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E2-81-520

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SELF ENERGIES OF THE ELECTRON AND PHOTON IN THE UNIFIED SPACE FIELD THEORY

1981

A Lagrangian of the interaction between spinor and vector fields in the unified space is considered. In the first approximation it is shown that the "naked" and physic masses of electron and photon are identical. Some effects are discussed.

The spinor equation in the unified space takes the form

$$i\Gamma_j \partial^j \psi(\xi) = 0 , \tag{1}$$

where $\psi(\xi)$ is the 16-component spinor. It is the field of the group of fermions (ν_e, e, p, n), $(\nu_\mu, \mu, N_1^+, N_1^o)$, etc.

The vector field in this space, $B_j(\xi)$, $(j = 1,2,\ldots 7,8)$, satisfies the equation

$$\partial^k \partial_k B_j(\xi) = 0 \tag{2}$$

and the generalized Lorentz condition

$$\partial^j B_j(\xi) = 0. \tag{3}$$

It is the field of photons and pions [2].

In the case of free particle motion, the unified space is divided into two invariant subspaces: the Minkowski space and Isospace. In this case, the spinor field of (1) can be transformed into the Dirac one by unitarity transformation: $e^{iz} = \frac{1}{\sqrt{2}}(1 - \frac{P^a \Gamma_a}{|P|})$, where P is the intrinsic momentum 4-vector, $m = |P|$ is the mass of the corresponding particle. The vector field of (2) by the variable separation

$$B(\xi) = (A_\mu(x)\rho(X), \phi(x) V_a(X)) \tag{4}$$

$$(\mu = 1,2,3,4; \quad a = 5,6,7,8)$$

is transformed into the fields of photons and pions. They satisfy the following equations:

For photons

$$(\partial^\nu \partial_\nu + m_\gamma^2) A_\mu(x) = 0, \qquad \partial^\mu A_\mu(x) = 0 \tag{5}$$

(In the case of free photon $m_\gamma = 0$, [1])

and

$$(\partial_a \partial_a + m_\gamma^2)\rho(X) = 0 . \tag{5'}$$

For pions

$$(\partial_\mu \partial^\mu + m_\pi^2)\phi(x) = 0 \qquad (6)$$

and

$$(\partial_\beta \partial_\beta + m_\pi^2)V_\alpha(x) = 0, \quad \partial_\alpha V_\alpha(x) = 0, \qquad (6')$$

where m_π^2 is the mass of pions.

In the intermediate states of the elements of S-matrix in unified space [4] the spinor and vector fields can be presented in terms of spectral expansion:

For the spinor fields

$$\psi_\nu^{(s)}(\xi) = \frac{1}{\sqrt{\pi}} \sum_{J_1, J_2} \int_0^\infty dm^2 < J_1, \nu, J_2 | m^2 > f(P, J_2, m^2)\psi_\nu(x, J_1, m^2), \qquad (7)$$

where J_1 and J_2 denote the ordinary and intrinsic quantum numbers, respectively

$$f(P, J_2, m^2) = A(P, J_2, m^2)\frac{P^\alpha \Gamma_\alpha - m}{\sqrt{2m}}. \qquad (8)$$

In (8) A^+ and A^- are the intrinsic operators of creation of an antiparticle and of annihilation of a particle, respectively. They belong to the Bose statistics, $\psi(x, J_2, m^2)$ is the Dirac field.

Then the Green function, according to (8), takes the form

$$S^c(\xi) = \frac{1}{(2\pi)^8} \int_{\circlearrowleft / \circlearrowright} d\lambda\, e^{-i\lambda\xi} \frac{\hat{\lambda}}{\lambda^2 + i\epsilon}\theta(\pm p_0), \qquad (9)$$

where $\circlearrowleft$ and $\circlearrowright$ denote the integral contours, which are closed respectively on the upper and under complex half-planes of the intrinsic energies [4], $\hat{\lambda} = \sum_{j=1}^{8} \lambda^j \Gamma_j$, $\lambda = (p, P)$, p is the ordinary momentum 4-vector and P is the intrinsic one.

