объединенный
институт
ядерных
исследований
дубна

E2-81-453

R.Lednický, V.L.Lyuboshitz

INFLUENCE OF FINAL-STATE INTERACTION ON CORRELATIONS OF TWO PARTICLES WITH NEARLY EQUAL MOMENTA

Submitted to ЯФ

1981

1. In terms of the model independent point-like one-particle sources emitting unpolarized particles with spin j, the density of emission probability of two identical particles with 4-momenta p_1 and p_2 in space-time points $x_1=(\vec{r}_1, t_1)$ and $x_2=(\vec{r}_2, t_2)$ is given by

$$W(p_1, p_2) = 1 + b(p_1, p_2), \tag{1}$$

$$b(p_1, p_2) = g_0 \cos(qx). \tag{2}$$

Here $q = p_1 - p_2$, $x = x_1 - x_2$, $qx = q_0 t - \vec{q}\vec{r}$ and $g_0 = (-1)^{2j}/N(j)$; $N(j)$ is the number of spin states: $N(j)=2j+1$ if the particle mass $m \neq 0$; $N(j)=2$ if $m=0$ and $j>0$ (e.g., for photons). Eq. (2) reflects the influence of Bose- or Fermi-statistics on identical particle correlations. It allows one to estimate space-time dimensions of multiparticle emission region. The corresponding method, generalizing the idea of the well-known Brown-Twiss interferometer in astrophysics[1], has been elaborated by Kopylov and Podgoretsky[2-4] and also by Cocconi [5] and is successfully applied in a number of experimental papers.

However, eq. (2) does not allow for the effect of final-state interaction of identical particles. In general aspect, the final-state interaction was first discussed by Watson [6] and Migdal[7]. Later on various manifestations of this effect have been considered by many authors. A consistent method allowing for the final-state interaction was developed by Baldin [8,9] in a study of the near threshold pion photoproduction on deuterium. It is important that, as in the case of Bose- or Fermi-statistics effect, the final-state interaction depends on space-time dimensions of particle emission region*.

The final-state interaction is absent for photons. Simple estimates show that its contribution is small for π^+-or π^--correlations, but it dominates in nucleon ones. Below we present analytical expressions for the correlation function $b(p_1, p_2)$ of arbitrary particles with $m \neq 0$ (like or unlike, charged or neutral) allowing for their final-state interaction.

---

* A dependence of such a kind was already pointed out by Baldin and Lebedev[10] in the case of slow pion photoproduction on nuclei.

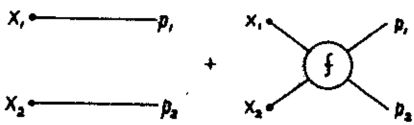

Fig. 1.

Following refs.[1-3] we assume the particles emitted by independent one-particle point-like sources (presumably, it is a reasonable approximation in the case of multiparticle processes). The influence of final-state interaction in terms of such a model was discussed previously by Kopylov [11] for pions (he allowed only for real intermediate states in the diagram of Fig. 1) and by Koonin [12] for nonrelativistic protons.

For a moment we assume that the production mechanism, as well as the particle interaction, is spin-independent. The function $b(p_1, p_2)$ can be then expressed through non-symmetrized Bethe-Salpeter amplitude $\psi_{p_1p_2}(x_1, x_2)$ corresponding to a given interaction potential (see, e.g., [13])

$$b(p_1, p_2) = \{ \frac{j+1}{2(2j+1)} |\psi^*_{p_1p_2}(x_1, x_2) + (-1)^{2j} \psi^*_{p_2p_1}(x_1, x_2)|^2 + \frac{j}{2(2j+1)} |\psi^*_{p_1p_2}(x_1, x_2) - (-1)^{2j} \psi^*_{p_2p_1}(x_1, x_2)|^2 \} - 1, \tag{3}$$

where $(j+1)/(2j+1)$ is the fraction of two-particle states with even and $j/(2j+1)$ with odd summary spin for integer j or the opposite for half-integer j. At equal emission times t^*_1 and t^*_2 in the two-particle rest frame the amplitude $\psi^*_{p_1p_2}(x_1, x_2)$, considered as a function of x_1 and x_2 coincides with the usual wave function of two interacting particles having the asymptotics corresponding to the superposition of plane and converging spherical wave [14]. In fact, the approximation $t^*_1 = t^*_2$ was used by Koonin when calculating the correlation function of two nonrelativistic protons[12]. Later on we require no special limitations on the emission times t^*_1 and t^*_2 and also on particle pair velocity in the rest frame of particle sources.

For noninteracting particles the amplitude is given by the product of plane waves: $\psi^*_{p_1p_2}(x_1, x_2) = e^{-ip_1x_1} e^{-ip_2x_2} = e^{-ip(x_1+x_2)} e^{-iqx/2}$, where $p = (p_1+p_2)/2$. It is seen that in this case the formula (3) yields the correlation function (2). The emission amplitude for interacting particles (see Fig. 1) is given by

$$\psi_{p_1p_2}(x_1, x_2) = e^{ip(x_1+x_2)} \{ e^{iqx/2} + \phi_{p_1p_2}(x) \}, \tag{4}$$

$$\phi_{p_1 p_2}(x) = \frac{8\pi\sqrt{p^2}}{(2\pi)^4 i} e^{ipx} \int \frac{e^{-i\kappa x} f(p_1, p_2, \kappa, 2p-\kappa)}{(\kappa^2 - m^2 + i0)((2p-\kappa)^2 + m^2 + i0)} d^4\kappa , \qquad (5)$$

where $f(p_1, p_2, \kappa, 2p - \kappa)$ is the nonsymmetrized scattering amplitude analytically continued to the unphysical region. We consider such small relative particle momenta that the contribution of s-wave interaction is dominant. Provided that the intrinsic range d of the interaction potential is smaller than the distance r^* between the emission points in the two-particle rest frame, we can put $f(p_1, p_2, \kappa, 2p-\kappa) \simeq f(k^*)$ (see also discussion of eq. (A.9) in Appendix) and take the amplitude f out of the integral in eq. (5):

$$\phi_{p_1 p_2}(x) = f(k^*)\Phi_{p_1 p_2}(x) . \qquad (6)$$

Here k^* is the momentum of one of the particles in their c.m.s.: $k^* = \sqrt{-q^2}/2 = \sqrt{p^2 - m^2}$, $p^2 = m_{12}^2/4$; $f(k^*) = (e^{2i\delta_0(k^*)} - 1)/2ik^*$ is the nonsymmetrized s-wave scattering amplitude* and $\Phi_{p_1 p_2}(x)$ is given by eq. (A.1) in Appendix. The integration in this formula can be performed in the approximation of small relative momenta of the particles in their c.m.s. ($k^* \ll m$). The result is given in (A.8).

