объединенный
институт
ядерных
исследований
дубна

E2-81-486

D.Yu.Bardin, P.Ch.Christova,[1] O.M.Fedorenko[2]

ON THE LOWEST ORDER ELECTROWEAK CORRECTIONS TO SPIN 1/2 FERMION SCATTERING.

II. The One-Loop Amplitudes

Submitted to "Nuclear Physics"

---

[1] The Department of Physics, High Pedagogical Institute, Shoumen, Bulgaria.

[2] State University of Petrozavodsk.

1981

1. INTRODUCTION

In this paper we calculate the one-loop electroweak corrections to scattering or annihilation of any two fermions with spin 1/2 within the framework of an extended Weinberg-Salam (WS) theory. The paper summarizes our efforts of many years in this direction [1]. The presentation of our final results was started in ref. [2] where we gave the total set of formulae for all one-loop diagrams necessary for the calculation of these corrections. Our main purpose is to give explicit expressions for the one-loop amplitudes of the considered processes which are free of all divergences and valid at any energies and momentum transfers.

At the end of 1979 several groups [3-8] studying the higher order effects in the WS theory discovered independently rather large radiative corrections to vector boson masses which can be measured in the nearest future. This would provide the desirable test for predictions of the WS theory not only at tree but also at the one-loop level.

This conclusion to a great extent came from the understanding of the important role of the renormalization scheme in such calculations.

It is well known, in the WS theory dealing with N_f fermion fields there are N_f+4 independent constants - N_f fermion masses (m_f), Higgs boson mass (M_χ) and three additional parameters which should be chosen arbitrary from the following set of parameters: e, g, g', M_W, M_Z, $\sin^2\theta_W = e^2/g^2$, $R = \cos^2\theta_W$, $\lambda = 2M_W/g$, $h = M_\chi^2/2\lambda^2$. Various renormalization schemes differ by a specific choice of these independent parameters, specific generation of counterterms, the way of calculations (gauge, regularization) and, what is most important, which physical processes are used to fix the renormalized parameters.

A number of renormalization schemes were studied and tested during last years. A scheme with input parameters e, M_W, M_Z, M_χ and m_f developed in refs. [9,10] is in our opinion unattractive because the renormalization performed in the 't Hooft-Feynman gauge was done off-mass-shell. As a consequence, the external lines of some particles give finite nonvanishing contributions after renormalization.

In refs. [11,12] a scheme with input parameters e, g, M_W, M_χ and m_f was investigated in the unitary gauge. We see two shortcomings in such an approach. First, the renormalized on-mass-shell weak constant g contains unphysical infrared and mass singularities. To remove them an additional renormalization should be applied. In refs.[11-13] the process $W \to \mu + \nu_\mu$ unmeasured up to now was used for this purpose. Second, the renormalization of two vector boson masses is performed unlikely in this scheme. The pole of the Z-boson propagator is not at the physical mass but at a mass which is calculated in the considered perturbation order.

Two of us[4,14] have developed a modification of this scheme using the requirement of zero one-loop corrections to the total muon lifetime as the first point for the fixation of physical parameters g and M_W and the experimentally measured[15] quantity for $\sin^2\theta_W$ as the second point.

During 1979-1980 two groups of authors[16,17] working in the ´t Hooft-Feynman gauge tested several renormalization schemes taking various sets of input parameters (either e , M_W, M_Z or g , $\sin\theta_W$, M_W) and various physical processes to fix them subsequently. The absence of a conventional renormalization scheme produces essential disadvantages for comparing results and conclusions obtained by different authors.

From 1980 one can notice a tendency to work in the unique on-mass-shell renormalization scheme[7,8,18,19] with the input parameters e , M_W , M_Z , M_χ and m_f. In this scheme one has only one independent coupling constant, the electric charge e for which the usual renormalization condition known from QED can be easily extended to the WS theory and conventional definition at the zero momentum transfer (Tompson formula) can also be adopted. In such a case all expansions in perturbative calculations are done in the only constant α which provide great advantages at higher order calculations. In the latter scheme the renormalization of all masses is done equivalently, the renormalized masses being physical particle masses. In this scheme the Weinberg parameter $\sin^2\theta_W$ is equal by definition to $1-M_W^2/M_Z^2$, and its renormalization is defined by subtractions on heavy-vector-boson mass-shell. Therefore, one can say that it is defined at $|q| \sim M_W$[7]. The weak charge g is also a depending quantity, by definition $g = e/\sin\theta_W$.

This renormalization scheme, which we consider as a more suitable for the higher order calculations in the WS theory, is supplemented in this paper by an arbitrary unitary mixing of fermion fields. Following an approach suggested in ref.[20] we consider mixing matrix elements as finite phenomenological parameters which should not be renormalized. As such parameters we consider also the fractional numbers defining quark charges. We introduce a simple matrix form to write down the renormalization procedure.

Using this renormalization scheme in the unitary gauge described in section 2 of this paper, we present in section 3 the example of general expressions for one-loop amplitudes of scattering or annihilation of any two fermions mediated by W-boson. All the calculations are done within the dimensional regularization scheme [21]. As in ref. [2] we keep the most economical way of exposition of results using as few words as possible. In section 3 each item begins with the figure of diagrams contributing to the considered part of the amplitude followed by a formula for the counterterm and for the corresponding self-energy or vertex function. At the end of each item we write down the contribution of diagrams to the form factors of the amplitude. References to formulae from our preceding paper [2] are indicated by Roman I.

All other one-loop fermion amplitudes are presented in journal version of this paper.

2. FERMION MIXING, RENORMALIZATION CONSTANTS

We consider an extended $SU(2) \otimes U(1)$ theory with arbitrary number of left-handed lepton doublets and with the same number times three of quark doublets. All right-handed components are corresponding singlets. To cancel Adler anomalies we adopt for charges of all qoark doublets $\binom{u}{d}$ the conventional requirement

$$\sum_{i=1}^{3} (Q_u^i + Q_d^i) = 1 , \tag{2.1}$$

where i is the colour index. We adopt also that weak and electromagnetic currents do not mix quarks with different colours.