For the vector field, in terms of spectral expansion, we have:

In case of $k = 1, 2, 3, 4$

$$B_\mu^{(s)}(\xi) = \frac{1}{\sqrt{2\pi}} \int_0^\infty dm^2 < \mu, J_1, J_1' | m^2 > A_\mu(x, J_1, m^2)\rho(x, J_1', m^2), \qquad (10)$$

where J_1 and J_1', as in the case of spinor field, are the usual and intrinsic quantum numbers, respectively. $A_\mu(x, J_1, m^2)$ and

$\rho(X, J_1', m^2)$ $(A_\mu = A_\mu^+ + A_\mu^-,\ \rho = \rho^+ + \rho^-)$

satisfy the equations (5) and (5') with $m: 0 \le m^2$. The Green function corresponding to $B_\mu^{(s)}(\xi)$ takes the form

$$D^{c}_{\mu\nu}(\xi) = \frac{\pm i}{(2\pi)^8} \int d\lambda\, H_{\mu\nu}(K^2) \frac{e^{-i\lambda\xi}}{\lambda^2 + i\epsilon} \theta(\pm k_0),$$

$$\lambda = (k, K), \quad H_{\mu\nu}(K^2) = \begin{cases} g_{\mu\nu} + \dfrac{\partial_\mu \partial_\nu}{K}, & K^2 \neq 0 \\ g_{\mu\nu}, & K^2 = 0 \ . \end{cases} \tag{11}$$

In case of $k = 5, 6, 7, 8$

$$A^{(s)}_{a}(\xi) = \frac{1}{\sqrt{2\pi}} \int_0^\infty dm^2 \langle a, J_2, J_2' | m^2 \rangle \phi(x, J_2, m^2) V_a(X, J_2', m^2) \ , \tag{12}$$

where ϕ and V_a are the fields satisfying (6) and (6') with $0 \le m^2$. Then the corresponding Green function is

$$D^{c}_{\alpha\beta}(\xi) = \frac{1}{(2\pi)^8} \int d\lambda\, \mathcal{H}_{\alpha\beta}(K^2) \frac{e^{-i\lambda\xi}}{\lambda^2 + i\epsilon} \theta(\pm k_0),$$

$$\lambda = (k, K), \quad \mathcal{H}_{\alpha\beta}(K^2) = \begin{cases} \delta_{\alpha\beta} + \dfrac{\partial_\alpha \partial_\beta}{K^2}, & K^2 \neq 0 \\ \delta_{\alpha\beta}, & K^2 = 0 \end{cases} \tag{13}$$

It can be shown that in terms of spectral expansion, the S-matrix with the common chronological order on ordinary and intrinsic times [4] is equivalent to the one with the usual chronological order on ordinary times. Thus, the Wick theorem is perfectly used. Moreover, the unitarity of the S-matrix is previously proved.

As a generalization of electromagnetic interaction, we consider the transformation

$$B_j(\xi) \to B_j(\xi) + \partial_j f(\xi)$$

and

$$\psi(\xi) \to e^{if(\xi)} \psi(\xi) .$$

With the compensation field method we have the interaction Lagrangian

$$\mathcal{L}_I(\xi) = g \sum_{j=1}^{8} \bar{\psi}(\xi) \Gamma^j \psi(\xi) B_j(\xi) . \tag{14}$$

The Lagrangian (14) is invariant under the group O(7,1) and particularly under the Lorentz group. From a Lagrangian of the type (14) some scattering effects of electromagnetic in-

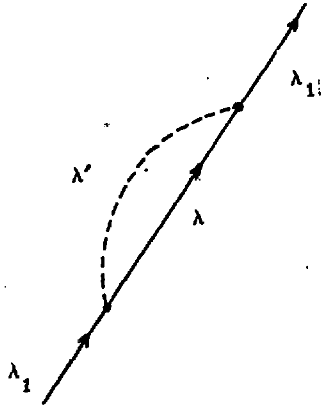

Fig. 1

teraction were calculated. It is also proved that with the propagators (9) and (11) and the S-matrix in the unified space the matrix elements take only finite values [4]. Here we illustrate some results for the self energies of the electron and photon in the first order approximation.