From eqs. (3-6) it follows that the correlation function takes the form

$$b(p_1, p_2) = b_0(p_1, p_2) + b_i(p_1, p_2) , \qquad (7)$$

where

$$b_0(b_1, p_2) = g_0 \cos(qx) \qquad (8)$$

is the contribution of Bose- or Fermi-statistics effect and $b_i(p_1, p_2)$ appears due to final state s-wave interaction:

$$b_i(p_1, p_2) = g_i \{ |f(k^*)\Phi_{p_1 p_2}(x)|^2 + 2\mathrm{Re}[f(k^*)\Phi_{p_1 p_2}(x)]\cos(qx/2)\} , \qquad (9)$$

$g_i = 1 + g_0$. When polarized (aligned) particles are emitted, the spin factors $g_0 = (-1)^{2j}/(2j+1)$ and g_i in eqs. (8-9) should be changed for $g_0 = \sum_{S\text{-even}} \rho_S - \sum_{S\text{-odd}} \rho_S$ and $g_i = 1 + g_0 = 2\sum_{S\text{-even}} \rho_S$. Here $\rho_S = \sum_m \rho_{Sm}^{Sm}$ is the normalized emission probability of identical particles with nearly equal momenta $p_1 \simeq p_2 \simeq p$ in a state with summary spin S; $\rho_{S\,m}^{S'm'}$ are the elements of the two-particle

---

* The corresponding scattering amplitude for identical particles in a state with summary spin S, is equal to $[1+(-1)^S]f(k^*)$. The cross sections of elastic scattering of two unpolarized identical particles is given by $\sigma = 4\pi(1 + g_0)|f(k^*)|^2$.

spin density matrix in the S, m -representation. If the particles are emitted independently with the same one-particle spin density matrix $\hat{\rho}_{(1)}$, then $\hat{\rho} = \hat{\rho}_{(1)} \otimes \hat{\rho}_{(1)}$ and $g_0 = (-1)^{2j} \mathrm{Sp}\,\hat{\rho}^2_{(1)}$. In the case of practical interest, when nucleons with polarization $\vec{P}$ are produced, we have $\hat{\rho}_{(1)} = (\hat{1} + \vec{P}\vec{\sigma})/2$ and $g_0 = -(1+P^2)/2$, $g_i = (1-P^2)/2$. We see that the contribution of s-wave interaction of two identical nucleons to the correlation function goes to zero at $P \to 1$ (identical nucleons with parallel spins cannot be in the s-wave state). When the final-state interaction is spin-dependent, the amplitudes $f(k^*)$ can be different in states with different summary spin. The generalization of eqs. (3-6) and (9) is quite transparent. In particular, the function b_i takes the form

$$b_i(p_1, p_2) = 2 \sum_{S-\mathrm{even}} \rho_S \{|f^S(k^*)\Phi_{p_1 p_2}(x)|^2 + 2\mathrm{Re}[f^S(k^*)\Phi_{p_1 p_2}(x)]\cos(qx/2)\}. \tag{10}$$

Note that $\rho_S = (2S+1)/(2j+1)^2$ if the production mechanism does not depend on spin projections.

2. The correlation function (7) should be averaged over a space-time distribution of particle sources. The result is denoted as follows:

$$B(q,p) = \langle b_0(p_1, p_2)\rangle + \langle b_i(p_1, p_2)\rangle = B_0(q,p) + B_i(q,p). \tag{11}$$

Below we consider in detail the correlations of two unpolarized neutrons. According to eq. (8), we have

$$B_0(q,p) = -\frac{1}{2}\langle\cos(qx)\rangle. \tag{12}$$

Using formula (6) also inside the range of the interaction potential, eq. (9) yields

$$B_i(q,p) = \frac{1}{2}\{|f(k^*)|^2 \langle|\Phi_{p_1 p_2}(x)|^2\rangle + 2\mathrm{Re}[f(k^*)\langle\Phi_{p_1 p_2}(x)\cos\frac{qx}{2}\rangle]\}. \tag{13}$$

The nn-interaction amplitude in the effective range approximation is of the form [14]

$$f(k^*) = (\frac{1}{f_0} + \frac{1}{2}d_0 k^{*2} - ik^*)^{-1}. \tag{14}$$

According to the experimental data (see, e.g., [15]), the scattering length f_0 and the effective radius d_0 are equal to 17 fm* and 2.7 fm, respectively.

The neutron emission points are assumed to be independent and distributed according to the Gaussian law

---

* The data on neutron-proton scattering yield the singlet scattering length equal to 23.7 fm[14]. Such a violation of isotopic invariance is not yet completely clarified.

$$W(x_1,x_2)=\frac{1}{(2\pi)^4 r_0^6\tau_0^2}\exp[-(r_1^2+r_2^2)/2r_0^2-(t_1^2+t_2^2)/2\tau_0^2]. \tag{15}$$

The distribution of $x=x_1-x_2$ is given by

$$W(x)=\frac{1}{(4\pi)^2 r_0^3\tau_0}\exp\left(-\frac{r^2}{4r_0^2}-\frac{t^2}{4\tau_0^2}\right). \tag{16}$$

Eqs. (15-16) should be considered only as a convenient way to introduce space-time parameters of the particle production region. The mean-square radius and emission time are equal to $\sqrt{3}\,r_0$ and τ_0, respectively (the mean-square space and time distance between the emission points are $\sqrt{2}$ times larger). Note that the parameters $2r_0,\tau_0$ in refs. [3,5] and $\sqrt{2}\,r_0$, $\sqrt{2}\tau_0$ in ref. [12] have been used as space-time characteristics of particle production. The averaging in (12) according to the distribution (16) yields the familiar Gaussian transform

$$B_0(q,p)=-\frac{1}{2}\exp(-r_0^2\vec{q}^{\,2}-\tau_0^2 q_0^2). \tag{17}$$

Using the azimuthal symmetry of the functions $W(x)$ in (16) and $\Phi_{p_1p_2}(x)$ with respect to the particle pair velocity $\vec{v}$ the integration over the azimuthal angle in eq. (13) can be performed:

$$B_i(q,p)=\frac{1}{2}\int 2\pi r_T dr_T dr_L\, dt\, W(x)\{|f(k^*)\Phi_{p_1p_2}(x)|^2 + 2\mathrm{Re}[f(k^*)\Phi_{p_1p_2}(x)]\,J_0\left(\frac{1}{2}q_T r_T\right)\cos[q_0(r_T-vt)/2v]\}, \tag{18}$$

where L and T denote the vector components parallel and perpendicular to the direction of the velocity $\vec{v}$; J_0 is the Bessel function.

