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WEAK CURRENTS IN THE FIELD THEORY OF UNIFIED SPACE

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In paper[1] an 8-dimensional unified space is considered, which is determined by the relation

$$dt^2 - dT^2 - (d\vec{x} + id\vec{X})(d\vec{x} - id\vec{X}) = dx^2 - dX^2 = d\xi^2 = dS^2 = 0, \qquad (1)$$

where $x = (\vec{x}, t)$ is the coordinate vector of the Minkowski space; $X = (\vec{X}, T)$ is the coordinate vector of the isospace (t and T are usual and intrinsic times, respectively); $\xi = (x, X)$.

Further, it has been proposed that in the case of free particle motion the unified space is divided into two invariant subspaces: Minkowski and intrinsic ones:

$$dx^2 = ds^2 = dX^2. \qquad (2)$$

The interval dS in (1) should be equal to zero as required due to the existence of maximum velocity (photon velocity). The Euclidean metric of the isospace is related to the time similarity requirement of ds in (2).

According to (1), we have the spinor equation

$$i\Gamma_j \partial^j \psi(\xi) = 0 \qquad (3)$$

and the Lafrangian

$$\mathfrak{L} = \bar{\psi}(\xi)\Gamma_j (\vec{\partial}^j - \overleftarrow{\partial}^j)\psi(\xi). \qquad (4)$$

The $\psi(\xi)$ in (3) is the 16-component spinor field and the matrices Γ_j (j =1,2 ... 7,8) satisfy the relations

$$\{\Gamma_j, \Gamma_k\} = 2g_{jk}, \qquad (5)$$

where g_{ik} is the metric of unified space. The matrices Γ_j can be choosen in the form

$$\Gamma_\mu = \begin{pmatrix} \gamma_\mu & 0 & 0 & 0 \\ 0 & \gamma_\mu & 0 & 0 \\ 0 & 0 & \gamma_\mu & 0 \\ 0 & 0 & 0 & \gamma_\mu \end{pmatrix}, \quad \Gamma_5 = \begin{pmatrix} 0 & 0 & 0 & \gamma_5 \\ 0 & 0 & \gamma_5 & 0 \\ 0 & \gamma_5 & 0 & 0 \\ \gamma_5 & 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_6 = \begin{pmatrix} 0 & 0 & 0 & -i\gamma_5 \\ 0 & 0 & i\gamma_5 & 0 \\ 0 & -i\gamma_5 & 0 & 0 \\ i\gamma_5 & 0 & 0 & 0 \end{pmatrix},$$

$$\Gamma_7 = \begin{pmatrix} 0 & 0 & \gamma_5 & 0 \\ 0 & 0 & 0 & -\gamma_5 \\ \gamma_5 & 0 & 0 & 0 \\ 0 & -\gamma_5 & 0 & 0 \end{pmatrix}, \quad \Gamma_8 = \begin{pmatrix} 0 & 0 & -i\gamma_5 & 0 \\ 0 & 0 & 0 & -i\gamma_5 \\ i\gamma_5 & 0 & 0 & 0 \\ 0 & i\gamma_5 & 0 & 0 \end{pmatrix}, \quad \Gamma_9 = \begin{pmatrix} \gamma_5 & 0 & 0 & 0 \\ 0 & \gamma_5 & 0 & 0 \\ 0 & 0 & -\gamma_5 & 0 \\ 0 & 0 & 0 & -\gamma_5 \end{pmatrix}. \qquad (6)$$

where γ_μ, γ_5 are the Dirac matrices and $\Gamma_9 = \Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5\Gamma_6\Gamma_7\Gamma_8$. It is shown[2] that the spinor field $\psi(\xi)$ can be one of the group of fermions: ν_e, e, p, n. The authors have proposed that leptons, as hadrons, have isospins and D-spins ($D_3 = Y/2$; Y is hypercharge). This results in the classification which is in accordance with that of Heisenberg[3]. Now we assume that the field $\psi(\xi)$ is also a field of ν_μ, μ, $N_1^+(1470)$, N_1° (1470).

