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LOCAL GAUGE INVARIANT QED WITH FUNDAMENTAL LENGTH

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I. In quantum field theory (QFT) one always considered as evident that the transition from x-representation of the theory to its p-representation and vice versa proceeds through the standard Fourier transform of the field operators

$$\phi(x) = \frac{1}{(2\pi)^{3/2}}\int e^{-ipx}\,\phi(p)\,d^4p\,, \tag{1.1}$$

$$\phi(p) = \frac{1}{(2\pi)^{5/2}}\int e^{ipx}\,\phi(x)\,d^4x. \tag{1.2}$$

The 4-dimensional x- and p-spaces, in which $\phi(x)$ and $\phi(p)$ are defined, are either mutually pseudo-Euclidean or Euclidean *, i.e., do not contain any universal scales of dimension of length or mass.

However, we can point out such an alternative to (1.1)-(1.2) transformation, in which x-space keeps flat but p-space acquires a constant curvature. In the 3-dimensional case such a "mixed" Fourier transform is rather familiar. It arises when considering the solutions of the Klein-Gordon equation. Indeed, if

$$(\Box + m^2)\,f(t,\vec{x}) = \left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\vec{x}^2} + m^2\right) f(t,\vec{x}) = 0\,, \tag{1.3}$$

then

$$f(0,\vec{x}) = \frac{1}{(2\pi)^{3/2}}\int e^{i\vec{p}\vec{x}}\,d^3\vec{p}\,\frac{f(|p_0|,\vec{p}) + f(-|p_0|,\vec{p})}{|p_0|}\,, \tag{1.4}$$

$$i\frac{\partial f}{\partial t}(0,\vec{x}) = \frac{1}{(2\pi)^{3/2}}\int e^{i\vec{p}\vec{x}}\,d^3\vec{p}\,f(|p_0|,\vec{p}) - f(-|p_0|,\vec{p}))\,, \tag{1.5}$$

$$\frac{f(|p_0|,\vec{p}) + f(-|p_0|,\vec{p})}{|p_0|} = \frac{1}{(2\pi)^{3/2}}\int e^{-i\vec{p}\vec{x}}\,f(0,\vec{x})\,d^3x\,, \tag{1.6}$$

$$f(|p_0|,\vec{p}) - f(-|p_0|,\vec{p}) = \frac{1}{(2\pi)^{3/2}}\int e^{-i\vec{p}\vec{x}}\left(i\frac{\partial f}{\partial t}(0,\vec{x})\right)d^3x\,,$$
$$|p_0| = \sqrt{m^2+\vec{p}^{\,2}}\,. \tag{1.7}$$

*In the Euclidean formulation of the theory.

We see that expansions (1.4)-(1.7) relate the pair of functions $f(0,\vec{x})$ and $i\frac{\partial f}{\partial t}(0,\vec{x})$ defined in the Euclidean $\vec{x}$-space with the pair of functions $f(|p_0|,\vec{p})$ and $f(-|p_0|,\vec{p})$ on the hyperboloid

$$p_0^2 - \vec{p}^2 = m^2. \tag{1.8}$$

The "curvature radius" m of the hyperboloid (1.8) plays here the role of a universal parameter.

One can easily construct a 4-dimensional analog of transformations (1.4)-(1.7). For this purpose we consider the hypersphere in the flat 5-dimensional p-space

$$p_0^2 - \vec{p}^2 + M^2 p_4^2 = M^2. \tag{1.9}$$

From here on the universal constant M is called the fundamental mass and the quantity $\frac{1}{M} = \ell$ the fundamental length.

Consider now the Fourier 5-integral

$$\Psi(x,\tau) = \frac{2M^2}{(2\pi)^{3/2}} \int e^{-ip_4\tau + ipx} \delta(p^2 + M^2 p_4^2 - M^2)\Psi(p,p_4)d^5p. \tag{1.10}$$

It is evident that

$$(\Box + M^2 \frac{\partial^2}{\partial\tau^2} + M^2)\Psi(x,\tau) = 0. \tag{1.11}$$

From (1.10) it also follows that

$$\Psi(x,0) = \frac{1}{(2\pi)^{3/2}} \int_{p^2 \le M^2} e^{ipx} \frac{dp}{\kappa_p}[\Psi(p,\kappa_p) + \Psi(p,-\kappa_p)], \tag{1.12}$$

$$i\frac{\partial\Psi}{\partial\tau}(x,0) = \frac{1}{(2\pi)^{3/2}} \int e^{ipx} dp[\Psi(p,\kappa_p) - \Psi(p,-\kappa_p)],$$

$$\kappa_p = |p_4| = \sqrt{1 - \frac{p^2}{M^2}}. \tag{1.13}$$

Inverting expansions (1.12)-(1.13), we get

$$\frac{\Psi(p,\kappa_p) + \Psi(p,-\kappa_p)}{\kappa_p} = \frac{1}{(2\pi)^{5/2}} \int e^{-ipx}\Psi(x,0)dx, \tag{1.14}$$

$$\Psi(p,\kappa_p) - \Psi(p,-\kappa_p) = \frac{1}{(2\pi)^{5/2}} \int e^{-ipx} (i\frac{\partial\Psi(x,0)}{\partial\tau})dx. \tag{1.15}$$