We note that in the case of spherically symmetrical distribution of the emission points the correlation function $B(q,p)$ depends only on three variables q_T, q_0 and v (the variables q_L and k^* are given by $q_L=q_0/v$ and $4k^{*2}=-q^2=q_T^2+q_0^2/\gamma^2v^2$; $\gamma=(1-v^2)^{-1/2}$ is the particle pair Lorentz factor). In the general case it also depends on the directions of the vectors $\vec{v}$ and $\vec{q}_T$ which, in principle, can be used to determine the form of the particle production region [3-5]. If this region moves with beforehand unknown velocity $\vec{v}_0$, this parameter will enter into eqs. (17-18) through the Lorentz transformation of 4-momenta q and p into the rest frame of particle sources.

Analysis of eqs. (18) and (A.8) shows and calculations confirm it that for small enough k^* ($k^* \ll m$ and $k^* \ll m r_0/\tau_0$) the function $\Phi_{p_1p_2}(x)$ can be approximated by formula (A.5) provided that the condition $\gamma\rho r_0/\tau_0 \gg m^{-1}$ is fulfilled; here

$\rho=(r_0^2+v^2\tau_0^2)^{1/2}$. In fact, this condition is fulfilled in many realistic cases of neutron production due to the small reverse neutron mass (m^{-1}=0.2 fm). Inserting (A.5) into eq.(18) and integrating over t* (see (A.2)), we obtain

$$B_i(q,p)=\frac{1}{4\sqrt{\pi}r_0^2\gamma\rho}\int r_T\,dr_T\,dr_L^*\exp(-r_T^2/4r_0^2-r_L^{*2}/4\gamma^2\rho^2)\,.\times$$
$$\times\left[\frac{|f|^2}{2r^{*2}}+\mathrm{Re}\left(f\frac{e^{ik^*r^*}}{r^*}\right)J_0\left(\frac{1}{2}q_Tr_T\right)\cos(q_0r_L^*/2\gamma v)\right], \tag{19}$$

where $r^*=(r_T^2+r_L^{*2})^{1/2}$.

Eqs.(18-19) yield upper estimates of the function B_i as the relation(6), together with (A.8), or (A.5), overestimates $|\phi_{p_1p_2}(x)|\simeq|\phi_{\vec{k}^*}(\vec{r}^*)|$ for $r^*<d$. It is interesting that the correction to eq. (19) can be found without knowledge of the explicit form of $\phi_{\vec{k}^*}(\vec{r}^*)$ inside the range of nuclear forces (a similar method has been used by Baldin[8,9] to calculate matrix elements for the near threshold pion photoproduction on deuterium). Using the simple relation which follows from the effective radius theory[16-18]

$$\frac{1}{4\pi}\int[|f\frac{e^{ik^*r^*}}{r^*}+\cos(\vec{k}^*\vec{r}^*)|^2-|\phi_{\vec{k}^*}(\vec{r}^*)+\cos(\vec{k}^*\vec{r}^*)|^2]\,d^3\vec{r}^*=$$
$$=|f|^2\frac{1}{2k^*}\frac{d}{dk^*}[\mathrm{Re}(\frac{1}{f})]\simeq\frac{1}{2}|f|^2d_0, \tag{20}$$

we obtain in the first order of d_0/r_0

$$B_i(q,p)=[\text{ expres.(19)}]-\frac{1}{8\sqrt{\pi}}|f|^2d_0/\gamma\rho r_0^2. \tag{21}$$

Actually, formula (21) underestimates the function B_i. However, for $r_0\gtrsim1.5$ fm the relative error is quite small and rapidly decreases with increasing r_0 (see fig. 2)* Formula (21) is unreliable for $r_0\le1$ fm when the function B_i is almost entirely determined by the behaviour of the neutron wave funtion inside the range of the interaction potential and is sensitive to the potential form.

At $k^*=0$ the integration in (19) can be performad and eq. (21) yields

$$B_i(0,p)=\frac{1}{\gamma\rho}[\frac{1}{4}\frac{|f|^2}{r_0}(1-\frac{1}{2\sqrt{\pi}}\frac{d_0}{r_0})A+\frac{1}{\sqrt{\pi}}\mathrm{Re}f\cdot C]. \tag{22}$$

---

* For the square well potential the correction to eq. (21) can be approximated as $\Delta B_i(q,p)\simeq(d_0/8.6r_0)^3|f|^2/\gamma\rho r_0$. For $r_0\gtrsim1.5$ fm $\Delta B_i/B_i<5\%$.

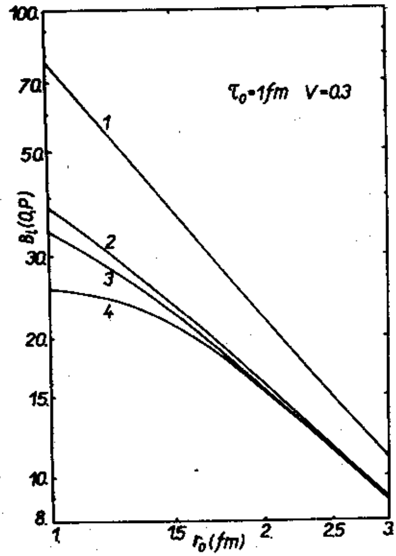

Fig.2. The function $B_i^{nn}(0,p)$. Curves 1 and 4 have been calculated according to eqs. (19) and (22), respectively. Curves 2 and 3 correspond to the exponential and square well potentials, respectively. The scattering length is equal to 17 fm and the effective radius 2.7 fm.

Here

$$A = \frac{1}{u}\arcsin(u), \qquad C = \frac{1}{2u}\ln\frac{1+u}{1-u}, \quad u = \frac{v}{\rho}\sqrt{r^2+\tau_0^2}. \tag{23}$$

$A=C=1$ at $v\to 0$, $A=\frac{\pi}{2}$, $C=\ln\left(2\gamma\sqrt{1+\frac{\tau_0^2}{r_0^2}}\right)$ for $\gamma\gg 1$ and $A=\pi/2$, $C=\ln(2\gamma v\tau_0/r_0)$ if $v\tau_0\gg r_0$.

We note that for small particle velocities, when $\gamma\rho\simeq r_0$, the function $B(q,p)$ becomes insensitive to the parameter τ_0 (see figs. 3,4). In this case

$$B_0(q,p) = -\frac{1}{2}\exp(-4k^{*2}r_0^2),$$
$$B_i(q,p) = \frac{1}{4}\frac{|f|^2}{r_0^2}\left(1-\frac{1}{2\sqrt{\pi}}\frac{d_0}{r_0}\right) + \frac{Ref}{\sqrt{\pi}\, r_0}F_1(2k^*r_0) - \frac{Im f}{2r_0}F_2(2k^*r_0), \tag{24}$$

where $F_1(z) = \frac{1}{z}e^{-z^2}\int_0^z e^{x^2}dx = 1-\frac{2z^2}{3}+\frac{(2z^2)^2}{3.5}-\dots$, $F_2(z)=\frac{1}{z}(1-e^{-z^2})$.

