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ядерных
исследований
дубна

E2-81-47

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Submitted to "Physics Letters"

1981

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Возможный метод проверки CP-инвариантности в осцилляциях нейтрино

Предложен метод проверки CP-инвариантности в осцилляциях нейтрино. Метод основан на сравнении /на некотором расстоянии от источника/ потоков нейтрино и антинейтрино, полученных от распадов K_L-мезонов. Показано, что в случае, если разности потоков нейтрино и антинейтрино отличны от нуля, их измерение позволило бы проверить CPT-инвариантность и получить информацию о числе типов нейтрино.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1981

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A Possible Test of CP Invariance in Neutrino Oscillations

A method is proposed for testing CP invariance in neutrino oscillations. The method is based on the comparison (at some distance from source) of fluxes of neutrinos and antineutrinos obtained from K_L-decays. Moreover, it is shown that in the case when properly normalized differences of neutrino and antineutrino fluxes are nonvanishing, their measurement would allow one to test CPT invariance and to get information on the number of neutrino types.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1981

1. The hypothesis of neutrino oscillations was made by B.Pontecorvo more than twenty years ago [1]. The verification of this hypothesis would allow us to answer such fundamental questions of neutrino physics as those of nonvanishing of neutrino masses, of neutrino mixing and of CP violation in the leptonic sector, etc.

All these problems become of special importance with the progress in grand unified theories.

Only recently experiments searching for neutrino oscillations have been carried out [2]. Quite sensitive experiments are being performed (and planned) at present [3]. Experiments on neutrino beams obtained from K_L decays are also in preparation [4]. In this note we would like to draw attention to a very favourable way of testing CP-invariance in neutrino oscillations with the help of such beams.

2. To start with let us introduce the necessary relations based on the phenomenological theory of neutrino oscillations. The charged current of the standard weak interaction theory is given by the expression

$$j_\alpha = \sum_{\ell = e,\mu,\tau,\dots} \bar{\nu}_{\ell L} \gamma_\alpha \ell_L . \tag{1}$$

If neutrino mixing takes place, we have

$$\nu_{\ell L} = \sum_i U_{\ell i} \nu_{iL} . \tag{2}$$

Here ν_i is the field operator of (Dirac or Majorana) neutrinos with mass m_i, and U is a unitary mixing matrix. For the amplitudes of $\nu_\ell \to \nu_{\ell'}$ and $\bar{\nu}_\ell \to \bar{\nu}_{\ell'}$ oscillations we have, respectively,

$$\mathcal{A}_{\nu_{\ell'};\nu_\ell}(t) = \sum_i U_{\ell' i}\, e^{-iE_i t}\, U^*_{\ell i} , \tag{3}$$

$$\mathcal{A}_{\bar{\nu}_{\ell'};\bar{\nu}_\ell}(t) = \sum_i U^*_{\ell' i}\, e^{-iE_i t}\, U_{\ell i} , \tag{4}$$

where $E_i = \sqrt{m_i^2 + \vec{p}^2}$, and $\vec{p}$ is the neutrino momentum ($|\vec{p}| \gg m_i$).

By comparing expressions (3) and (4) it is easy to see that

$$P_{\nu_{\ell'};\nu_\ell}(R,p) = P_{\bar{\nu}_\ell;\bar{\nu}_{\ell'}}(R,p) . \tag{5}$$

Here $P_{\nu_{\ell'};\nu_\ell}(R,p)$ is the probability of finding $\nu_{\ell'}$ at a distance R from the source of ν_ℓ. Relation (5) is the consequence of CPT invariance /5/.

If CP invariance holds, the matrix U is real, and from (3) and (4) it follows /5/:

$$P_{\nu_{\ell'};\nu_\ell}(R,p) = P_{\bar\nu_{\ell'};\bar\nu_\ell}(R,p), \qquad (\ell' \neq \ell) \tag{6}$$

(for $\ell' = \ell$ this relation is satisfied due to CPT invariance).

