объединенный
институт
ядерных
исследований
дубна

4808/2-81

28/9-81

E2-81-499

R.P.Zaikov

QUANTUM NONLOCAL CONSERVED CHARGES FOR SUPERSYMMETRIC GENERALIZED NONLINEAR SIGMA MODELS

Submitted to "Болгарский физический журнал"

1981

1. INTRODUCTION

In paper [1] it is shown that the classical conserved nonlocal charges [2,3] survive in the case of quantization for any of the generalized nonlinear sigma models. For the O(N) sigma model this has been shown earlier in paper [4]. The existence of this infinite number of quantum nonlocal conserved charges provides the factorization of the S-matrix and the absence of the particle production [4]. Consequently, we are able to compute exactly the S-matrix [4], when there exist higher local quantum conserved charges [5,6] and the asymptotical states.

In the present paper the infinite number of quantum nonlocal conserved charges are found in the case of generalized supersymmetric nonlinear sigma models. The classical spinor conserved supercurrents for these models were found in ref. [7,8]. To find corresponding quantum supercurrents the same method is used for computing the classical conserved currents as given in ref. [9] in which the regularization procedure is performed.

This procedure consists in subtracting the singular terms from the currents and from the equations for generating functions. These singular terms are determined nonperturbatively using the supersymmetric short distance operator product expansion (SOPE). From the analysis of SOPE of one conserved spinor suppercurrent and one scalar superfield with zero scale dimension it follows that there is only one singular term (up to logarithmic terms) at short distances.

The explicit form of the second quantum conserved nonlocal charge is found. This charge is written down also in terms of the asymptotical fields. Any other nonlocal charges can be found as solutions of the equations for generating functions or from multiple commutators of the second charge.

2. QUANTUM EQUATION FOR THE GENERATING FUNCTIONS OF NONLOCAL SUPERCURRENTS

In papers [9] it is shown that the classical conserved spinor supercurrents have the following form

$$J^{(k)}_{\alpha}(x;\theta) = [A_{\alpha}(x;\theta), X^{(k)}(x;\theta)] + [(\gamma_5 A)_{\alpha}(x;\theta), X^{(k-1)}(x;\theta)], \qquad (2.1)$$

$$(k = 1,2,\dots)$$

where the functions $X^{(k)}$ ($k=0, 1, \ldots$) are $N\times N$ matrices with matrix elements transforming as scalar superfields. These functions are determined as solutions of the following supersymmetric equations

$$D_\alpha X^{(k)}(x;\theta) = (\gamma_5 D)_\alpha X^{(k-1)}(x;\theta) - i[(\gamma_5 A)_\alpha (x;\theta), X^{(k-1)}(x;\theta)], \quad (k=1,2,\ldots) \tag{2.2}$$

where we start with $X^{(0)}=0$,

$$D_\alpha = i\frac{\partial}{\partial\theta^\alpha} + (\bar\theta\gamma^\mu)_\alpha \partial_\mu \tag{2.3}$$

is the spinor supersymmetric covariant derivative and

$$A_\alpha(x;\theta) = iG^{-1}(x;\theta) D_\alpha G(x;\theta). \tag{2.4}$$

Here $G(x;\theta)$ is an $N\times N$ matrix with scalar superfield elements. As it has been shown in ref.[9], $X^{(k)}$ are generators of infinite parametric nonlocal and nonlinear transformations with respect to which the action is invariant. According to the last transformations the supercurrents (2.1) have Noether character [9]. For the particular sigma models (2.4) can be written also in the form:

$$A^{jk}_\alpha(x;\theta) = 2i[n_j(x;\theta) D_\alpha n_k(x;\theta) - D_\alpha n_j(x;\theta) n_k(x;\theta)], \tag{2.5}$$

for the $O(N)$ sigma model,

$$A^{jk}_\alpha(x;\theta) = 2i[\bar z_j(x;\theta) D_\alpha z(x;\theta) - D_\alpha \bar z_j(x;\theta) z_k(x;\theta)], \tag{2.6}$$

for the CP^{N-1} models, and

$$A^{jk}_\alpha(x;\theta) = 2i[\bar v_{j\kappa}(x;\theta) D_\alpha v_{\kappa k}(x;\theta) - D_\alpha \bar v_{j\kappa}(x;\theta) v_{\kappa k}(x;\theta)] \tag{2.7}$$

for the Grassmannian models.