For two neutrons with $2k^*r_0\leq 1$ the function B_i in (24) can be approximated by

$$B_i(q,p) = B_i(0,p)\left|\frac{f(k^*)}{f(0)}\right|^2\exp(-4k^{*2}r_e^2),$$
$$r_e^2 = r_0^2\left[1-\frac{4}{3\sqrt{\pi}}\frac{r_0}{f_0}\Big/\left(1+\frac{4}{\sqrt{\pi}}\frac{r_0}{f_0}-\frac{1}{2\sqrt{\pi}}\frac{d_0}{r_0}\right)\right]. \tag{25}$$

We see that for small particle velocities the correlation function $B(q,p)$ depends only on the momentum $k^*=\frac{1}{2}\sqrt{-q^2}$ and this dependence is determined by the scattering amplitude and the parameter r_0.

It should be pointed out that the effect of final-state interaction becomes stronger with decreasing space-time dimensions of the production region. For r_0 and τ_0 of about 2-3 fm the contribution of final-state interaction to the neut-

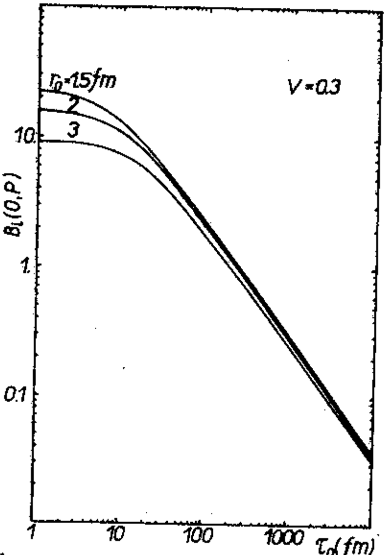

Fig.3. The function $B_i^{nn}(0,p)$ calculated according to eq. (22); f_0 =17 fm, d_0 =2.7 fm.

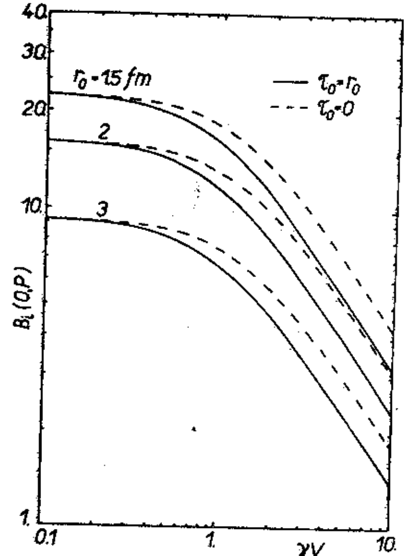

Fig.4. The energy dependence of the function $B_i^{nn}(0,p)$; $\gamma v = p/m$.

ron correlation function at small $|q|$ is several tens as large as the contribution of Fermi-statistics effect; see the curves in figs. 3-6 calculated according to formulae (21-22). The final-state interaction is unessential in the case of large distances between particle sources or large emission time, e.g., in the case of neutron evaporation by excited heavy nuclei [19] (if $r_0 \sim 3$ fm and $v\tau_0 \sim 10^4$ fm, the ratio B_i/B_0 is about 2% at $q=0$). Note that the role of final-state interaction becomes less important also with increasing neutron energy (fig. 4); in the ultrarelativistic case $B_i(0,p) \sim 1/\gamma$.

As can be seen from fig. 5 , the q-dependence of the function $B_i(q,p)$ at small $|q|$ is mainly determined by the neutron scattering amplitude (due to the large neutron scattering length). Besides, even at $v=0.3$ the k^{*2}-dependence of the ratio $B_i(q,p)/|f(k^*)|^2$ is approximated by the exponential in eq. (25) (dashed line in fig. 5) with a relative error smaller than 10% for $2k^*r_0 \le 1$ (note that $q_T^2 = 4k^{*2}$ at $q_0=0$).

It should be noted that for polarized neutrons the peak of the correlation function at $q=0$ can diminish or even turn into a dip. We recall that in the case of independent neutron emission with polarization $\vec{P}$ the functions B_0 and B_i in eqs (17) and (18) should be multiplied by $(1+\vec{P}^2)$ and $(1-\vec{P}^2)$, respectively.

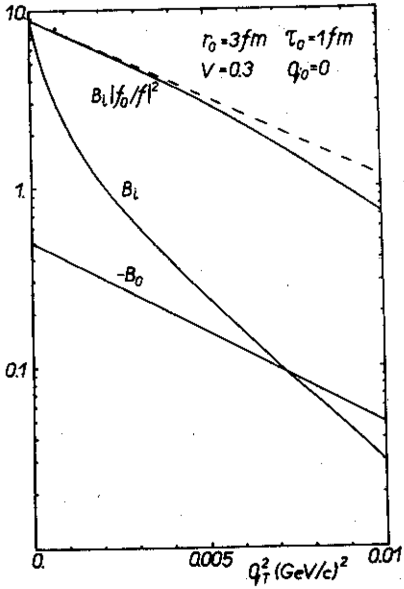

Fig.5. The functions $B_i^{nn}(q,p)$ $B_i^{nn}|f_0/f|^2$ and $-B_0^{nn}(q,p)$ calculated according to eqs. (21) and (17). The dashed line corresponds to the approximate formula (25).

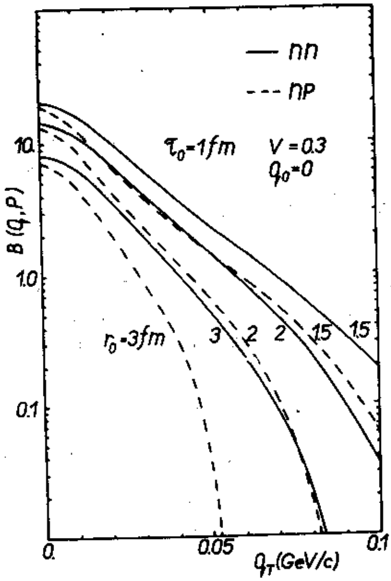

Fig.6. The neutron-neutron and neutron-proton correlation functions.

We thus see that two-neutron correlations can be used to determine space-time dimensions of their production region only if the final-state interaction is correctly taken into account; the formulae of refs.[2-5] cannot be directly applied here.