Let f be the column of all fermions

$$f = \binom{f^u}{f^d} , \tag{2.2}$$

with f^u and f^d to be columns of all "up" and "down" fermions. Let K be an arbitrary unitary mixing matrix

$$K = \begin{pmatrix} I_a & 0 & 0 & 0 \ldots \\ 0 & K_b & 0 & 0 \ldots \\ 0 & 0 & I_{a'} & 0 \ldots \\ 0 & 0 & 0 & K_{b'} \ldots \\ \ldots & \ldots & \ldots & \ldots \end{pmatrix}, \qquad K K^+ = K^+ K = I , \tag{2.3}$$

mixing for definiteness down-fermions

$$f^{d'} = K f^d , \qquad \bar{f}^{d'} = \bar{f}^d K^+ . \tag{2.4}$$

It has a quasidiagonal structure where $I_a, I_{a'}, \ldots$ are unit matrices acting in subspaces (if exist) of nonmixed down fermions and $K_b, K_{b'}, \ldots$ are unitary submatrices mixing fermions in separate subspaces (e.g., lepton and quark mixing).

To obtain the fermion mass term in the Lagrangian, it is necessary to consider the interaction of fermions with the scalar doublet before spontaneous breaking of $SU(2)\otimes U(1)$ symmetry

$$L_{ff\phi} = -(\bar{f}^u, \bar{f}^d K^+)_L [\phi^c G^u f^u_R + \phi G^d f^d_R] + \text{h.c.} \qquad (2.5)$$

Here the matrix element G_{ij} is the Yukawa coupling of an i-th left-handed doublet and j-th right-handed singlet with the scalar doublet ϕ. In the unitary gauge spontaneous symmetry breaking leads to the substitution

$$\phi \to \frac{\lambda+\chi}{\sqrt{2}} \binom{0}{1}, \qquad \phi^c \to \frac{\lambda+\chi}{\sqrt{2}} \binom{1}{0}, \qquad (2.6)$$

where $\lambda/\sqrt{2}$ is the vacuum expectation value of ϕ and χ is the Higgs boson field. Thus, fermions acquire masses

$$L_{ff\phi} \to -(1+\frac{\chi}{\lambda}) [\bar{f}_L M f_R + \bar{f}_R M^+ f_L], \qquad (2.7)$$

with the following mass matrix

$$M = \frac{\lambda}{\sqrt{2}} \begin{pmatrix} G^u & 0 \\ 0 & K^+ G^d \end{pmatrix}, \qquad (2.8)$$

which is in general nondiagonal.

We introduce further the fermion-field renormalization matrices

$$f_{0L} = \sqrt{Z_{fL}}\, f_L, \qquad f_{0R} = \sqrt{Z_{fR}}\, f_R, \qquad (2.9)$$

and the fermion mass renormalization matrix Z_{m_f} with the dimension of mass

$$M_0 = (\sqrt{Z_{fL}}^+)^{-1} Z_{m_f} (\sqrt{Z_{fR}})^{-1}. \qquad (2.10)$$

Let the renormalized mass term in the Lagrangian contain the diagonal mass matrix m_f of physical fermion masses

$$-\bar{f} m_f f. \qquad (2.11)$$

Hence, the mass counterterm is

$$-[\bar{f}_L Z_{m_f} f_R + \bar{f}_R Z^+_{m_f} f_L - \bar{f} m_f f]. \qquad (2.12)$$

Other renormalization constants are introduced in the following manner

$$W_0 = \sqrt{Z}_W W, \qquad e_0 = (\sqrt{Z}_A)^{-1} e,$$

$$Z_0 = \sqrt{Z}_Z Z, \qquad M^2_{0W} = Z_{M_W} Z^{-1}_W M^2_W, \tag{2.13}$$

$$A_0 = \sqrt{Z}_A A + \sqrt{Z}_M Z, \qquad M^2_{0Z} = Z_{M_Z} Z^{-1}_Z M^2_Z,$$

$$\chi_0 = \sqrt{Z}_\chi \chi. \qquad M^2_{0\chi} = Z_{M_\chi} Z^{-1}_\chi M^2_\chi.$$

The renormalizations of the other parameters will be a consequence of the above renormalizations. Defining

$$R_0 = M^2_{0W} / M^2_{0Z}, \qquad R = M^2_W / M^2_Z, \tag{2.14}$$

we derive the renormalization of the parameter $R = \cos^2\theta_W$

$$R_0 = R + \delta R, \quad \frac{\delta R}{R} = \frac{Z_{M_W} Z^{-1}_W}{Z_{M_Z} Z^{-1}_Z} - 1. \tag{2.15}$$

Similarly, defining

$$g_0 = e_0 (1 - R_0)^{-1/2}, \quad g = e(1-R)^{-1/2}, \tag{2.16}$$

we obtain the renormalization of the weak charge

$$g_0 = g + \delta g, \quad \frac{\delta g}{g} = Z_A^{-1/2} (1 - \frac{\delta R}{1-R})^{-1/2} - 1. \tag{2.17}$$

Within the one-loop approximation we find the following set of expressions for the introduced renormalization constants

$$\sqrt{Z}^+_{f_L} \sqrt{Z}_{f_L} - I = \frac{g^2}{16\pi^2} \{ (1-R) Q^2_f [2P + 4P_{IR} - 4 + 3\,\mathrm{Ln}\frac{m^2_f}{M^2_W}] +$$
$$+ [(1 + (1-R) | Q_f |) \frac{1}{M^2_W} m^2_f - \frac{3}{2} \frac{1}{M^2_W} \begin{pmatrix} K m^2_d K^+ & 0 \\ 0 & K^+ m^2_u K \end{pmatrix}] P + \tag{2.18}$$
$$+ \frac{3}{2} \frac{(1-R)^2}{R} Q^2_f - \frac{3}{2} \frac{1-R}{R} | Q_f | + (\frac{3}{4} + \frac{3}{8} \frac{1}{R}) I \},$$

