,q), \quad (3.18)$$

where $A^{\pm}(p,q)$ and $B^{\pm}(p,q)$ satisfy the conditions

$$A^{\pm}(p,0)=A^{\pm}(p), \quad \frac{\partial}{\partial q}A^{\pm}(p,q)|_{q=0}=0, \quad (3.19)$$

$$B^{\pm}(0,q) = B^{\pm}(q), \quad \frac{\partial}{\partial p} B^{\pm}(p,q)|_{p=0} = 0. \tag{3.20}$$

Then using the standard transformation laws for $A^{\pm}(p,q)$ and $B^{\pm}(p,q)$, i.e., $A^{\pm}(p,q) \to e^{ipa_+ + iqa_-} A^{\pm}(p,q)$ and $B^{\pm}(p,q) \to e^{ipa_+ + iqa_-} B^{\pm}(p,q)$, we can restore the manifest translational invariance for the dipole ghost fields $\Phi_0^{\pm}(x)$ and $\tilde{\Phi}_0^{\pm}(x)$.

4. LORENTZ INVARIANCE

The aim of this section is to define the proper Lorentz transformations of the fields $F^{\pm}(x)$, $\tilde{F}^{\pm}(x)$, $\Phi^{\pm}(x)$ and $\tilde{\Phi}^{\pm}(x)$ under which the commutators of the fields and the equations are invariant. At the same time this would serve as a check of the self-consistency of the results that are already obtained.

For the purpose it looks natural* to adopt for the operators $A^{\pm}(p)$, $B^{\pm}(p)$, $C^{\pm}(p)$ and $G^{\pm}(p)$ the following transformation properties:

$$U_\chi^{-1} A^{\pm}(p) U_\chi = A^{\pm}(e^{-\chi} p) \qquad U_\chi^{-1} A'^{\pm}(0) U_\chi = e^{-\chi} A'^{\pm}(0), \tag{4.1}$$

$$U_\chi^{-1} B^{\pm}(p) U_\chi = B^{\pm}(e^{\chi} p) \qquad U_\chi^{-1} B'^{\pm}(0) U_\chi = e^{\chi} B'^{\pm}(0), \tag{4.2}$$

$$U_\chi^{-1} C^{\pm}(p) U_\chi = C^{\pm}(e^{-\chi} p), \tag{4.3}$$

$$U_\chi^{-1} G^{\pm}(p) U_\chi = G^{\pm}(e^{\chi} p). \tag{4.4}$$

Then having in mind the explicit expressions (3.1), (3.2), (3.9)-(3.12) one can immediately obtain for the fields $F^{\pm}(x)$, $\tilde{F}^{\pm}(x)$, $\Phi^{\pm}(x)$ and $\tilde{\Phi}^{\pm}(x)$ the following transformation laws:

$$U_\chi^{-1} F^{\pm}(x) U_\chi = F^{\pm}(\Lambda_\chi x) \pm i\sqrt{\frac{2}{\pi}}\,\chi\,\tilde{Q}^{\pm}, \tag{4.5}$$

$$U_\chi^{-1} \tilde{F}^{\pm}(x) U_\chi = \tilde{F}^{\pm}(\Lambda_\chi x) \pm i\sqrt{\frac{2}{\pi}}\,\chi\,Q^{\pm}, \tag{4.6}$$

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*An analogous method has been used in paper /11/ in order to obtain the correct Lorentz transformations of the fields $F^{\pm}(x)$ and $\tilde{F}^{\pm}(x)$.

$$U_\chi^{-1}\Phi^\pm(x)U_\chi = \Phi^\pm(\Lambda_\chi x) \pm \frac{i\chi}{\sqrt{2\pi}}\left(\frac{\lambda_1 x^2}{2} + \frac{c_1}{\lambda_1}\right)Q^\pm +$$
$$\pm i\frac{\lambda_1\chi}{2\sqrt{2\pi}}[e^{-\chi}x_- A'^{\pm}(0) - e^{\chi}x_+ B'^{\pm}(0)] \qquad (4.7)$$
$$\equiv \Phi^\pm(\Lambda_\chi x) \pm i\chi K^\pm(\Lambda_\chi x),$$