3. The final-state interaction turns out to be essential also for the correlations of two neutral kaons (the experimental data and theoretical discussion of the latter see in refs. [20,21]). Provided that K°- and $\bar{K}^\circ$-mesons are emitted by independent point-like sources, the correlation function $B^{K_s^\circ K_s^\circ}$ of two K_s°-mesons differs from the neutron one only by the spin factors (see (8-9)). Thus, $B_0^{K_s^\circ K_s^\circ} = \langle\cos(qx)\rangle$ describes the effect of Bose-statistics for spin-zero particles[21], and $B_i^{K_s^\circ K_s^\circ}$ is given by eqs. (13) or (18-19) multiplied by 4 with the scattering amplitude approximated by

$$f^{K^\circ\bar{K}^\circ}(k^*) \simeq \frac{i}{2}[f^{(0)}/(1+k^*f^{(0)}) + f^{(1)}/(1+k^*f^{(1)})].$$

Here $f^{(0)}=1.7$ fm corresponds to the S* -contribution with isospin $T=0$ and $f^{(1)}=0.84$ fm - to the δ-contribution with $T=1$ (see [22]). The correction term in eq. (21) is small as $\mathrm{Re}f \simeq 0$; see eq. (20). According to (22), $B_i^{K^\circ_s K^\circ_s}(0,p) = A|f|^2/\gamma\rho r_0$, where $|f|=1.27$ fm and $1 \leq A \leq \frac{\pi}{2}$. We see that for $\rho \sim 1$ fm and not too large γ-factors the contributions of final-state interaction and Bose-statistics effect to the correlation function become comparable.

The correlation functions for other identical spin-zero particles have a similar structure, in particular, for two neutral or charged pions (provided that the Coulomb interaction can be neglected, i.e., for $k^* > \pi m_\pi/137 \simeq 3$ MeV/c; see Section 5). Experimental data and theoretical calculations give the $\pi^\pm\pi^\pm(\pi^\circ\pi^\circ)$ -scattering length equal to $\sim -0.03(01)$ fm[23]. According to formula (22) multiplied by 4, the contribution of final-state interaction to the $\pi^\pm\pi^\pm(\pi^\circ\pi^\circ)$ -correlation function is smaller than 7%(23%) for $r_0=1$ fm*. Therefore it is possible to determine space-time dimensions of charged pion production region taking into account only the Bose-statistics effect (see refs. [2-5]) provided that $\gamma\rho \gtrsim 1$ fm.

4. Final-state interaction has an influence also on the correlations of nonidentical particles. In this case it is convenient to consider the symmetrized correlation function $b = \frac{1}{2}[b(p_1,p_2) + b(p_2,p_1)]$. Then for the spin-independent interaction

$$b = |f\Phi_{p_1p_2}(x)|^2 + 2\mathrm{Re}[f\Phi_{p_1p_2}(x)]\cos(qx/2), \tag{26}$$

---

* Formulae (19), (21-22) are reliable under the condition $\gamma\rho r_0/\tau_0 \gg m^{-1}$. As $m_\pi^{-1}=1.4$ fm, this condition is not always fulfilled for space-time dimensions of ~ 1 fm in contrast to the case of nucleons or kaons. E.g., at $r_0=\tau_0=1$ fm and $v=0.3$ formula (22) leads to a $\sim$20% overestimation of the function $B_i^{\pi\pi}(0,p)$. Note that in the realistic cases the terms in eqs. (21-22) proportional to $|f^{\pi\pi}|^2$ are very small (the effective radius of the two-pion interaction is determined by the ρ-meson mass: $d_0 \simeq 2m_\rho^{-1}=0.5$ fm).

i.e., the correlation function is entirely determined by the function $b_i (b_0 = 0)$. In the general case, when the s-wave final-state interaction of particles with spins j_1 and j_2 depends on their summary spin, and the particles are produced with normalized probability ρ_S in a state with summary spin S

$$b = \sum_S \rho_S \{ |f^S \Phi_{p_1 p_2}(x)|^2 + 2\mathrm{Re}[f^S \Phi_{p_1 p_2}(x)] \cos(qx/2) \} . \tag{27}$$

If the production process does not depend on the particle spin projections, $\rho_S = (2S+1)/(2j_1+1)(2j_2+1)$. In particular, the proton-neutron correlation function averaged over the distribution of the sources of unpolarized nucleons is of the form

$$B^{np}(q,p) = \frac{1}{2}[B^{(0)}(q,p) + 3B^{(1)}(q,p)], \tag{28}$$

where the functions $B^{(S)}(q,p)$, $S=0,1$ corresponding to the contributions of singlet and triplet spin states are given by eqs. (13), (18) or (19), (21-22). The np-scattering length and the effective radius of np-interactions in the singlet (triplet) state appearing in eq. (14) are equal to 23.7 fm and 2.7 fm (-5.4 fm and 1.7 fm), respectively [14]. As is seen from fig. 6, the neutron-proton correlations are slightly weaker than the ones of two neutrons.

Note that in the case of independent neutron and proton production with polarizations $\vec{P}_n$ and $\vec{P}_p$, the functions $B^{(0)}$ and $B^{(1)}$ in (28) should be multiplied by $(1-\vec{P}_n\vec{P}_p)$ and $(1+\frac{1}{3}\vec{P}_n\vec{P}_p)$, respectively.

According to eq. (26), the $\pi^+\pi^-$-correlation function is given by (18) or (19), (21-22) multiplied by 2 with the $\pi^+\pi^-$-scattering amplitude equal to $f_0^{\pi^+\pi^-} = \frac{2}{3} f_0^{(T=0)} + \frac{1}{3} f_0^{(T=2)} \simeq 0.2$ fm at $q=0$ [23]. For slow $\pi^+\pi^-$-pairs $(p \leq m)$ and $r_0 \sim 1$ fm we have $B^{\pi^+\pi^-}(0,p) \sim 0.25$. Thus, $\pi^+\pi^-$-pairs can be used as a background in a study of charged-like pion correlations [2-5] provided that the dimensions of the pion production region are large enough ($\gamma\rho \gtrsim 1$-2 fm).

5. It is well-known that the contribution of Coulomb interaction is determined by the factor [14]

$$A_c^{(\pm)}(k^*) = \pm \frac{2\pi}{k^* a_c} [\exp(\pm \frac{2\pi}{k^* a_c}) - 1]^{-1} , \tag{29}$$

where a_c is the Bohr radius and the sign +(-) corresponds to repulsion (attraction). The factor $A_c(k^*)$ is essentially different from 1 only for $k^*a_c/2\pi \leq 1$. For particles with unit charges and equal masses $a_c = 2\hbar^2/me^2$; for protons a_c =57.5 fm. We thus see that the higher is the particle mass the wider is the region of momenta k^* where the Coulomb interaction is important.

Assuming again that the intrinsic range of the strong interaction potential is smaller than the distance between the emission points in the particle c.m.s., we can find the explicit expression for the amplitude $\psi_{p_1p_2}(x_1,x_2)$ in eq. (3) allowing for both the Coulomb interaction and the s-wave short-range interaction of identical particles; see eqs. (4) and (A.9-11). For equal emission times in the two-particle rest frame this expression coincides with the solution of Schroedinger equation outside the range of strong interaction having the asymptotics corresponding to the superposition of plane and diverging spherical wave distorted by the Coulomb interaction (see eq. (A.13)).