$$\sqrt{Z}^{+}_{fR}\sqrt{Z}_{fR}-I=\frac{g^2}{16\pi^2}\{(1-R)Q_f^2[2P+4P_{IR}-4+3\,Ln\frac{m_f^2}{M_W^2}]+$$

$$+(1-(1-R)|Q_f|)\frac{1}{M_W^2}m_f^2P+\frac{3}{2}\frac{(1-R)^2}{R}Q_f^2\}, \qquad (2.19)$$

$$Z_{m_f}-m_f=m_f\frac{g^2}{16\pi^2}\{(1-R)Q_f^2[8P+4P_{IR}-8+6\,Ln\frac{m_f^2}{M_W^2}]+$$

$$+[6\frac{(1-R)^2}{R}Q_f^2-3\frac{1-R}{R}|Q_f|+\frac{1}{4}\frac{1}{M_W^2}m_f^2]P+\frac{1}{2}t-\frac{(1-R)^2}{R}Q_f^2+ \qquad (2.20)$$

$$+\frac{1}{2}\frac{1-R}{R}|Q_f|+[-3\frac{(1-R)^2}{R}Q_f^2+\frac{3}{2}\frac{1-R}{R}|Q_f|]\ell n R\}.$$

Here m_u^2 and m_d^2 are diagonal matrices of the fermion mass squared, Q_f is the diagonal matrix of fermion charges and $Ln(m_f^2/M_W^2)$ is an $N_f\times N_f$ – diagonal matrix with elements $\ell n(m_i^2/M_W^2)$. In eq. (2.20) "t" stands for the contribution of tadpoles. By tradition we include also the constants coming from bubble diagrams. Both contributions cancel, however, in all calculated below renormalized amplitudes.

It is seen from eqs. (2.18)-(2.20) that matrices Z_{m_f} and $\sqrt{Z}_{f_R}$ are real and diagonal in the one-loop approximation but it is not the case for the matrix $\sqrt{Z}^{+}_{f_L}\sqrt{Z}_{f_L}$. In the one-loop approximation we can accept [20] that $\sqrt{Z}_{f_L}$ is a hermitian matrix. Then

$$\sqrt{Z}_{f_L}-I=\frac{1}{2}(\sqrt{Z}_{f_L}\sqrt{Z}_{f_L}-I). \qquad (2.21)$$

Further we write down the other renormalization constants

$$\frac{\delta M_W^2}{M_W^2}=Z_{M_W}-Z_W=\frac{g^2}{16\pi^2}[(\frac{34}{3}-\frac{1}{3}N_f-\frac{3}{2}\frac{1}{R}+\frac{1}{M_W^2}Tr\,m_f^2)P+t+W(-1)]. \qquad (2.22)$$

$$\frac{\delta M_Z^2}{M_Z^2}=Z_{M_Z}-Z_Z=\frac{g^2}{16\pi^2}[(14R-\frac{7}{3}-\frac{11}{6}\frac{1}{R}-\frac{8}{3}\frac{(1-R)^2}{R}Tr\,Q_f^2- \qquad (2.23)$$

$$-\frac{1}{3}(2-\frac{1}{R})N_f+\frac{1}{M_W^2}Tr\,m_f^2)P+t+Z(-1)].$$

$$Z_W - 1 = \frac{g^2}{16\pi^2}[(-2R - \frac{20}{3} + \frac{1}{3}N_f)P + 4(1-R)P_{IR} + W^F(-1)]. \tag{2.24}$$

$$Z_Z - 1 = \frac{g^2}{16\pi^2}[(-14R + \frac{14}{3} + \frac{2}{3}\frac{1}{R} + \frac{8}{3}\frac{(1-R)^2}{R}TrQ_f^2 + \frac{1}{3}(2 - \frac{1}{R})N_f)P + Z^F(-1)], \tag{2.25}$$

$$Z_A - 1 = \frac{e^2}{16\pi^2}[(-14 + \frac{8}{3}TrQ_f^2)P + \frac{2}{3}(1 + 2TrQ_f^2 Ln\frac{m_f^2}{M_W^2})]. \tag{2.26}$$

$$\frac{\delta R}{R} = \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} = \frac{g}{16\pi^2}\{[14(1-R) + \frac{1}{3}\frac{1-R}{R}(1-N_f) +$$

$$+ \frac{8}{3}\frac{(1-R)^2}{R}TrQ_f^2]P + W(-1) - Z(-1)\}. \tag{2.27}$$

$$\frac{\delta g}{g} = \frac{1}{2}[\frac{\delta R}{1-R} - (Z_A - 1)] = \frac{1}{2}\frac{g^2}{16\pi^2}[\frac{43}{3} - \frac{1}{3}N_f)P -$$

$$-\frac{2}{3}(1-R)(1 + 2TrQ_f^2 Ln\frac{m_f^2}{M_W^2}) + \frac{R}{1-R}(W(-1) - Z(-1))]. \tag{2.28}$$

$$\sqrt{Z}_M = \frac{eg}{16\pi^2}R^{-1/2}[(14R - \frac{7}{3} - \frac{1}{6}\frac{1}{R} - \frac{1}{3}N_f + \frac{8}{3}(1-R)TrQ_f^2)P + M(-1)]. \tag{2.29}$$

$$\frac{\delta M_\chi^2}{M_\chi^2} = Z_{M_\chi} - Z_\chi = \frac{g^2}{16\pi^2}[(3 + \frac{3}{2}\frac{1}{R} - \frac{15}{4}r_W - 9r_W^{-1} - \frac{9}{2}R^{-1}r_Z^{-1} - \tag{2.30}$$

$$-\frac{1}{M_W^2}Trm_f^2 + 6\frac{1}{M_W^2 M_\chi^2}Tr\,m_f^4)P + \frac{3}{2}t + \chi(-1)],$$

$$Z_\chi - 1 = \frac{g^2}{16\pi^2}[(-3 - \frac{3}{2}\frac{1}{R} + \frac{3}{2}r_W + \frac{1}{M_W^2}Tr\,m_f^2)P + \chi^F(-1)]. \tag{2.31}$$

Here $r_W = M_\chi^2/M_W^2$ and $r_Z = M_\chi^2/M_Z^2$. Throughout the calculations we have used the simple equality $Tr|Q_f| = N_f/2$. Explicit equations for finite constants $W(-1)$, $W^F(-1)$, $Z(-1)$, $Z^F(-1)$, $M(-1)$, $\chi(-1)$ and $\chi^F(-1)$ are listed in journal version.