In the quantum case, when $A_\alpha(x;\theta)$ and $X^{(k)}(x;\theta)$ are operator-valued functions, the currents (2.1) as well as the equations for the generating functions (2.2) are needed to be correctly determined. For this purpose the nonperturbative method of supersymmetric operator product expansions at short distances (SOPE) is applied. With this method the singular terms at short distances of the product of two operators in (2.1) and (2.2) can be determined nonperturbatively. In our case when $A_\alpha(x;\theta)$ is a conserved spinor supercurrent, i.e., $D^\alpha A_\alpha(x;\theta)=0$ its scale dimension is 1/2. For the function $X^{(k)}(x;\theta)$ we restrict ourselves to solutions of eq. (2.2) with zero scale dimensions. Then we define

$$J^{(k)}_\alpha(x_1,x_2;\theta) = [A_\alpha(x_1;\theta), X^{(k)}_\delta(x_2;\theta)] + [(\gamma_5 A)_\alpha(x_1;\theta), X^{(k-1)}_\delta(x_2;\theta)] -$$

$$- C_{\alpha}(x_1 - x_2;\theta)\tilde{X}^{(k)}_{\delta}(x_2;\theta) - (\gamma_5 C)_{\alpha}(x_1 - x_2;\theta)\tilde{X}^{k-1}_{\delta}, \qquad (2.8)$$

$$D_{\alpha} X^{(k)}_{\delta}(x_1;\theta) = (\gamma_5 D)_{\alpha} X^{(k-1)}_{\delta}(x_1;\theta) - i[(\gamma_5 A)_{\alpha}(x_1;\theta), X^{(k-1)}_{\delta}(x_2,\theta)] +$$

$$+ (\gamma_5 C)_{\alpha}(x_1 - x_2;\theta)\tilde{X}^{(k-1)}_{\delta}(x_2;\theta), \qquad (k = 1,2,\dots) \qquad (2.9)$$

and $X^{(0)} = 0$. Here $\delta = \sqrt{-(x_1 - x_2)^2}$, and A_{α} are given by formulas (2.4-7) where the classical fields $G(x;\theta)$ are replaced by the corresponding renormalized quantum fields, $C_{\alpha}(x;\theta)$ are the singular c -number functions which will be determined later from the short distance SOPE. The thus determined current (2.8) and equations for generating functions (2.9) are regular at $x_2 = x_1$ and consequently the limit $x_2 \to x_1$, when $x_1 - x_2$ is space-like interval, exists there.

The corresponding conserved charges are determined from the vector components of the spinor supercurrent (2.8). These currents are given by

$$J^{(k)}_{\mu}(x_1, x_2) = [A_{\mu}(x_1), \chi^{(k)}_{\delta}(x_2)] + \epsilon_{\mu\nu}[A^{\nu}(x_1), \chi^{(k-1)}_{\delta}(x_2)] -$$

$$- C_{\mu}(x_1 - x_2)\tilde{\chi}^{(k)}_{\delta}(x_2) - \epsilon \quad C^{\nu}_{\mu\nu}(x_1 - x_2)\tilde{\chi}^{(k-1)}_{\delta}(x_2), \qquad (2.10)$$

$$(k = 1,2,\dots)$$

and

$$\partial_{\mu}\chi^{(k)}_{\delta}(x_1) = \epsilon_{\mu\nu}\partial^{\nu}\chi^{(k-1)}_{\delta}(x_1) - i\epsilon_{\mu\nu}[A^{\nu}(x_1), \chi^{(k-1)}_{\delta}(x_2)] +$$

$$+ i\epsilon_{\mu\nu} C^{\nu}(x_1 - x_2)\tilde{\chi}^{(k-1)}_{\delta}(x_2), \quad (k = 1,2,\dots) \qquad (2.11)$$

where

$$\chi^{(0)}_{\delta} = 0.$$

The corresponding conserved charges are given by

$$Q^{(k)} = \lim_{\delta \to 0} Q^{(k)}_{\delta} = \lim_{\delta \to 0} \int^{\infty} dx_1 J^{(k)}_0(x_0, x_1). \qquad (2.12)$$

It can be checked that the limit in (2.12) exists and the charges $Q^{(k)}$ are conserved, indeed.

From (2.10), (2.11) and (2.12) it follows that

$$Q_{\delta}^{(k)} = \chi_{\delta}^{(k+1)}(x_1 = \infty), \tag{2.13}$$

consequently, the generating functions for $x_j \to \infty$ coincide with conserved charges.