It is shown that the Dirac field can be obtained from the spinor field in the unified space by unitary transformation. In fact, from (3) we have the equation

$$\Gamma^j \lambda_j v^{\mu\pm}(\vec{\lambda}) = 0 , \qquad (7)$$

where $v^{\mu\pm}(\vec{\lambda})$ are the state amplitudes in the Fourier expansion

$$\psi^{\pm}(\xi) = (2\pi)^{-7/2} \int e^{\pm i\lambda\xi} \sum_{\mu=1}^{8} a_\mu^{\pm}(\vec{\lambda}) v^{\mu\pm}(\vec{\lambda}) d\vec{\lambda} . \qquad (8)$$

In (7) λ is an 8-dimansional momentum: $\lambda = (p, P)$, where p is the usual momentum 4-vector and P is the intrinsic one; $d\vec{\lambda} = d\vec{p}\, d\vec{P}$.

Let us consider the unitarity transformation

$$e^{\pm iz(|P|)} = \frac{1}{\sqrt{2}}\left(1 \pm \frac{\Gamma_a P_a}{|P|}\right). \qquad (9)$$

Operating on (6) by (8), we have

$$(\Gamma^k p_k \pm |P|) v^{\mu\pm}(\vec{p}, |P|) = 0 , \qquad (10)$$

which takes the Dirac equation form. The quantity P takes values in the interval: $0 \le |P| \le \infty$. It is equal to a fixed mass for the case of a free particle[2]. In (9) $v^\mu(\vec{p}, |P|)$ is called the amplitude in terms of e^{iz}. In this presentation the spinor field in the unified space can be written in the form (ψ_{ν_e}, ψ_e, ψ_p, ψ_n) in which the spinor fields ψ_{ν_e}, ψ_e, ψ_p, ψ_n, are the 4-component spinors.

Consider transformations in the intrinsic space. It is clear that the transformation generators satisfy the following commutation relations

$$[\tau_j, \tau_k] = 2i\epsilon_{jk\ell}\tau_\ell , \quad [\tau_j, D_k] = 2i\epsilon_{jk\ell} D_\ell , \quad [D_j, D_k] = 2i\epsilon_{jk\ell}\tau_\ell , \qquad (11)$$
$$(i, k, \ell = 1, 2, 3) ,$$

where τ_1, τ_2 and τ_3 are the components of the vactor $\vec{\tau}$ formed by rotations on the planes (ξ_6, ξ_7), (ξ_7, ξ_5) and (ξ_5, ξ_6); the components D_1, D_2 and D_3 of the vector $\vec{D}$ correspond to rotations on the planes (ξ_8, ξ_5), (ξ_8, ξ_6) and (ξ_8, ξ_7), respectively.

It can be shown that the fields $\psi(\vec{\xi}, t)$ and $\psi(\vec{\xi}', t)$ satisfy the commutation relations

$$[\psi_j(\vec{\xi}, t), \overset{*}{\psi}_k(\vec{\xi}', t)] = \delta_{jk}\,\delta(\vec{\xi}-\vec{\xi}'). \tag{12}$$

Note that algebra in (11) is not changed in the transformation $\vec{D} \to \Gamma_9 \vec{D} \equiv \vec{\tau}^{(5)}$.

Introducting the operators

$$\vec{T} = \frac{i}{2}\int \overset{*}{\psi}(\xi)\vec{\tau}\psi(\xi)\,d\vec{\xi}, \quad \vec{T}^{(5)} = \frac{i}{2}\int \overset{*}{\psi}(\xi)\,\vec{\tau}^{(5)}\psi(\xi)\,d\xi \tag{13}$$

and using (11) and (12), we obtain

$$[T_j, T_k] = i\epsilon_{jk\ell}T_\ell,\ [T_j, T_k^{(5)}] = i\epsilon_{jk\ell}\vec{T}_\ell^{(5)},\ [T_j^{(5)}, T_k^{(5)}] = i\epsilon_{jk\ell}T_\ell, \tag{14}$$

which completely agree with the Gell-Mann commutators [4].