3. ELECTROWEAK CORRECTIONS TO SCATTERING PROCESSES MEDIATED BY $W^{\mp}$

In the tree approximation the scattering (annihilation) of any two fermions through the W-exchange is described only by one Feynman diagram

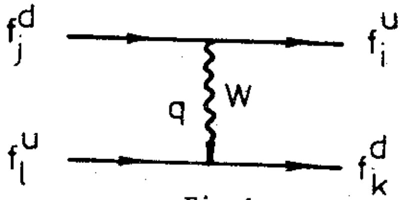


Fig. 1

with the amplitude

$$M^{CC}_{0\,ij,k\ell} = C^{ij,k\ell}_{M_0^W} \left(\delta_{\alpha\beta} + \frac{q_\alpha q_\beta}{M_W^2} \right) O_\alpha \otimes O_\beta \,, \tag{3.1}$$

where $q = p_j - p_i$, $O_\alpha = \gamma_\alpha (1+\gamma_5)$ and

$$C^{ij,k\ell}_{M_0^W} = -i \frac{g^2}{8} (2\pi)^4 \frac{1}{q^2 + M_W^2} K_{ij} K^+_{k\ell} \,. \tag{3.2}$$

It is convenient to include the one-loop corrections using the representation in terms of form factors

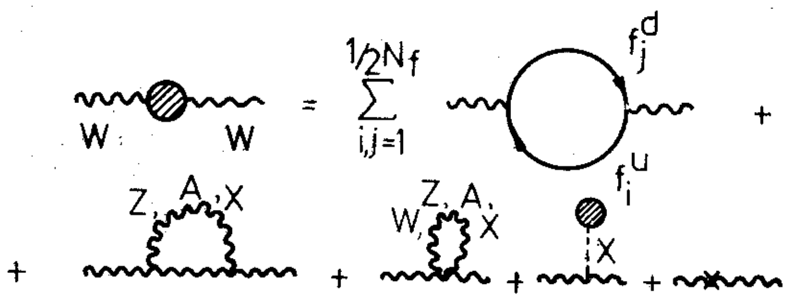
$$M^{CC}_{ij,k\ell} = C^{ij,k\ell}_{M_0^W} [O_\alpha \otimes O_\alpha F_1^W (q^2, S) + \frac{1}{M_W^2} \hat{q}(1+\gamma_5) \otimes \hat{q}(1+\gamma_5) F_2^W (\hat{q}^2, S)], \tag{3.3}$$

where

$$F_i^W(q^2, S) = 1 + \text{one-loop corrections}$$

$$(S = -(p_j + p_\ell)^2). \tag{3.4}$$

3.1. W-Boson Self-Energy Function and its Contribution to the Amplitude



$$\Pi^W_{\alpha\beta}(q)|_{Ren} = \Pi^W_{\alpha\beta}(q) + c.t.\,, \tag{3.5}$$

$$c.t. = -i\delta_{\alpha\beta}[\delta M_W^2 + (Z_W - 1)(q^2 + M_W^2)] + i(Z_W - 1)\, q_\alpha q_\beta\,, \tag{3.6}$$

$$\Pi^W_{\alpha\beta}(q) = \delta_{\alpha\beta}\,\Pi^W(q^2) + q_\alpha q_\beta\, \Theta^W(q^2)\,, \tag{3.7}$$

$$\Pi^W(q^2) = i\frac{g^2}{16\pi^2} M_W^2 \{ [((\frac{1}{6}R^2 \frac{q^2}{M_W^2} - \frac{3}{2}R^2 + \frac{1}{2}R - \frac{5}{3}) \frac{q^2 + M_W^2}{M_W^2} - 2R - \frac{20}{3} + \tag{3.8}$$

$$+ \frac{1}{3}N_f) \frac{q^2 + M_W^2}{M_W^2} + \frac{34}{3} - \frac{1}{3}N_f - \frac{3}{2}\frac{1}{R} + \frac{1}{M_W^2}\mathrm{Tr}\, m_f^2] P + t + W(\frac{q^2}{M_W^2}) \},$$

$$\Theta^W(q^2) = i\frac{g^2}{16\pi^2} [(-\frac{1}{6}R^2 \frac{q^2}{M_W^2} + \frac{4}{3}R^2 - \frac{1}{2}R + \frac{5}{3}) \frac{q^2 + M_W^2}{M_W^2} + R^2 + \frac{3}{2}R + \frac{25}{3} - \frac{1}{3}N_f] P. \tag{3.9}$$

Contributions to form factors:

$$F_1^W(q^2) = \frac{(-i)\Pi^W(q^2) - \delta M_W^2}{q^2 + M_W^2} - (Z_W - 1) = \frac{g^2}{16\pi^2} \{ [(\frac{1}{6}R^2 \frac{q^2}{M_W^2} - \frac{3}{2}R^2 + \frac{1}{2}R - \tag{3.10}$$

$$- \frac{5}{3}) \frac{q^2 + M_W^2}{M_W^2} - 2R - \frac{20}{3} + \frac{1}{3}N_f] P + D_W(\frac{q^2}{M_W^2}) \} - (Z_W - 1),$$

$$F_2^W(q^2) = \frac{(-i)\Pi^W(q^2) - \delta M_W^2}{q^2 + M_W^2} - (Z_W - 1) + \frac{(-i)\Pi^W(q^2) - \delta M_W^2}{M_W^2} +$$