3. SUPERSYMMETRIC SHORT-DISTANCE OPERATOR-PRODUCT EXPANSION

Consider the supersymmetric short-distance expansion of the product of two local operators

$$A(x;\theta)\,B(y,\theta) \sim \Sigma\, C_{\chi}(x-y,\theta)\, O_{\chi}(y,\theta), \tag{3.1}$$

where C_χ are singular c-number functions and O_χ are composite local operators. In the case when fields $A(x;\theta)$ and $B(x;\theta)$ have isotopic indices, i.e., if they are transformed under some internal symmetry group G, then C_χ are scalar with respect to G. The singular terms of (3.1) can be determined only from dimensional considerations. In the case under consideration, when we have a product of one conserved spinor supercurrent with scale dimension 1/2 and one scalar field $X(x;\theta)$ with zero scale dimension, we have

$$C_{\chi}(\lambda x;\lambda^{1/2}\theta) = \lambda^{-1/2+d_\chi}(x,\theta), \tag{3.2}$$

where d_χ is the scale dimension of the composite field O_χ, λ is an arbitrary parameter and it is taken into account that the dimension of the anticommuting variable θ is 1/2. Consequently, from (3.2) it follows that the singular terms at short distances correspond only to the composite (which can be constructed from A_α and X) fields with scale dimensions $d_\chi = 0$ and 1/2. The terms with dimension 1/2 give logarithmic singularity which is cancelled in the charges and consequently such will not be considered here.

The supersymmetric c-number coefficient function $C_\alpha(x_1-x_2;\theta)$ corresponding to the terms with zero scale dimensional can be written in the following form

$$C_{\alpha}(x;\theta) = (\gamma_5 D)_{\alpha} \ln\mu|x|, \tag{3.3}$$

where μ is a parameter with the dimension of mass and D_α is the spinor covariant derivative (2.3). It can be checked that (3.3) is a conserved spinor, i.e.,

$$D^{\alpha} C_{\alpha} = 0,$$

and is invariant with respect to the supertransformation, i.e.,

$$(S^1_\alpha + S^2_\alpha) C_\beta(x_1 - x_2;\theta) = (\gamma_5 D)_\beta (\bar\theta\gamma^\mu)_\alpha (\partial^1_\mu + \partial^2_\mu) \ln\mu|x_{12}| = 0 .$$

Here $S^{1,2}_\alpha$ are the generators of the supertransformations anticommuting with D_α.

Substituting (3.3) into (3.1) we have

$$J_\alpha(x_1,\theta) X(x_2,(\theta)) = (\gamma_5 D^1)_\alpha \ln\mu|x_{12}| \tilde X(x_2,\theta) + \text{ reg. terms}, \qquad (3.4)$$

where $\tilde X$ is a scalar or pseudoscalar superfield with zero scale dimension. From (3.4) for vector component we have

$$J_\mu(x_1)\chi(x_2) = \epsilon_{\mu\nu}\partial^\nu_1 \ln\mu|x_{12}|\tilde\chi(x_2) + \text{ reg. terms}, \qquad (3.5)$$

where $\tilde\chi(x_2)$ is a pseudoscalar component of $\tilde X(x,\theta)$. The first nontrivial function $\tilde X$ is given by the formula

$$\tilde\chi^{(2)} = \int_{-\infty}^{x_1} dy_1 A_1(x_0, y_1) . \qquad (3.6)$$

Consequently, the l.h.s. of (3.5) is transformed also as a vector under spice reflections as the r.h.s. The normalization coefficient in (3.5) has the same value as in paper [4].

4. EXPLICIT FORM OF QUANTUM CONSERVED CHARGES

The explicit form of conserved charges (2.12) to be found requires the generating function to be derived as a solution of eqs. (2.9). If we find the second charge $Q^{(2)}$, then using the commutation relation we are able also to derive and other conserved charge. From (2.10) it follows that it is necessary to determine the function $\chi^{(2)}(x)$ from (2.11). It can be checked that this function is given by

$$\chi^{(2)}(x) = \int_{-\infty}^{x_1} dy_1 [g^{-1}\partial_0 y(x_0, y_1) - \frac{i}{2}\bar\psi^{-1}\gamma_0\psi(x_0,y_1)], \qquad (4.1)$$

where

$$A_\mu(x) = i g^{-1}(x)\partial_\mu g(x) + \frac{1}{2}\bar\psi^{-1}(x)\gamma_\mu\psi(x) \qquad (4.2)$$

is the vector component of the first conserved spinor supercurrent $A_\alpha(x,\theta)$ is given by (2.4-7); $g(x)$ and $\psi(x)$, respectively, are scalar and spinor components of chiral superfield $G(x;\theta)$. Substituting (4.1) into (2.10) we have

$$J^{(2)}_\mu(x_1, x_2) = [A_\mu(x_1), \chi^{(2)}_\delta(x_2)] + \epsilon_{\mu\nu}[A^\nu_\nu(x_1), C] - \frac{N-2}{2\pi}\epsilon_{\mu\nu}\partial^\nu \ln\mu|x|\tilde\chi^{(2)}(x_2) . \qquad (4.3)$$