In the realistic cases the distance between emission points is small as compared to the Bohr radius. In such an approximation the expression for the amplitude $\psi_{p_1p_2}(x_1,x_2)$ is essentially simplified (see eq. (A.15)) and the correlation function takes the form

$$B(q,p) = A_c(k^*)[1 + B_0(q,p) + B_i(q,p)] - 1, \tag{30}$$

where the functions B_0 and B_i should be calculated according to eqs. (8) and (10-11). The scattering amplitude for neutral particles should be replaced here by the effective amplitude $f_c(k^*)$ or $f_c^S(k^*)$, renormalized by Coulomb interaction, and the function $\Phi_{p_1p_2}(x)$ by

$$\Phi^c_{p_1p_2}(x) = \Phi_{p_1p_2}(x) + i(A_c^{(+)}(k^*) - 1)\sin k^*r^*/r^*. \tag{31}$$

In the case of two protons the functions $B_0(q,p)$ and $B_i(q,p)$ are given by eqs. (12) and (13) with the effective amplitude

$$f_c(k^*) = [\frac{1}{f_0} + \frac{1}{2}d_0k^{*2} - \frac{2}{a_c}h(k^*a_c) - ik^*A_c^{(+)}(k^*)]^{-1}, \tag{32}$$

where f_0 =7.77 fm, d_0 =2.77 fm and the function h(x) takes the form /14/

$$h(x) = \frac{1}{x^2}\sum_{n=1}^{\infty}\frac{1}{n(n^2+x^{-2})} - C + \ln x, \quad C = 0.5772\ldots . \tag{33}$$

If the emission points are distributed according to the Gauss law (see (16)), the function B_0 is given by eq. (17) and the function B_i, under the conditions $m\gamma\rho r_0/\tau_0 \gg 1$, $k^* \ll m$, $k^* \ll m r_0/\tau_0$, by eqs. (19), (21) modified by $f(k^*) \to f_c(k^*)$ and $\Phi_{p_1 p_2}(x) \to \Phi^c_{p_1 p_2}(x)$. As is shown in Appendix (see (A.16)), the correction term, proportional to the effective radius d_0, connected with the behaviour of the wave function inside the range of the strong interaction potential is correctly taken into account by modified eqs. (19), (21).

It is clear from eq. (30) that the correlation function tends to the value $B=-1$ when $q \to 0$ (due to Coulomb repulsion). The joint effect of Coulomb and short-range forces leads to a peak in the correlation function, which, in contrast to the case of neutral particles, is shifted to $q \neq 0$ ($k^* \simeq 2\pi/a_c$ = =21.6 MeV/c, see fig. 7).

Eq. (30), together with (12-13) and (A.8), represents the analytical solution of the problem taking account of relativistic effects and nonequal emission times in the two-particle c.m.s. (if they appear to be essential). Previously the corresponding calculations for nonrelativistic protons have been done by Koonin[12] (see also experimental papers [24,25]). Our calculations at $v\tau_0=0$, $r_0=1.4$-2.8 fm yield the cross section ($\sigma \sim 1+B$) in the peak smaller by ~20% than that in ref.[12]. It should be noted that the use of the approximation $r^* \ll a_c$ (see (A.14)) lowers the peak cross-section by about 10(20)% for $r_0=2(3)$ fm. Nevertheless, our approach makes it possible to look over, with enough accuracy, the analytical dependence of two-proton correlation function on the space-time parameters and does not require a large computer time.

In conclusion we note that the s-wave approximation for the strong final-state interaction is quite reliable for $k^* \lesssim 50$ MeV/c; the contribution of higher orbital angular momenta to the two-nucleon cross sections is less than 1%.

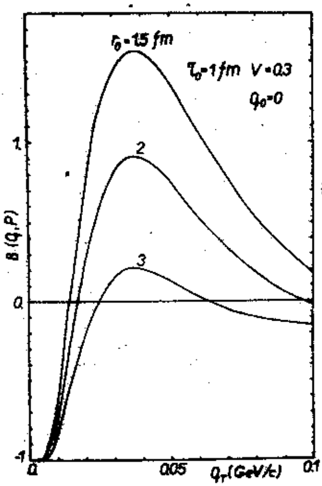

Fig. 7. The proton-proton correlation functions calculated according to formulae (29)-(33); $f_0^{pp}=7.77$ fm, $d_0^{pp}=2.77$ fm.

We wish to thank M.I.Podgoretsky for his interest to our work and worth advices. We also acknowledge useful discussions with G.A.Leksin and A.V.Stavinsky.

APPENDIX

According to eq. (5), the function $\Phi_{p_1p_2}(x)$ introduced in eq. (6) is of the form

$$\Phi_{p_1p_2}(x)=\frac{8\pi\sqrt{k^{*2}+m^2}}{i(2\pi)^4}\int\frac{\exp[i(\sqrt{k^{*2}+m^2}t^*-\kappa_0^*t^*+\vec{\kappa}^*\vec{r}^*)]d^3\vec{\kappa}^*d\kappa_0^*}{(\kappa_0^{*2}-\vec{\kappa}^{*2}-m^2+i0)[(2\sqrt{k^{*2}+m^2}-\kappa_0^*)^2-\vec{\kappa}^{*2}-m^2+i0]}. \tag{A.1}$$

Here $\vec{r}^*$ and t^* are the space and time components of the 4-vector $x=\{x,t\}$ in the two-particle rest frame. They are related to $\vec{r}$ and t in the rest frame of the particle sources by the Lorentz transformation

$$\vec{r}^*_T=\vec{r}_T,\quad r^*_L=\gamma(r_L-vt),\quad t^*=\gamma(t-vr_L). \tag{A.2}$$

Here $\vec{v}=\vec{p}/p_0$ is the particle pair velocity, $\gamma=(1-v^2)^{-1/2}$ is the Lorentz factor, $r_L(r_T)$ is the component of the vector $\vec{r}$ longitudinal (transverse) to $\vec{v}$. After integrating over the energy variable κ_0^* and over the angles of the vector $\vec{\kappa}^*$, we get

$$\Phi_{p_1p_2}(x)=\frac{1}{2\pi r^*}\int_{-\infty}^{\infty}\frac{\kappa\sin\kappa r^*}{\kappa^2-k^{*2}-i0}\{(1+\sqrt{\frac{k^{*2}+m^2}{\kappa^2+m^2}})\exp[-i(\sqrt{\kappa^2+m^2}-\sqrt{k^{*2}+m^2})|t^*|]-$$
$$-(1-\sqrt{\frac{k^{*2}+m^2}{\kappa^2+m^2}})\exp[-i(\sqrt{\kappa^2+m^2}+\sqrt{k^{*2}+m^2})|t^*|]\}d\kappa, \tag{A.3}$$

where $r^*=(r_T^2+r_L^{*2})^{1/2}$ and $\kappa=|\vec{\kappa}^*|$. For $k^*\ll m$ the formula (A.3) yields

$$\Phi_{p_1p_2}(x)=\frac{1}{\pi r^*}\int_{-\infty}^{\infty}\frac{\kappa\sin\kappa r^*}{\kappa^2-k^{*2}-i0}\exp(-i\frac{\kappa^2-k^{*2}}{2m}|t^*|)d\kappa. \tag{A.4}$$