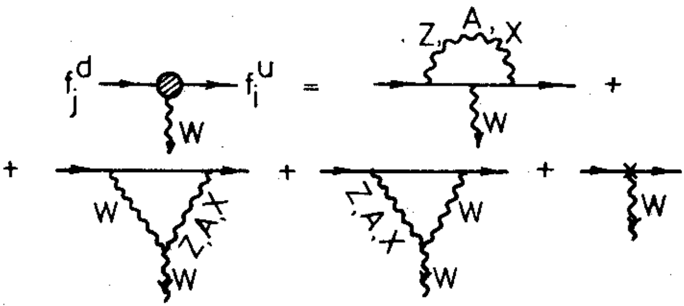
$$+ \frac{(-i)\Theta^W(q^2)}{M_W^2}(q^2 + M_W^2) = \frac{g^2}{16\pi^2} [-\frac{2}{3}R^2 \frac{q^2 + M_W^2}{M_W^2} - 2R - \frac{20}{3} + \frac{1}{3}N_f] P - (Z_W - 1), \tag{3.11}$$

where

$$D_W(\frac{q^2}{M_W^2}) = [W(\frac{q^2}{M_W^2}) - W(-1)] \frac{M_W^2}{q^2 + M_W^2}\,. \tag{3.12}$$

The function $W(q^2/M_W^2)$ is given by formula (A.1).

3.2. Vertex Function and its Contribution to the Amplitude



$$(\Gamma_\rho^W(q))_{ij}^{Ren} = (\Gamma_\rho^W(q))_{ij} + c.t., \tag{3.13}$$

$$c.t. = -\frac{g}{2\sqrt{2}} O_\rho \left[\left(\frac{1}{2}(Z_W - 1) + \frac{\delta g}{g} \right) K + \sqrt{Z}_{u_L} K \sqrt{Z}_{d_L} - K \right]_{ij}, \tag{3.14}$$

$$(\Gamma_\rho^W(q))_{ij} = -\frac{g}{2\sqrt{2}} K_{ij} \frac{g^2}{16\pi^2} \{ O_\rho [((-\frac{1}{6} R^2 \frac{q^2}{M_W^2} + \frac{3}{2} R^2 - \frac{1}{2} R + \frac{11}{6} - \frac{1}{4} R \frac{m_i^2 + m_j^2}{M_W^2}) \times$$

$$\times \frac{q^2 + M_W^2}{M_W^2} + 2R - \frac{29}{6} + (\frac{1}{4} - \frac{1}{2}(1-R)|Q_i|) \frac{m_i^2}{M_W^2} + (\frac{1}{4} - \frac{1}{2}(1-R)|Q_j|) \frac{m_j^2}{M_W^2}) P +$$

$$+ (1-R)|Q_i||Q_j| (2P + \ell n \frac{m_i m_j}{M_W^2} + 2(q^2 + m_i^2 + m_j^2) J(q^2, m_i^2, m_j^2) P_{IR} +$$

$$+ q^2 K(q^2, m_i^2, m_j^2) - \frac{3}{2} q^2 J(q^2, m_i^2, m_j^2)) - (1-R)|Q_i| (2\ell n \frac{m_i^2}{M_W^2} + u_1(q^2, m_i^2, m_j^2)) - \tag{3.15}$$

$$-(1-R)|Q_j| (2\ell n \frac{m_j^2}{M_W^2} + u_1(q^2, m_j^2, m_i^2)) - (1-R) u(q^2, M_W^2) + (-\frac{1}{2} + \frac{1}{4} \frac{1}{R} -$$

$$- \frac{(1-R)^2}{R} |Q_i||Q_j|) V_1(q^2, M_Z^2) + R V_2(q^2, M_W^2, M_Z^2)] + [-\frac{1}{2} R \frac{m_i m_j}{M_W^2} \frac{q^2 + M_W^2}{M_W^2} \gamma_\rho (1-\gamma_5) +$$

$$+ (\frac{1}{6} R^2 \frac{q^2}{M_W^2} - \frac{5}{6} R^2 - \frac{4}{3} + \frac{1}{4} R \frac{m_i^2 + m_j^2}{M_W^2}) \frac{1}{M_W^2} q_\rho \hat{q} (1+\gamma_5) + \frac{1}{2} R \frac{m_i m_j}{M_W^4} q_\rho \hat{q} (1-\gamma_5)] P \}.$$

Functions $J(q^2,m^2,m^2)$ and $K(q^2,m^2,m^2)$ are defined by formulae (I.2.16) and (I.3.4); functions $u_1(q^2,m_1^2,m_2^2)$ and $u_2(q^2,m_1^2,m_2^2) = -u_1(q^2,m_2^2,m_1^2)$, by formulae (I.3.20), (I.3.21) and (I.3.24), (I.3.25); and function $u(q^2,M^2)$, by (I.3.22). Functions $V_1(q^2,M_V^2)$ and $V_2(q^2,M_1^2,M_2^2)$ are defined by expressions in brackets in formulae (I.3.1) and (I.3.14).

The contribution of this vertex function to the amplitude

$$i\frac{g}{2\sqrt{2}}(2\pi)^4\frac{1}{q^2+M_W^2}(\delta_{\rho\alpha}+\frac{q_\rho q_\alpha}{M_W^2})(\Gamma_\rho^W(q))_{ij}^{Ren}\otimes K_{k\ell}^+O_\alpha. \qquad (3.16)$$

is splitted into three parts:

1. Contributions to form factors

$$F_1^W(q^2)=\frac{g^2}{16\pi^2}\Big\{[(-\frac{1}{6}R^2\frac{q^2}{M_W^2}+\frac{3}{2}R^2-\frac{1}{2}R+\frac{11}{6}-\frac{1}{4}R\frac{m_i^2+m_j^2}{M_W^2})\frac{q^2+M_W^2}{M_W^2}+$$

$$+R+\frac{10}{3}-\frac{1}{6}N_f]P-\frac{1}{3}(1-R)[1+2\mathrm{Tr}Q_f^2\mathrm{Ln}\frac{m_f^2}{M_W^2}]+\frac{1}{2}\frac{R}{1-R}(W(-1)-Z(-1))-$$

$$-(1-R)|Q_i|(\frac{3}{2}\ell n\frac{m_i^2}{M_W^2}+u_1(q^2,m_i^2,m_j^2))-(1-R)|Q_j|(\frac{3}{2}\ell n\frac{m_j^2}{M_W^2}+ \qquad (3.17)$$

$$+u_1(q^2,m_j^2,m_i^2))-(1-R)(\frac{5}{4}+u(q^2,M_W^2))+(-\frac{1}{2}+\frac{1}{4}\frac{1}{R}-$$

$$-\frac{(1-R)^2}{R}|Q_i||Q_j|)[V_1(q^2,M_Z^2)+\frac{3}{2}]+R[V_2(q^2,M_W^2,M_Z^2)+\frac{3}{2}]\Big\}+\frac{1}{2}(Z_W-1),$$

$$F_2^W(q^2)=\frac{g}{16\pi^2}[(\frac{2}{3}R^2-\frac{1}{2}R+\frac{1}{2})\frac{q^2+M_W^2}{M_W^2}+R+\frac{10}{3}-\frac{1}{6}N_f]P+\frac{1}{2}(Z_W-1). \qquad (3.18)$$

2. The factorized pure electromagnetic term with genuine infrared divergences

$$M_{0_{ij,k\ell}}^{CC}\ \frac{e^2}{16\pi^2}\{Q_i^2(2P_{IR}+\ell n\frac{m_i^2}{M_W^2})+Q_j^2(2P_{IR}+\ell n\frac{m_j^2}{M_W^2})+|Q_i||Q_j|[2(q^2+ \qquad (3.19)$$

$$+m_i^2+m_j^2)J(q^2,m_i^2,m_j^2)P_{IR}+q^2K(q^2,m_i^2,m_j^2)-\frac{3}{2}q^2J(q^2,m_i^2,m_j^2)+4]\}.$$

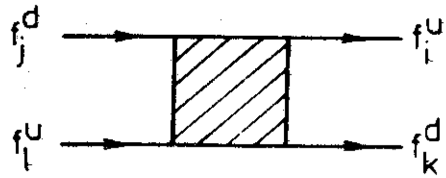
3. The rest which is not reduced to two form factors

$$C_{M_0^W}^{ij,k\ell}\ \frac{g^2}{16\pi^2}[-\frac{1}{2}R\frac{m_i m_j}{M_W^2}\frac{q^2+M_W^2}{M_W^2}]P\gamma_\alpha(1-\gamma_5)\otimes O_\alpha. \qquad (3.20)$$

3.3. The second vertex function $(\Gamma_\rho^W)^+_{k\ell}$ gives the analogous contribution to the amplitude which can be obtained from the previous one by changing $m_i \to \mu_k$ and $m_j \to \mu_i$.

$$i\frac{g}{2\sqrt{2}}(2\pi)^4\frac{1}{q^2+M_W^2}(\delta_{\alpha\rho}+\frac{q_\alpha q_\rho}{M_W^2})K_{ij}\,O_\alpha \otimes (\Gamma_\rho^W(q))^{+\,Ren}_{k\ell}\,. \qquad (3.21)$$

3.4. Two-particle exchange diagrams



(see formulae (I.4.2), (I.4.12), (I.4.14), (I.4.23), (I.4.27), (I.4.30) and (I.4.32)).

1. Contributions to form factors

$$F_1^W(q^2,S)=\frac{g^2}{16\pi^2}\{[\frac{1}{6}R^2\frac{q^2}{M_W^2}-\frac{3}{2}R^2+\frac{1}{2}R-2+\frac{1}{4}R\frac{m_i^2+m_j^2+\mu_k^2+\mu_\ell^2}{M_W^2}]\frac{q^2+M_W^2}{M_W^2}P+$$

$$(1-R)[2\frac{q^2}{M_W^2}+6-\frac{q^2+M_W^2}{M_W^2}(1+\frac{M_W^2}{q^2})\ell n|1+\frac{q^2}{M_W^2}|]-R(q^2+M_W^2)\omega(q^2,M_W^2M_Z^2)+ \qquad (3.22)$$

$$+(1-R)|Q_i|(\frac{3}{2}\ell n\frac{m_i^2}{M_W^2}+u_1(q^2,m_i^2,m_j^2))+(1-R)|Q_j|(\frac{3}{2}\ell n\frac{m_j^2}{M_W^2}+u_1(q^2,m_j^2,m_i^2))+$$

$$+(1-R)|Q_k|(\frac{3}{2}\ell n\frac{\mu_k^2}{M_W^2}+u_1(q^2,\mu_k^2\cdot\mu_\ell^2))+(1-R)|Q_\ell|(\frac{3}{2}\ell n\frac{\mu_\ell^2}{M_W^2}+u_1(q^2,\mu_\ell^2\cdot\mu_k^2))+$$

$$+B_1^W(q^2,S)\}.$$

where

$$B_1^W(q^2,S)=2[1-\frac{1}{2R}+\frac{(1-R)^2}{R}(|Q_i||Q_k|+|Q_j||Q_\ell|)]S(q^2+M_W^2)B(q^2,S;M_W^2,M_Z^2)-$$

$$-2[1-\frac{1}{2R}+\frac{(1-R)^2}{R}(|Q_i||Q_\ell|+|Q_j||Q_k|)]\{(q^2+M_W^2)(S-q^2)B(q^2,q^2-S;M_W^2M_Z^2)+$$

$$+A(q^2,q^2-S,S;M_W^2,M_Z^2)]-2(1-R)(|Q_i||Q_\ell|+|Q_j||Q_k|)[\frac{3}{2}\ell n|\frac{q^2-S}{S}|+$$

$$+\Phi\left(1+\frac{q^2-S}{M_W^2}\right)-\Phi\left(1+\frac{S}{M_W^2}\right)+2\ln\left|1+\frac{q^2}{M_W^2}\right|\ln\left|\frac{q^2-S}{S}\right|+(q^2+M_W^2)\times$$

$$\times A_0(q^2,q^2-S,S;M_W^2)]+2(1-R)\left[-\frac{3}{2}\ln\frac{|S|}{M_W^2}-2\Phi\left(-\frac{q^2}{M_W^2}\right)-2\ln\left|1+\frac{q^2}{M_W^2}\right|\times\right.$$

$$\left.\times\ln\frac{|S|}{M_W^2}+\Phi(1)-\Phi\left(1+\frac{S}{M_W^2}\right)\right], \tag{3.23}$$

$$F_2^W(q^2)=\frac{g^2}{16\pi^2}\left(-\frac{2}{3}R^2+R-1\right)\frac{q^2+M_W^2}{M_W^2}P, \tag{3.24}$$