The obtained 4-dimensional Fourier transforms (1.12)-(1.13) and (1.14)-(1.15) give rise to a relation of the form

$$\begin{pmatrix} \Psi(x,0) \\ i\dfrac{\partial\Psi(x,0)}{\partial\tau} \end{pmatrix} \leftrightarrow \begin{pmatrix} \Psi(p,\kappa_p) \\ \Psi(p,-\kappa_p) \end{pmatrix} . \tag{1.16}$$

Thus, in contrast with (1.1)-(1.2) the pseudo-Euclidean x-space becomes associated with the de Sitter p-space (1.9).*

Basic Hypothesis

We assume that QFT will provide an adequate description of the processes at high energies (at small distances) if and only if the transition to momentum representation of the field operators is based on expansions (1.2)-(1.15).

In this way there naturally arises a new Lagrangian formulation of QFT, local in terms of $\Psi(x,0)$ and $i\dfrac{\partial\Psi(x,0)}{\partial\tau}$. The new Lagrangian contains terms depending on the fundamental length $\ell = \frac{1}{M}$, which do not survive in the limit $\ell \to 0$.

In the present letter we briefly discuss the corresponding version of quantum electrodynamics, which we call "QED with a fundamental length".

II. The Lagrangian of the new QED (in the Euclidean formulation) is

$$\mathcal{L}(x) = -\frac{1}{4}F_{KL}(x,0)F^{KL}(x,0) - \frac{1}{2}\left[M\left(\frac{\partial A_4(x,0)}{\partial\tau} - iA_4\right) + \frac{\partial A_n}{\partial x_n}\right]^2 + \tag{2.1}$$

$$+\frac{1}{2}\Big\{ M\bar{\Psi}(x,0)\gamma^5\Psi + \frac{1}{M}\left(\frac{\partial\bar{\Psi}}{\partial x_n} + ieA_n\bar{\Psi}\right)\gamma^5\left(\frac{\partial\Psi}{\partial x_n} - ieA_n\Psi\right) +$$

$$+ M\left[\left(\overline{i\frac{\partial\Psi}{\partial\tau}}\right) + \frac{e}{M}A_4\bar{\Psi}\right]\gamma^5\left[i\frac{\partial\Psi}{\partial\tau} - \frac{e}{M}A_4\Psi\right]$$

*Correspondingly the Euclidean 4-dimensional x-space will be associated with the following de Sitter p-space

$$M^2p_4^2 - p_E^2 = M^2 . \tag{1.17}$$

$$-[\overline{(i\frac{\partial\Psi}{\partial\tau})}+\frac{e}{M}A_4\bar{\Psi}][M(\gamma^5+2\sin\frac{\mu}{2})+(i\frac{\partial}{\partial x_n}+eA_n)\Gamma^n]\Psi -$$

$$-\bar{\Psi}[M(\gamma^5+2\sin\mu/2)+(i\frac{\partial}{\partial x_n}+eA_n)\Gamma^n][i\frac{\partial\Psi}{\partial\tau}-\frac{e}{M}A_4\Psi], \tag{2.1}$$

$$K,L = 0,1,2,3,4, \qquad \cos\mu = \sqrt{1-\frac{m^2}{M^2}}.$$

Let us explain the notation. Here the electromagnetic potential is a 5-vector

$$A_K(x,0) = \{A_k(x,\tau),\ A_4(x,\tau)\}_{\tau=0}. \tag{2.2}$$

The 5-dimensional field strength $F_{KL}(x,0)$ is defined by /1/:

$$F_{KL}(x,0) = \{\frac{\partial}{\partial x^L}(e^{i\tau}A_K(x,\tau)) - \frac{\partial}{\partial x^K}(e^{i\tau}A_L(x,0))\}_{\tau=0}, \qquad K,L = 0,1,2,3,4. \tag{2.3}$$

Lagrangian (2.1) is invariant with respect to the group of local gauge transformations

$$A_n(x,0) \to A_n(x,0) + \frac{\partial}{\partial x^n}\lambda(x,0)$$

$$A_4(x,0) \to A_4(x,0) - iM(\lambda(x,0) - i\frac{\partial\lambda(x,0)}{\partial\tau}) \tag{2.4}$$

$$\Psi(x,0) \to e^{ie\lambda(x,0)}\Psi(x,0)$$

$$i\frac{\partial\Psi(x,0)}{\partial\tau} \to e^{ie\lambda(x,0)}[i\frac{\partial\Psi(x,0)}{\partial\tau} - ie(\lambda(x,0) - i\frac{\partial\lambda(x,0)}{\partial\tau})\Psi(x,0)],$$

where

$$\lambda(x,0) = \lambda(x,\tau)|_{\tau=0}$$
$$(M^2\frac{\partial^2}{\partial\tau^2} - \Box + M^2)\lambda(x,\tau) = 0. \tag{2.5}$$