We mention that relations (6) could hold even for a complex matrix U. Indeed, let us assume that

$$m_i^2 - m_1^2 \ll m_n^2 - m_1^2, \qquad i = 1,2,\dots,n-1,$$

$$(m_1 \le m_2 \le \dots \le m_n)$$

and that

$$\frac{(m_i^2-m_1^2)R}{2p} \ll 1, \qquad (i \neq n). \tag{7}$$

In this case for the oscillation probabilities we have

$$P_{\nu_{\ell'};\nu_\ell}(R,p) = 2|U_{\ell' n}|^2 |U_{\ell n}|^2 \left(1-\cos\frac{(m_n^2-m_1^2)R}{2p}\right) = P_{\bar\nu_{\ell'};\bar\nu_\ell}(R,p), \quad (\ell' \neq \ell). \tag{8}$$

Consequently, relations (6) are valid if inequalities (7) are satisfield. In the case of two neutrino types only one neutrino mass squared difference enters into the amplitudes, and relation (6) is always satisfied. We also mention that if the cosine terms do vanish on averaging over the neutrino spectra, over the distance from the neutrino source to the detector, etc., we have

$$P_{\nu_{\ell'};\nu_\ell} = \sum_i |U_{\ell' i}|^2 |U_{\ell i}|^2 = P_{\bar\nu_{\ell'};\bar\nu_\ell}. \tag{9}$$

Large CP violating effects in neutrino oscillations could be expected provided the phases of U are not small. We shall consider just such a case.

3. The neutrino beam obtained from K_L decay is a mixture of $\nu_\mu, \nu_e, \bar\nu_\mu$ and $\bar\nu_e$. Neglecting small ($\sim 10^{-3}$) CP violating effects in the K_L decays, the number of $\nu_e(\nu_\mu)$ is equal to the number of $\bar\nu_e(\bar\nu_\mu)$. It is just the CP-symmetry of the initial state of such a beam which makes it an ideal tool in searching for CP violation in neutrino oscillations. For this we have to compare the fluxes of ν_ℓ's and $\bar\nu_\ell$'s in the detector. The corresponding asymmetry is

$$A_\ell(R,p) = \frac{I_{\nu_\ell}(R,p) - I_{\bar\nu_\ell}(R,p)}{I_{\nu_\ell}(R,p) + I_{\bar\nu_\ell}(R,p)} = \frac{\sum_{\ell'=e,\mu}\left(P_{\nu_\ell;\nu_{\ell'}}(R,p) - P_{\bar\nu_\ell;\bar\nu_{\ell'}}(R,p)\right) I^0_{\nu_{\ell'}}(p)}{\sum_{\ell'=e,\mu}\left(P_{\nu_\ell;\nu_{\ell'}}(R,p) + P_{\bar\nu_\ell;\bar\nu_{\ell'}}(R,p)\right) I^0_{\nu_{\ell'}}(p)}. \tag{10}$$

Here $I_{\nu_\ell}(R,p)$ ($I_{\bar\nu_\ell}(R,p)$) is the intensity of ν_ℓ's ($\bar\nu_\ell$'s), $\ell = e, \mu, \tau, \dots$ with momentum p, at a distance R from the neutrino source, and $I^0_{\nu_\ell}(p)$, ($\ell = e,\mu$) is the initial intensity of ν_ℓ's.

Should it turn out that for any ℓ (e or μ or τ, ...) the asymmetry A_ℓ is different from zero, this would mean that:

1) Neutrino oscillations take place (if $A_e \neq 0$ or $A_\mu \neq 0$, then $\nu_e \rightleftarrows \nu_\mu$ oscillations, if $A_\tau \neq 0$, then $\nu_e \rightleftarrows \nu_\tau$ and/or $\nu_\mu \rightleftarrows \nu_\tau$ oscillations);

2) CP invariance is violated in the oscillations;

3) The number of types of effectively oscillating neutrinos is 3 or larger and at least two neutrino mass differences enter into the ossillation probabilities.

Should the asymmetry $A_e(R,p)$ be different from zero, the asymmetry $A_\mu(R,p)$ would have to be different from zero as well. Using CPT invariance (relation (5)), we get

$$\frac{I_{\nu_e}(R,p) - I_{\bar\nu_e}(R,p)}{I_{\nu_\mu}(R,p) - I_{\bar\nu_\mu}(R,p)} = -\frac{I^0_{\nu_\mu}(p)}{I^0_{\nu_e}(p)}. \tag{11}$$

Hence relation (11) is a test of CPT invariance.