The function $\omega(q^2,M_W^2,M_Z^2)$ is defined by eq. (I.4.6) and $B(q^2,x;M_1^2,M_2^2)$ by eq. (I.4.7). Here we introduce the notation

$$A(q^2,x,y;M_1^2,M_2^2)=\frac{1}{y}\left[-\ln\frac{|x|}{M_1M_2}-\frac{1}{2}\frac{M_1^2-M_2^2}{q^2}\ln\frac{M_1^2}{M_2^2}+\frac{1}{2}\frac{1}{q^2}L(q^2,M_1^2,M_2^2)-\right. \tag{3.25}$$

$$\left.-M_1^2M_2^2B(q^2,x;M_1^2,M_2^2)+\left(1-\frac{1}{2}\frac{q^2+M_1^2+M_2^2}{y}\right)C(q^2,x;M_1^2,M_2^2)\right],$$

$$A_0(q^2,x,y;M^2)=\frac{1}{y}\left[-\ln\frac{|x|}{M^2}+\left(1+\frac{M^2}{q^2}\right)\ln\left|1+\frac{q^2}{M^2}\right|+\left(1-\frac{1}{2}\frac{q^2+M^2}{y}\right)C_0(q^2,x;M^2)\right], \tag{3.26}$$

with $C(q^2,x;M_1^2,M_2^2)$ and $C_0(q^2,x;M^2)$ given by eqs. (I.4.9) and (I.4.18) respectively.

2. Factorized electromagnetic term

$$M_{0_{ij,k\ell}}^{CC}\frac{e^2}{16\pi^2}\{-|Q_i||Q_k|[2(-S+m_i^2+\mu_k^2)J(-S;m_i^2,\mu_k^2)P_{IR}-SK(-S,m_i^2,\mu_k^2)+$$

$$+\frac{3}{2}SJ(-S;m_i^2,\mu_k^2)+4]-|Q_j||Q_\ell|[2(-S+m_j^2+\mu_\ell^2)J(-S;m_j^2,\mu_\ell^2)P_{IR}-$$

$$-SK(-S;m_j^2,\mu_\ell^2)+\frac{3}{2}SJ(-S;m_j^2,\mu_\ell^2)+4]-|Q_i||Q_\ell|[2(k^2+m_i^2+\mu_\ell^2)\times \tag{3.27}$$

$$\times J(k^2;m_i^2,\mu_\ell^2)P_{IR}+(S-q^2)K(S-q^2,m_i^2,\mu_\ell^2)-\frac{3}{2}(S-q^2)J(S-q^2;m_i^2,\mu_\ell^2)+4]-$$

$$-|Q_j||Q_k|[2(k^2+m_j^2+\mu_k^2)J(k^2;m_j^2,\mu_k^2)P_{IR}+(S-q^2)K(S-q^2,m_j^2,\mu_k^2)-$$

$$-\frac{3}{2}(S-q^2)K(S-q^2;m_j^2,\mu_k^2)+4]\},$$

where $k^2=(p_j-p_k)^2=S-q^2-m_i^2-m_j^2-\mu_k^2-\mu_\ell^2$.

3. The rest, irreducible to form factors, is equal exactly to the sum of corresponding third-kind contributions of two vertices with opposite sign, i.e., they cancel each other.

We notice that all expressions for two-particle exchange diagrams are valid everywhere except for the point $q^2=-M_W^2$.

3.5. Finite amplitude of the reaction with W-exchange. While summing all contributions to the amplitude we shall separate the pure electromagnetic factorized term containing genuine infrared divergences. The latter will cancel at the level of calculation of the reaction cross-section when real-photon bremsstrahlung diagrams are involved.

Summing all contributions to form factors F_1^W and F_2^W we observe that all ultraviolet divergences cancel each other. The finite part of the form factor F_2^W could be neglected, because we work in the region (I.1.1). Finally we are left with the finite part of form factor F_1^W only where there are cancelled also all nonunitary (increasing with q^2 faster than $(\ell n(q^2))^2$) terms.

$$F_1^W(q^2,S)=1+\frac{g^2}{16\pi^2}\{-\frac{2}{3}(1-R)[1+2\,\mathrm{Tr}\,Q_f^2\mathrm{Ln}\frac{m_f^2}{M_W^2}]+\frac{R}{1-R}[W(-1)-Z(-1)]+$$

$$+[-1+\frac{1}{2R}-\frac{(1-R)^2}{R}(|Q_i||Q_j|+|Q_k||Q_\ell|)][V_1(q^2,M_Z^2)+\frac{3}{2}]+\bar{D}_W(\frac{q^2}{M_W^2})+ \qquad (3.28)$$

$$+2R[\bar{V}_2(q^2,M_W^2,M_Z^2)+\frac{3}{2}]+(1-R)[\frac{7}{2}-(1+\frac{M_W^2}{q^2})\ell n|1+\frac{q^2}{M_W^2}|-$$

$$-2\bar{u}(q^2,M_W^2)]-R\overline{(q^2+M_W^2)\omega}(q^2,M_W^2,M_Z^2)+B_1^W(q^2,S)\}.$$

Here the bar above symbols means that from the corresponding expression all nonunitary terms are thrown out.