As to the structure of the Lagrangian we make the following remarks:

1. Performing in (2.1) the Euclidean version of the Fourier transform (1.12)-(1.15) one can pass to momentum representation. A crucial point is that the momentum 4-space is now de Sitter space (1.17) with a curvature radius equal to the fundamental mass

2. Expression (2.1) for $\mathcal{L}(x)$ is obtained in the standard way from the sum of two free Lagrangians, describing the electromagnetic[1] and Dirac fields[2], performing the "minimal" but 5-dimensional substitution

$$\frac{\partial}{\partial x^M} \to \frac{\partial}{\partial x^M} + ie\, A_M(x,0) \qquad M = 0,1,2,3,4. \tag{2.6}$$

Therefore, the gauge invariant Lagrangian of the new QED is constructed as unambiguously as in the ordinary QED.

3. The new gauge group (2.4) is larger than the ordinary localized U(1) -group. The latter is picked up from (2.4) if one sets

$$\lambda(x,0) = i\, \frac{\partial \lambda(x,0)}{\partial r}\,. \tag{2.7}$$

At first sight, the theory based on Lagrangian (2.1) looks unrenormalizable. However, the wider gauge symmetry of Lagrangian (2.1) gives rise to additional Ward identities. The preliminary analysis has shown that this fact seems to ensure renormalizability of the "QED with the fundamental length".

III. The developed theory of electromagnetic interactions puts forward a number of specific predictions. We shall discuss some of them. Using (2.1) one can easily see that the effective interaction Lagrangian expressed in terms of the fields $\bar{\Psi}(x,0)$ and $\Psi(x,0)$ contains the term

$$-\frac{e}{4M}\,\mathrm{ch}\,\theta\, \bar{\Psi}\gamma^5\sigma^{mn}\Psi F_{mn} - \frac{e}{4M}\,\mathrm{sh}\,\theta\, \bar{\Psi}\sigma^{mn}\Psi F_{mn}\,, \qquad \mathrm{sh}\,\theta = \mathrm{tg}\,\mu/2\,. \tag{3.1}$$

Hence it follows that:

1. P- and CP-symmetry violation takes place[3] due to the existence of electric dipole moment of charged particles: $d = \frac{e\ell}{2}$. Consequently, at least $\ell \lesssim 10^{-24}$ cm[3].

2. Interaction (3.1) gives an additional contribution to $(g-2)$-anomaly/3/. We can show that $\frac{g-2}{2} \sim m^2 \ell^2$, where m is the fermion mass. From the data on $(g-2)_{muon}$ it follows that $\ell \lesssim 2.6 \cdot 10^{-17}$ cm.

3. Interaction (3.1) manifests itself in the reactions with polarized high-energy particles/4/. For instance, the asymmetry combination for the process $e^+e^- \to e^+e^-$

$$A = \frac{\left(\sin^8\frac{\theta}{2}+\cos^8\frac{\theta}{2}\right)\left(\frac{d\sigma}{d\Omega}\right)_{\uparrow\downarrow} - \left(\frac{d\sigma}{d\Omega}\right)_{\uparrow\uparrow}}{\left(\sin^8\frac{\theta}{2}+\cos^8\frac{\theta}{2}\right)\left(\frac{d\sigma}{d\Omega}\right)_{\uparrow\downarrow} + \left(\frac{d\sigma}{d\Omega}\right)_{\uparrow\uparrow}} \tag{3.2}$$

in the usual quantum electrodynamics is different from zero due to the radiation corrections but decreases at large E^2 as $\frac{\alpha m^2}{E^4}\ell n\frac{E^2}{m^2}$. At the energies of the large electron-positron storage ring accelerator, to be constructed at CERN, the radiation corrections will be negligibly small. Then the main contribution to the quantity A will come from the new interaction provided the fundamental length varies from 1/300 ~ 1/1000 $(GeV)^{-1}$

In conclusion we should like to emphasize that applications of the Fourier transform (1.12)-(1.15) associated with the non-Euclidean momentum space result in a local gauge invariant QED containing the fundamental length ℓ as a new universal parameter. It is obvious that "QED with fundamental length" corresponds to new rich physics.

A corresponding formulation of the unified electroweak theory does not confront principle difficulties.

REFERENCES

1. Kadyshevsky V.G. Nucl.Phys.B, 1978, v. 141, p. 447.
2. Volobuyev I.P. et al. Theoretical and Mathematical Physics, 1979, v. 40, N3, p. 363-372.
3. Kadyshevsky V.G. Physics of Elementary Particles and Atomic Nuclei, 1980, 11, N. 1, p. 5.
4. Mateev M.D., Chizhov M.V. JINR, P2-80-61, Dubna, 1980.

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