Let us consider the currents connected with the operators $\vec{\tau}$ and $\vec{\tau}^{(5)}$ .Considering the gauge transformation

$$\psi(\xi) \to \exp(i\Lambda(x)\,Q)\psi(\xi), \tag{15}$$

where (x) is a function of usual coordinates, $\Lambda(x) \ll 1$ (here we consider 4-dimensional currents). If Q is a hermitian operator, then we obtain the following currents

$$J_\mu = \frac{\delta \mathfrak{L}}{\delta\Lambda(x)_{,\mu}} = -\frac{1}{2}\overset{*}{\psi}\Gamma_4\Gamma_\mu Q\psi - \frac{1}{2}\overset{*}{\psi}Q\Gamma_4\Gamma_\mu\psi\,. \tag{16}$$

If we put $Q=\vec{\tau}$, we have the currents

$$V_\mu = -\bar{\psi}\Gamma_\mu\vec{\tau}\psi\,, \tag{17}$$

and if $Q=\vec{\tau}^{(5)}$,the currents are

$$A_\mu = -\bar{\psi}\Gamma_\mu\mathcal{J}_5\vec{\tau}\psi, \tag{18}$$

where the matrix $\mathcal{J}_5$ takes the form

$$\mathcal{J}_5 = \begin{pmatrix} \gamma_5 & 0 & 0 & 0 \\ 0 & \gamma_5 & 0 & 0 \\ 0 & 0 & \gamma_5 & 0 \\ 0 & 0 & 0 & \gamma_5 \end{pmatrix}. \tag{19}$$

The currents are vector ones in (17) and axial vector ones are in (18).

Thus, the gauge transformation $\psi(\xi) \to \exp\{[\vec{\tau} + \vec{\tau}^{(5)}]\Lambda(x)\}\psi(\xi)$ gives the vector-axial vector currents $J^{V-A} \equiv J$

$$\vec{J} = -\bar{\psi}\Gamma_\mu(1+\mathcal{J}_5)\vec{\tau}\psi\,. \tag{20}$$

From (20) we have the charge currents

$$J_k^{\pm} = -2[\bar{\nu}_e o_k e + \bar{\nu}_\mu o_k \mu + \bar{p}\, o_k n + \bar{N}_1^{+} o_k N_1^{\circ}], \tag{21}$$

where $o_k = \gamma_k (1+\gamma_5)$, and the neutral currents

$$J^3_k = -\bar{\nu}_e o_k \nu_e + \bar{e} o_k e - \bar{\nu}_\mu o_k \nu_\mu + \bar{\mu} o_k \mu - \bar{p} o_k p + \bar{n} o_k n - \\ - \bar{N}^+_1 o_k N_1 + \bar{N}^o_1 o_k N_1 . \qquad (22)$$

For the expressions (21) and (22) the relations between $\vec{?}$ and Γ_j (in (5)) are used.

Eliminating the lepton currents from (21) and (22), the lepton weak interaction Lagrangian can be formed as follows

$$\mathfrak{L}_w = \frac{G}{\sqrt{2}} \frac{(J\ \ )^2}{4} = \frac{G}{\sqrt{2}} \{[\bar{\nu}_e o_k e + \bar{\nu}_\mu o_k \mu][\bar{e} o_k \nu_e + \bar{\mu} o_k \nu_\mu] + \\ + \frac{1}{4}[\bar{e} o_k e - \bar{\nu}_e o_k \nu_e + \bar{\mu} o_k \mu - \bar{\nu}_\mu o_k \nu_\mu]^2 \}, \qquad (23)$$

which is in good agreement with the results of other authors /5/.

Generalizing (23), we have the weak interaction Lagrangian involving the participation of hadrons.

$$\mathfrak{L}_w = \frac{G}{\sqrt{2}} \{ J^+ J^- + (J^3)^2 \} . \qquad (24)$$

Thus, the field of the unified space involves the above-mentioned currents. Besides ordinary momenta, the intrinsic momenta of observed (free) particles are also figured in the matrix elements of such currents. The masses of such particles are equal to the modules of the corresponding vectors of the intrinsic momenta ($|P| = m$). Hence due to a great number of particles in experiment, it is necessary to average the result over all possible directions of the intrinsic momenta. (averaging on the mass surfaces). Doing this, the result of the matrices γ_μ and γ_5 can appear with coefficients different from unity. In the general case with hadron processes, form factors also appear if we take into account types of interactions /6/.

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