For equal emission times in the c.m.s. of particles ($t^*=0$, $t=vr_L$) $\Phi_{p_1p_2}(x)$ coincides with the diverging spherical wave:

$$\Phi_{p_1p_2}(x)=e^{ik^*r^*}/r^*. \tag{A.5}$$

In the case of arbitrary t^* it is convenient to transform (A.4) into the form

$$\Phi_{p_1p_2}(x)=\frac{e^{ik^*r^*}}{r^*}\int_{-\infty}^{r^*}e^{-ik^*r'}f(r',|t^*|)dr'- \tag{A.6}$$
$$-\frac{e^{-ik^*r^*}}{r^*}\int_{r^*}^{\infty}e^{ik^*r'}f(r',|t^*|)dr',$$

where

$$f(r',|t^*|) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\kappa r'} \exp(-i\frac{\kappa^2-k^{*2}}{2m}|t^*|)\,d\kappa =$$

$$= \frac{1}{2}\sqrt{\frac{m}{\pi|t^*|}}(1-i)\exp[\,i(\frac{k^{*2}|t^*|}{2m}+\frac{r'^2 m}{2|t^*|})]\,. \tag{A.7}$$

From here

$$\Phi_{p_1p_2}(x) = i\frac{\sin k^*r^*}{r^*}+\frac{1}{2r^*}(1-i)\{e^{ik^*r^*}[C_1(z_-)+iS_1(z_-)] + \tag{A.8}$$

$$+e^{-ik^*r^*}[C_1(z_+)+iS_1(z_+)]\},$$

where $z_\pm = \sqrt{\frac{m}{2|t^*|}}(r^*\pm\frac{k^*|t^*|}{m})$ and $C_1(z)$, $S_1(z)$ are the Fresnel integrals: $C_1(z)+iS_1(z) = \sqrt{\frac{2}{\pi}}\int_0^z e^{iy^2}dy$. If $z_- \gg 1, S_1=C_1=1/2$ and (A.8) is reduced to the spherical wave (A.5). If $\sqrt{\frac{m}{2|t^*|}}r^* \ll 1$, the function $\Phi_{p_1p_2}(x)$ is close to zero for arbitrary k^*-values.

We can present the function (5) also in the form of 3-dimensional integral

$$\phi_{p_1p_2}(x) = \int \phi_{\vec{k}^*}(\vec{r})\,g(\vec{r}^*-\vec{r},|t^*|)\,d^3\vec{r}\,, \tag{A.9}$$

where $e^{-i\vec{k}^*\vec{r}}+\phi_{\vec{k}^*}(\vec{r})$ satisfies the corresponding Schroedinger equation, $\phi_{\vec{k}^*}(\vec{r}) = f(k^*)e^{ik^*r}/r$ outside the range of strong interaction. The function $g(\vec{r}',|t^*|)$ represents a 3-dimensional analog of the function $f(r',|t^*|)$ in eq. (A.7):

$$g(r',|t^*|) = \frac{1}{(2\pi)^3}\int e^{i\vec{\kappa}\vec{r}'}\exp(-i\frac{\kappa^2-k^{*2}}{2m}|t^*|)\,d^3\vec{\kappa} =$$

$$= \frac{1}{8}(\frac{m}{\pi|t^*|})^{3/2}(1-i)^3\exp[\,i(\frac{k^{*2}|t^*|}{2m}+\frac{r'^2 m}{2|t^*|})]\,, \tag{A.10}$$

$g(r',0) = \delta^3(r')$. In the s-wave approximation the formulae (A.9)-(A.10) yield the previous result (6)-(A.8) neglecting the deviation of the function $\phi_{\vec{k}^*}(\vec{r})$ from the diverging wave at $r<d$. It is clear that for $r^*<d$ the function $\phi_{p_1p_2}(x)$ at arbitrary t^*-values is determined by the behaviour of $\phi_{\vec{k}^*}(\vec{r})$ inside the range of strong interaction. To estimate the corresponding correction to the function B_i in (18), we can use the square well potential and put for $r<d$

$$\phi_{\vec{k}^*}(\vec{r}) = \frac{1}{\sin Kd}[\,f(k^*)e^{ik^*d}\frac{\sin Kr}{r}+\frac{1}{k^*r}(\sin k^*d\cdot\sin Kr-\sin Kd\cdot\sin k^*r)]$$

where $K=(K_0^2+k^{*2})^{1/2}$ and $K_0 \simeq \pi/2d$, $d \simeq d_0$ (for neutrons $K_0 = 110.4$ MeV and $d = 0.983\, d_0$ [26]). Presumably, such a correction is only weakly dependent on the potential form if $d/r_0 \lesssim 2$; see fig. 2 and eqs. (20)-(21) corresponding to the approximation $t^* = 0$.

For two charged particles with $k^* < 2\pi/a_c$ the scattering amplitude cannot be taken out of the integral in eq. (5). In this case it is convenient to use the representation (A.9)-(A.10). Outside the range of the strong interaction the function $\phi_{\vec{k}^*}(\vec{r})$ is determined by the well-known Coulomb wave functions; for particles with equal charges (see, e.g., refs. [8,9]).

$$\phi_{\vec{k}^*}(\vec{r}) = \sqrt{A_c^{(+)}}\, e^{i\delta_0^c} \Big[e^{-i\vec{k}^*\vec{r}} F\Big(-\frac{i}{k^* a_c}, 1, i(\vec{k}^*\vec{r} + k^* r)\Big) + \frac{e^{2i\delta_0} - 1}{2ik^*} \cdot \frac{\tilde{G}}{A_c^{(+)}\, r} \Big] - e^{-i\vec{k}^*\vec{r}} \, . \tag{A.11}$$

Here $\delta_0^c = \arg\Gamma(1 + i/k^* a_c)$ is the Coulomb phase corresponding to zero orbital angular momentum, δ_0 is the additional s-wave phase shift due to short range forces, F is the confluent hypergeometric function and $\tilde{G}$ is the combination of confluent hypergeometric functions:

$$\tilde{G} = \sqrt{A_c^{(+)}}\, e^{i(k^* r - \frac{1}{k^* a_c}\ln 2k^* r + \delta_0^c)}\, G\Big(1 + \frac{i}{k^* a_c}, \frac{i}{k^* a_c}, 2ik^* r\Big) \, .$$

The function $G(\alpha, \beta, z)$ is defined in § d of ref. [14]; see also ref. [27]. For $k^* r \gg 1$ $G \simeq 1$. The phase shift $\delta_0(k^*)$ is determined by the equation [14]

$$\frac{e^{2i\delta_0} - 1}{2ik^*} = \Big\{ \Big[\frac{1}{f_0} + \frac{1}{2} d_0 k^{*2} - h(k^* a_c)\Big] / A_c^{(+)} - ik^* \Big\}^{-1} , \tag{A.12}$$

where $A_c^{(+)}(k^*)$ and $h(k^* a_c)$ are given by (29) and (32), respectively.