Substituting (4.3) in (2.12) we have

$$Q_{\delta}^{(2)} = \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{x_1-\delta} dy_1 [A_0(x_0, x_1), [A_0(x_0, y_1), C]] -$$
$$- \frac{N-2}{2\pi} \ln\mu\delta \int_{-\infty}^{\infty} dx_1 [A_1(x_0, x_1), C] \tag{4.4}$$

and

$$Q^{(2)} = \lim_{\delta \to 0} Q_{\delta}^{(2)} . \tag{4.5}$$

The current (4.3) and the charge (4.4) have the same form as corresponding quantities for ordinary sigma models [1,4], however, here in $A_\mu(x)$ given by (4.2) there is the term $\bar{\psi}\sigma_\mu\psi$ found from the spinor component of the chiral field. By the method given in paper [4] it can be checked that the charge is indeed conserved, i.e.,

$$dQ^{(2)}/dx^{\circ} = 0.$$

Any of other higher conserved charges can be found from (4.4) by the multiple commutator.

5. CONSERVED CHARGES IN TERMS OF ASYMPTOTICAL FIELD

From conservation of the charges $Q^{(k)}$ if follows that they do not depend on x_0, i.e., they have the same form at any x_0. Then suppose that there exist the asymptotical fields $G_{out(in)}(x;\theta)$ with the following components for the O(N) sigma model

$$(n_j(x))_{\substack{in\\(out)}} = \frac{1}{2\pi} \int \frac{d\underline{p}}{\sqrt{m^2 + p_1^2}} [e^{i\underline{p}\underline{x}} a_j^+(\underline{p}) + e^{-ipx} a_j^-(\underline{p})] , \tag{5.1}$$

$$(\psi_j(x))_{\substack{in\\(out)}} = \frac{1}{2\pi} \int dp [e^{ipx} u_{js}^+(p) b_{sj}^+(p) + e^{-ipx} u_{sj}^-(\underline{p}) a_{sj}(\underline{p})], \tag{5.2}$$

where m is a dynamically generated mass, and for the spinor we have the following normalization conditions

$$\bar{u}_s^{\pm} \gamma_0 u_r^{\mp} = \pm \frac{m}{p_0} \delta_{sr} , \tag{5.3}$$

Substituting (5.1) and (5.2) into (4.4) using (5.3), after some computation (see [4]) and taking the limit we have

$$Q_{jk}^{(2)} = \frac{1}{N} \int \frac{d\underline{p}}{2p_0} \frac{dq}{2q_0} \epsilon(\underline{p}-\underline{q}) : \{ a_j^+(p) a_k^-(\underline{p}) - a_\ell^+(\underline{p}) a_j^-(\underline{p}) +$$

$$+ 2p_0[b_j^+(\underline{p})\, b_\ell^-(\underline{p}) - b_\ell^+(\underline{p})\, b_j^-(\underline{p})]\}\{a_\ell^+(\underline{q})\, a_k^-(\underline{q}) - a_k^+(\underline{q})\, a_\ell^-(\underline{q}) +$$

$$+ 2q_0[b_\ell^+(\underline{q})\, b_k^-(\underline{q}) - b_\ell^+(\underline{q})\, b_k^-(\underline{q})]\} + \tag{5.4}$$

$$+ \frac{N-2}{\pi N}\int \frac{dp}{p_0} \ln\left(\frac{p_0+p_1}{m}\right)\{a_j^+(\underline{p})\, a_k^-(\underline{p}) - j \leftrightarrow k + 2p_0[b_j^+(\underline{p})\, b_k^-(\underline{p}) - j \leftrightarrow k]\} .$$

The charge (5.4) and higher conserved charges which can be found by multiple commutators of (5.4) can be used for computations of the S matrix (see ref./4/).

Acknowledgements are due to V.Krivoshchekov for useful discussions.

REFERENCES

1. Zaikov R.P. JINR, E2-80-655, Dubna, 1980.
2. Luscher M., Pohlmeyer L. Nucl.Phys., 1978, B137, p. 54.
3. Brezen E. et al. Phys.Lett., 1979, 82B, p. 442.
4. Luscher M. Nucl.Phys., 1978, B135, p. 1.
5. Kulish P.P. Theor.Math.Fis., 1976, 26, p. 132.
6. Zamolodchikov A.B., Zamolodchikov Al.V. Nucl.Phys., 1978, B133, p. 525.
7. Corrigan E., Zachos C.K. Phys.Lett., 1979, 88B, p. 273.
8. Curtright T. Phys.Lett., 1979, 88B, p. 276.
9. Zaikov R.P. Bulg.J.Phys., 1980, 7, p. 155; Zaikov R.P. JINR, E2-80-118, Dubna, 1980; Zaikov R.P. JINR, E2-80-197, Dubna, 1980.

Received by Publishing Department
on July 21 1981.