According to (14), we have the expression for the free electron self energy (fig. 1)

$$W^{(I)} = g^2 \bar{u}_\alpha^+ (\vec{\lambda}_1) \int_{\mathfrak{F}} \frac{\Gamma_\mu \hat{\lambda} \Gamma_\nu}{\lambda^2 + i\epsilon} \times \tag{15}$$

$$\times \frac{g_{\mu\nu}}{\lambda'^2 + i\epsilon} \theta(k_0)\theta(k_0') u_\alpha^- (\vec{\lambda}_1) d\Lambda, \quad \lambda' = \lambda_1 - \lambda,$$

where $\lambda = (k, K)$, $u(\vec{\lambda}_1)$ is the spinor amplitude of electron. After the integration over K_0, from (15) we obtain

$$W^{(I)} = -6g^2 \bar{u}_\alpha^+ (\vec{\lambda}_1) \int \frac{\hat{\lambda}_1 \theta(k_0)\theta(k_{10} - k_0)}{2 \mathcal{H}_0 (\lambda_1 - \lambda)^2} dk\, d\vec{K}\, u_\alpha^- (\vec{\lambda}_1) . \tag{16}$$

Because $\hat{\lambda}_1 u_\alpha (\vec{\lambda}_1) = 0$, hence $W^{(I)} = 0$.

Thus, in the present approach, in the first order approximation the "naked" and physic masses of a free electron are identical. Such a result is clearly obtained also for ν_e, ν_μ, μ, p, n, N_1^+, N_1°, etc.

Let us prove that the unitarity condition in this consideration is satisfied. In fact, if we put $S = 1 + iT$, we have from $SS^* = I$ the relation

$$2I_m \langle f | T | f \rangle = \langle f | T | \ell \rangle \langle f | T | \ell \rangle^* , \tag{17}$$

where $|f\rangle$ is a free fermion state and $|\ell\rangle$ is an observable one. In the first order approximation the matrix element $\langle f | T | f \rangle$ coincides with $W^{(I)}$ and is equal to zero. The states $|\ell\rangle$ are the ones of a fermion and a boson. By the quantum number and energy-momentum conservation the matrix element $\langle f | T | \ell \rangle$ is also equal to zero. Thus, the unitarity condition is satisfied.

Consider now the self energy of a free photon. Using the above-mentioned propagator, we have

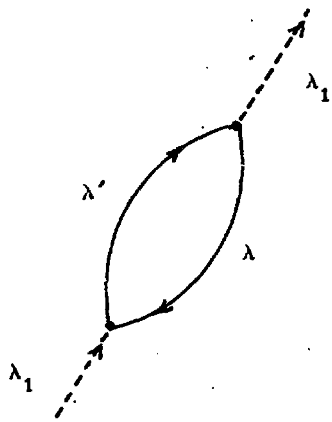

Fig.2

$$I = g^2 \int \frac{Sp[\hat{\lambda}\Gamma_{\mu} (\hat{\lambda}_1 - \hat{\lambda})\Gamma_{\nu}]\, g_{\mu\nu}}{\sqrt{k^2 - K^2(\lambda_1 - \lambda)^2}}\, dV, \quad (18)$$

which is proportional to the free photon self energy described by fig. 2.

In (18) the volume element dV takes the form

$$dV = k_0^6 dk_0 \sin^5\theta_6 \sin^4\theta_5 \sin^3\theta_4 \sin^2\theta_3 \sin\theta_2 \times$$

$$\times d\theta_1\, d\theta_2\, d\theta_3\, d\theta_4\, d\theta_5 d\theta_6, \quad (19)$$

$$0 \le k_0 \le k_{10},\ 0 \le \theta_1 \le 2\pi,$$

$$0 \le \theta_2, \theta_3, \theta_4, \theta_5 \le \pi,$$

$$\arccos\frac{\pi}{2} \ge \theta_6 \ge \frac{m_{\gamma}}{k_0}.$$

Because the mass of free photon m_{γ} equals zero, from (19) the integral (18) and hence the self energy of a free photon is equal to zero. As in the case of fermions, it can be shown that the unitarity condition in the considered approximation is also satisfied.

Thus, the electron and photon correction masses, proportional to the considered self energies are equal to zero and the "naked" and physic masses of these particles are identical.

The Lagrangian (14) can give the probabilities of the processes $\ell + N_i \to \ell' + N_i'$, $\ell + \ell' \to \Pi + \Pi'$, where ℓ is the lepton, $N_i = (p, n, N_1^+, N_1^0$ of the photon decay of pion, etc.

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Received by Publishing Department
on July 29 1981.