If there exist three types of neutrinos (ν_e, ν_μ, ν_τ), then the difference from zero of asymmetries A_e (or A_μ) means that the asymmetry A_τ is also nonvanishing. In this case we have:

$$\frac{I_{\nu_\tau}(R,p) - I_{\bar\nu_\tau}(R,p)}{I_{\nu_\mu}(R,p) - I_{\bar\nu_\mu}(R,p)} = \frac{I^0_{\nu_\mu}(p) - I^0_{\nu_e}(p)}{I^0_{\nu_e}(p)}. \tag{12}$$

This relation is the consequence of unitarity of the mixing matrix U. Indeed, we have

$$\sum_{\ell'} D_{\ell'\ell}(R,p) = 0, \tag{13}$$

where

$$D_{\ell'\ell}(R,p) = P_{\nu_{\ell'};\nu_\ell}(R,p) - P_{\bar\nu_{\ell'};\bar\nu_\ell}(R,p). \tag{14}$$

For the case under consideration it follows from (13) that *

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* After having completed this work we got the preprint of V.Barger et al. DOE-ER/00881-177 where these relations were obtained using the Kobayashi-Maskawa parametrization of the mixing matrix.

$$D_{\mu e}(R,p) = -D_{\tau e}(R,p) = D_{\tau\mu}(R,p) \qquad (15)$$

(in the general case of oscillations between n types of neutrinos the number of independent quantities $D_{\ell'\ell}$ is equal to (n-1) (n-2)/2, and it coincides with the number of phases of the matrix U entering into the oscillation probabilities). Making use of (15) the relation (12) is easily obtained. A test of (12) in principle yields some information on the number of neutrino types effectively oscillating.

We emphasize that the relation (12) is based on assumption that there are only three types of oscillating neutrinos ν_e, ν_μ, ν_τ. However, even if, say, oscillations $\nu_e \rightleftarrows \nu_\mu$ were absent, oscillations $\nu_e \rightleftarrows \nu_\tau$ might take place. Should in this case the asymmetry A_τ be different from zero, this would mean that there exists a fourth type of neutrino and/or the so-called second class oscillations $\nu_{\ell L} \rightleftarrows \bar{\nu}_{\ell L}$ occur /6/.

4. Summing up, we would like to emphasize once more that the measurement of asymmetries in neutrino beams obtained from K_L decays would be a direct method of searching for CP violating effects in neutrino oscillations. For the ratio of the number of ℓ^+'s to the number of ℓ^-'s ($\ell = e, \mu, \tau, \ldots$) we have

$$\frac{N_{\ell+}}{N_{\ell-}} = \frac{\int \sigma_{\bar{\nu}_\ell}(E) \sum_{\ell'=e,\mu} P_{\bar{\nu}_\ell;\bar{\nu}_{\ell'}}(R,E)\, I^0_{\nu_{\ell'}}(E)\, dE}{\int \sigma_{\nu_\ell}(E) \sum_{\ell'=e,\mu} P_{\nu_\ell;\nu_{\ell'}}(R,E)\, I^0_{\nu_{\ell'}}(E)\, dE}. \qquad (16)$$

Here $\sigma_{\bar{\nu}_\ell}$ (σ_{ν_ℓ}) is the cross section of the process ν ($\bar{\nu}$) $+N \to \ell^+(\ell^-)+X$. The ratio $N_{\ell+}/N_{\ell-}$ could be different from $R = \sigma_{\bar{\nu}\ell}/\sigma_{\nu\ell}$ only if CP invariance is violated in neutrino oscillations.

In conclusion we mention that the above considerations apply also to neutrinos in beam dump experiments (if the main source of neutrinos are the decays of D and $\bar{D}$ mesons). In the last beam dump experiments at CERN the ratio $N_{\mu+}/N_{\mu-}$ has been measured /7/. The errors of data do not allow one to draw definite conclusions, but one should keep in mind that the CDHS group observed an excess of negative muons over positive ones (whereas the expected value was 0.48 ($=\sigma_{\bar{\nu}\mu}/\sigma_{\nu\mu}$), the Steinberger group obtained the value 0.06±0.10±0.06 by the method of extrapolation to infinite target density and the value 0.28±0.05±0.07 by the subtraction method).

We wish to express our deep gratitude to B.Pontecorvo, S.Petcov and G.Micelmacher for fruitful discussions.

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Received by Publishing Department
January 23 1980.