$$U_\chi^{-1}\tilde\Phi^\pm(x)U_\chi = \tilde\Phi^\pm(\Lambda_\chi x) \pm \frac{i\chi}{\sqrt{2\pi}}\left(\frac{\lambda_2 x^2}{2} + \frac{c_2}{\lambda_2}\right)\tilde Q^\pm +$$
$$\pm\frac{i\lambda_2\chi}{2\sqrt{2\pi}}[e^{-\chi}x_- A'^{\pm}(0) + e^{\chi}x_+ B'^{\pm}(0) = \qquad (4.8)$$
$$\equiv \tilde\Phi^\pm(\Lambda_\chi x) \pm i\chi\tilde K^\pm(\Lambda_\chi x),$$

where $Q^\pm$ and $\tilde Q^\pm$ are defined by eqs. (3.3) and (3.4) and Λ_χ is the matrix of the vector representation of the Lorentz group defined by formula (2.24).

Although these transformation laws look quite unusual, we shall prove now that the whole problem is invariant under their action. For the purpose we first note that the following equations take place:

$$\Box K^\pm(\Lambda_\chi x) = \pm i\lambda_1\sqrt{\frac{2}{\pi}}\,\chi\,\tilde Q^\pm, \qquad (4.9)$$

$$\Box\tilde K^\pm(\Lambda_\chi x) = \pm i\lambda_2\sqrt{\frac{2}{\pi}}\,\chi\,Q^\pm. \qquad (4.10)$$

But now it is evident that the r.h. sides of these equations coincide up to the multiplicative constants λ_1 and λ_2 with the additive term in the transformation laws of $F^\pm(x)$ and $\tilde F^\pm(x)$, respectively. Therefore, we can conclude that eqs. (1.1),(3.5) and (3.6) are covariant under the action of these transformations.

A little but more complicated is the proof of the invariance of the commutators. Having in mind eqs. (3.13)-(3.15), it is not difficult to see that the only nontrivial commutators involving $Q^\pm$, $\tilde Q^\pm$, $K^\pm(x)$ and $\tilde K^\pm(x)$ are exactly the following:

$$[Q^\pm, \Phi^\mp(x)] = \mp\frac{i\lambda_1}{2\sqrt{2\pi}}, \qquad (4.11)$$

$$[\tilde Q^\pm, \tilde\Phi^\mp(x)] = \mp\frac{i\lambda_2}{2\sqrt{2\pi}}, \qquad (4.12)$$

$$[\Phi^\pm(x), K^\mp(y)] = -[K^\pm(x), \Phi^\mp(y)] = \frac{\lambda_1^2}{16\pi}(x_- y_+ - x_+ y_-), \qquad (4.13)$$

$$[\tilde{\Phi}^{\pm}(x), \tilde{K}^{\mp}(y)] = -[\tilde{K}^{\pm}(x), \tilde{\Phi}^{\mp}(y)] = \frac{\lambda_2^2}{16\pi}(x_- y_+ - x_+ y_-), \tag{4.14}$$

$$[\Phi^{\pm}(x), \tilde{K}^{\mp}(y)] = \mp \frac{i}{8\pi} c_2 \frac{\lambda_1}{\lambda_2} \mp i \frac{\lambda_1 \lambda_2}{16\pi}(y^2 - x_+ y_- - x_- y_+), \tag{4.15}$$

$$[K^{\pm}(x), \tilde{\Phi}^{\mp}(y)] = \mp \frac{i}{8\pi} c_1 \frac{\lambda_2}{\lambda_1} \mp i \frac{\lambda_1 \lambda_2}{16\pi}(x^2 - x_+ y_- - x_- y_+). \tag{4.16}$$

Now having in mind formulae (2.14)-(2.16), (2.24), (2.25) and (3.16), we arrive at the conclusion that the additive terms in the Lorentz transformations of the fields exactly cancel the additive terms in the transformations of the functions $D^{\pm}(x)$ and $\tilde{D}^{\pm}(x)$. In fact, the commutators of the fields are invariant under the action of the transformations (4.5)-(4.18). Thus, we see that the whole problem is invariant with respect to the latter transformations. This in its turn can be regarded as a check of the consistency of the regularization procedure.

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