APPENDIX A

1.
$$
\begin{aligned}
W\left(\frac{q^2}{M_W^2}\right) = & -\frac{2}{9}R^2\frac{q^6}{M_W^6}+\left(\frac{35}{36}R^2-\frac{7}{12}R+\frac{14}{9}\right)\frac{q^4}{M_W^4}+\left(\frac{91}{36}R^2+\frac{103}{12}-\frac{5}{18}N_f\right)\times \\
& \times\frac{q^2}{M_W^2}+\frac{5}{4}R^2+\frac{3}{4}R-\frac{59}{6}+\frac{5}{4}\frac{1}{R}-\frac{1}{3}r_W+\left(-\frac{1}{12}R^2+\frac{1}{6}R+\frac{7}{12}-\frac{2}{3}\frac{1}{R}-\frac{1}{12}\frac{1}{R^2}+\frac{1}{6}r_W\right. \\
& \left.-\frac{1}{12}r_W^2\right)\frac{M_W^2}{q^2}+\frac{1}{6}N_f\frac{q^2}{M_W^2}\ln\frac{|q^2|}{M_W^2}+(1-R)\left(\frac{5}{6}+\frac{11}{3}\frac{M_W^2}{q^2}+\frac{5}{6}\frac{M_W^2}{q^4}\right)\left(1-\frac{q^4}{M_W^4}\right)\ln\left|1+\frac{q^2}{M_W^2}\right|+ \\
& +\left[-\frac{1}{24}R^2\frac{q^6}{M_W^6}+\frac{7}{24}R(R+1)\frac{q^4}{M_W^4}+\left(\frac{13}{12}R^2+\frac{25}{12}R+\frac{1}{3}\right)\frac{q^2}{M_W^2}+\frac{13}{12}R^2-\frac{3}{4}-\frac{1}{3}\frac{1}{R}+\right. \\
& \left.+\frac{1}{24}\frac{1-R^2}{R^2}(-7R^2+50R-7)\frac{M_W^2}{q^2}+\frac{1}{24}\frac{(1-R)^3}{R^3}(R^2+10R+1)\frac{M_W^4}{q^4}\right]\ln R+ \\
& +\left[\frac{1}{24}\frac{q^2}{M_W^2}-\frac{3}{8}-\frac{1}{8}r_W+\left(-\frac{3}{8}+\frac{1}{2}r_W-\frac{1}{8}r_W^2\right)\frac{M_W^2}{q^2}+\frac{1}{24}(1-r_W)^3\frac{M_W^4}{q^4}\right]\ln r_W+ \\
& +\left[\frac{1}{24}R^2-\frac{5}{12}R(R+1)\frac{M_W^2}{q^2}+\frac{1}{24}(R^2+10R+1)\frac{M_W^4}{q^4}\right]\frac{\lambda(q^2,-M_W^2,-M_Z^2)}{M_W^6}L(q^2;M_W^2,M_Z^2)+ \\
& +\left[-\frac{1}{2}\frac{M_W^2}{q^2}+\frac{1}{24}\frac{\lambda(q^2,-M_W^2,-M_\chi^2)}{q^4}\right]\frac{1}{M_W^2}L(q^2;M_W^2,M_\chi^2).
\end{aligned}
\tag{A.1}
$$

(see eqs. (I.2.14)-(I.2.16) for functions $\lambda(q^2,-M_1^2,-M_2^2)$ and $L(q^2;M_1^2,M_2^2)$).

The authors are indebted very much to A.A.Vladimirov, V.M.Dubovik, D.V.Shirkov, M.Mueller-Preussker and D.Ebert for helpful discussions, support and critical comments in reading manuscript.

REFERENCES

1. Bardin D.Yu., Fedorenko O.M. JINR, P2-11413, P2-11414, Dubna, 1978.

2. Bardin D.Yu., Christova P.Ch., Fedorenko O.M. Nucl.Phys., 1980, B175, p.435.
3. Marciano W.J. Phys.Rev., 1979, D20, p.274; Marciano W.J., Sirlin A. Phys.Rev.Lett., 1981, 46, p.163.
4. Bardin D.Yu., Fedorenko O.M., Shumeiko N.M. Jad.Fiz., 1980, 32, p.782.
5. Veltman M. Phys.Lett., 1980, 91B, p.95.
6. Antonelli F., Consoli M., Corbo G. Phys.Lett., 1980, 91B, p.90.
7. Sirlin A. Phys.Rev., 1980, D22, p.971.
8. Aoki K.I. et al. Progr.Theor.Phys., 1980, 64, p.707; 1981, 65, p.1001.
9. Ross D.A., Taylor J.C. Nucl.Phys., 1973, B51, p.125.
10. Sakakibara S. Preprint DO-TH- 80/1, Dortmund, 1980.
11. Appelquist T.W., Primack J.R., Quinn H.R. Phys.Rev., 1973, D7, p.2998.
12. Salomonson P., Ueda Y. Phys.Rev., 1975, D11, p.2606.
13. Marciano W.J., Sirlin A. Phys.Rev., 1973, D8, p.3612.
14. Bardin D.Yu., Fedorenko O.M. Jad.Fiz., 1979, 30, p.811.
15. Prescott C. et al. Phys.Lett., 1978, 77B, p.347; 1979, 84B, p.524.
16. Veltman M. Nucl.Phys., 1977, B123, p.89; Passarino G., Veltman M. Nucl.Phys., 1979, B160, p.151; Lemoine M., Veltman M. Nucl.Phys., 1980. B164, p.445; Green M., Veltman M. Nucl.Phys., 1980, B169, p.137.
17. Consoli M. Nucl.Phys., 1979, B160, p.208.
18. Fleischer J., Jegerlehner F. Preprint BI-TP 80/18, Bielefeld, 1980; Marciano W.J., Sirlin A. Phys.Rev., 1980, D22, p.2695.
19. Aoki K.I. Soryushiron Kenkyu (Kyoto), 1979, 59, p.383.
20. Marciano W.J., Sirlin A. Nucl.Phys., 1975, B93, p.303.
21. t'Hooft G., Veltman M. Nucl.Phys., 1972, B44, p.189; Marciano W.J. Nucl.Phys., 1975, B84, p.132; Chanowitz M., Furman M., Hinchliffe I. Nucl.Phys., 1979, B159, p.225.

Received by Publishing Department
on July 14 1981.