In the approximation of equal emission times in the two-particle rest frame (at $|t^*| \ll m r^{*2}$) $\phi_{p_1 p_2}(x) = \phi_{\vec{k}^*}(\vec{r}^*)$. In this case the amplitude $\psi_{p_1 p_2}$ is given by $\psi_{p_1 p_2}(x_1, x_2) =$

$= e^{ip(x_1 + x_2)} [e^{-i\vec{k}^*\vec{r}^*} + \phi_{\vec{k}^*}(\vec{r}^*)]$ and at $k^* r^* \gg 1$ has the asymptotics

$$\psi_{p_1 p_2}(x_1, x_2) \simeq e^{ip(x_1+x_2)} \Big\{ e^{-i\vec{k}^*\vec{r}^* + \frac{i}{k^* a_c}\ln(\vec{k}^*\vec{r}^* + \vec{k}^*\vec{r}^*)} \times \tag{A.13}$$

$$\times\left[1+\frac{1}{ik^{*2}a_c^2(\vec{k}^*\vec{r}^*+k^*r^*)}\right]+$$

$$+\left[\tilde{f}_c(\pi-\theta)+\frac{e^{2i\delta_0}-1}{2ik^*}e^{2i\delta_0^c}\right]\frac{e^{ik^*r^*-\frac{i}{k^*a_c}\ln 2k^*r^*}}{r^*}\Big\}.$$

Here $\tilde{f}_c(\pi-\theta)$ is the amplitude of Coulomb scattering, θ is the angle between $\vec{k}^*$ and $\vec{r}^*$.

The expression for the amplitude $\psi_{p_1p_2}(x_1,x_2)$ is essentially simplified when $r^*/a_c \ll 1$. In this case

$$F=1,\ \tilde{G}=\cos k^*r^*+iA_c^{(+)}\sin k^*r^* \tag{A.14}$$

and eqs. (4), (A.9)-(A.11) yield

$$\psi_{p_1p_2}(x_1,x_2)=\sqrt{A_c^{(+)}}\,e^{i\delta_0^c}\,e^{ip(x_1+x_2)}[e^{iqx/2}+f_c(k^*)\Phi^c_{p_1p_2}(x)], \tag{A.15}$$

where $f_c(k^*)=\dfrac{e^{2i\delta_0}-1}{2ik^*}\cdot\dfrac{1}{A_c^{(+)}(k^*)}$ coincides with eq. (32)

and $\Phi^c_{p_1p_2}(x)$ is given by eq. (31). Eq. (30) immediately follows from (A.15) and the definition (3) of the correlation function.

The correction to the correlation function connected with the deviation of the "true" wave function $e^{-i\vec{k}^*\vec{r}}+\phi_{\vec{k}^*}(\vec{r})$ inside the range of strong interaction from the Coulomb one $e^{-i\vec{k}^*\vec{r}}+\tilde{\phi}_{\vec{k}^*}(\vec{r})$, given by eq. (A.11), can be taken into account in a similar way as in the case of two neutral particles. According to the effective radius theory /16-18/, there is the integral relation analogous to eq. (20):

$$\frac{1}{4\pi}\int[\,|\phi_{\vec{k}^*}(\vec{r})+e^{-i\vec{k}^*\vec{r}}|^2-|\phi_{\vec{k}^*}(\vec{r})+e^{-i\vec{k}^*\vec{r}}|^2]\,d^3\vec{r}= \tag{A.16}$$

$$=A_c^{(+)}(k^*)\,|f_c(k^*)|^2\,d_0/2\,.$$

From (A.16) it follows that the function $B_i(q,p)$ in eq. (30) includes the correction term proportional to the effective radius (see eq. (21)).

REFERENCES

1. Hanbury-Brown R., Twiss R.G. Phil.Mag., 1954, 45, p. 633.
2. Grishin V.G., Kopylov G.I., Podgoretsky M.I. Yad.Fiz., 1971, 13, p. 1116.

3. Kopylov G.I., Podgoretsky M.I. Yad.Fiz., 1973, 18, p. 656.
4. Kopylov G.I. Phys.Lett., 1974, 50B, p. 472.
5. Cocconi G. Phys.Lett., 1974, 49B, p. 459.
6. Watson K.W. Phys.Rev., 1952, 88, p. 1163.
7. Migdal A.B. JETP, 1955, 28, p. 1.
8. Baldin A.M. Nuovo Cim., 1958, 8, p. 569; JETP, 1960, 38, p. 579.
9. Baldin A.M. Near Threshold Pion Photoproduction. Proceedings of the P.N.Lebedev Physical Institute, v. 19, p. 3-36, ed. Acad. Nauk USSR, Moscow, 1963 (in Russian).
10. Baldin A.M. Lebedev A.I. JETP, 1957, 33, p. 1221.
11. Kopylov G.I. Yad. Fiz., 1977, 25, p. 1091.
12. Koonin S.E. Phys.Lett., 1977, 70B, p. 43.
13. Styulassy M. et al. Phys.Rev., 1979, C20, p. 2267.
14. Landau L.D., Lifshitz E.M. Quantum Mechanics. §§130, 133, 136, 138. Moscow, Nauka, 1974 (in Russian).
15. von Witsch W. et al. Phys.Lett., 1979, 80B, p. 187.
16. Landau L.D., Smorodinsky Ya.A. JETP, 1944, 14, p. 269.
17. Smorodinsky Ya.A. Dokl. Acad.Nauk USSR, 1948, 60, p. 217.
18. Bethe H.A. Phys.Rev., 1949, 76, p. 38.
19. Kopylov G.I. Podgoretsky M.I. Yad.Fiz., 1972, 15, p. 392.
20. Cooper A.M. et al. Nucl.Phys., 1978, B139, p. 45.
21. Lyuboshitz V.L., Podgoretsky M.I. Yad.Fiz., 1979, 30, p. 789.
22. Martin A.D., Ozmutha E.N., Squires E.J. Nucl.Phys., 1977, B121, p. 514.
23. Belkov A.A., Bunyatov S.A. Yad.Fiz., 1979, 29, p.1295; Yad. Fiz., 1981, 33, p. 727.
24. Bayukov Yu.D. et al. Preprint ITEF-80, Moscow, 1980; Preprint ITEF-86, Moscow, 1980.
25. Angelov N. et al. Yad.Fiz., 1980, 32, p. 1357.
26. Blatt J.M., Jackson J.D. Phys.Rev., 1949, 76, p. 18.
27. Mott N., Massey H. The Theory of Atomic Collisions. ch.3- Oxford, 1965.

Received by Publishing Department
on July 23 1981.