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AUTOMODEL ASYMPTOTICS AND LIGHT CONE BEHAVIOUR IN QUANTUM FIELD THEORY

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1. Within theoretical explanation of the experimental data on the automodel behaviour of some quantities of quantum field theory at high energies and large momentum transfers in particular form-factors for deep inelastic lepton-hadron scattering it was established that this phenomenon does not contradict the general principles of local quantum field theory [1], that it is closely connected with a singular structure of the Fourier-transform of these quantities in the neighbourhood of the light cone, that there are some restrictions upon possible asymptotics. This problem has been extensively discussed in literature starting from the Björken paper [2] 1967 (see also [3-9], a detailed list of involving papers up to 1979 is contained in survey [10]; in mathematical aspect-after papers by Bogolubov, Vladimirov and Tavkhelidze [11] 1972 (see also [12-14]).

2. Although explicit forms of quantities are unknown some information about them follows from the general properties of the local quantum field theory [1]. So an arbitrary matrix elements of the current commutators

$$F(q) = \int \langle A|[J_1(\tfrac{x}{2}), J_2(-\tfrac{x}{2})]|B\rangle e^{iqx}\, dx$$

obeys the following properties: o) $F \in S'(R^4)$, 1) $F(q)=0$, $|q_0| < |\mathbf{q}| - \Lambda\ (\Lambda > 0)$ (spectrality), 2) $\tilde{F}(x) = 0$, $x^2<0$ (local commutativity).

Here

$$q = (q_0, \mathbf{q}),\ \mathbf{q} = (q_1, q_2, q_3),\quad qx = q_0 x_0 - (\mathbf{q}, \mathbf{x}),\quad x^2 = x_0^2 - |\mathbf{x}|^2 .$$

Let $\rho(k)$ be an automodel (proper varying) function of an order α (for this and other definitions see [14]). If $F(q)$ satisfies an asymptotic relations

$$F(q) \sim \rho(|\nu|) F_{\pm}(\xi, \mathbf{n}),\quad \nu \to \pm\infty, \tag{1}$$

where

$$\xi = -\frac{q^2}{2|q_0|\Delta},\quad \nu = \frac{2q_0}{\Delta},\quad \mathbf{n} = \frac{\mathbf{q}}{|\mathbf{q}|} \tag{2}$$

so we shall say that $F(q)$ has an automodel asymptotics in the Björken domain with respect to ρ (or briefly automodel asymptotics). The exact meaning of the relations (1) will be given later.

In this paper we formulate a theorem of the tauberian type on one-to-one correspondence between automodel asymptotics of $F(q)$ satisfying conditions 0)-2) and an asymptotic behaviour of this Fourier-transform $\tilde{F}(x)$ in the neighbourhood of the light cone $x^2 \sim 0$; we derive also formulas connecting these asymptotics in particular "sum rules" for $F_\pm(\xi, n)$. Note that in paper[14] the authors established these similar results under additional assumptions that $F(q)$ is odd and rotation invariant. Here we get rid of these assumptions, so our results can be applied to the study of many-particles scattering at high energies[5]. A detailed exposition of the results with proofs will be printed in one of the nearest issues of the journal TMP.

3. A cut factor $\chi(\xi,\nu)$ is called admissible if $\chi \in C^\infty(R^2)$, $|D^\alpha \chi(\xi,\nu)| \le C_\alpha$ and

$$\chi(\xi,\nu) = \begin{cases} 0, & \xi < -\dfrac{|\nu|}{4} + \dfrac{a^2}{|\nu|}, \\ 1, & \xi > -\dfrac{|\nu|}{4} + \dfrac{b^2}{|\nu|}, \quad b > a > 2. \end{cases}$$

For $F(q)$ from $\mathcal{S}'(R^4)$ satisfying conditions 1)-2) of §2 we introduce a distribution $\mathcal{F}_\chi(\xi,\nu,n)$ from $\mathcal{S}'(R^2) \otimes \mathcal{E}'(S^2)$ by Eq.

$$\mathcal{F}_\chi(\xi,\nu,n) = \chi\left(-\frac{q^2}{2|q_0|\Delta}, \frac{2q_0}{\Delta}\right) F(q). \qquad (3)$$

It was proved that if $\phi \in \mathcal{S}(R^1)$, then $(\mathcal{F}_\alpha(\cdot,\nu,n), \phi(\cdot)) \in C^\infty(R^1 \times S^2)$.

Definition 1. We say that $F(q)$ has an automodel asymptotics with respect to an automodel function ρ along the direction $n_0 \in S^2$ if for some admissible cut factor χ there exists a neighbourhood $U \subset S^2$ of n_0 such that

$$\frac{1}{\rho(|\nu|)} \mathcal{F}_\chi(\xi,\nu,n) \xrightarrow{n \subset U} F_\pm(\xi,n), \quad \nu \to \pm\infty \quad \text{in } \mathcal{S}'(R^1_\xi) \qquad (4)$$

provided $(F_+, F_-) \neq 0$. If (4) take place for all $n \in S^2$, then we say that $F(q)$ has an automodel asymptotics with respect to ρ. This fact formally will be written in the form (2).

Remark. In fact the limits (4) exist for all admissible cut factors and they do not depend on χ.

4. Definition 2. We say that a distribution $f(x_0, x)$ from $\mathcal{S}'_+ \otimes \mathcal{S}'(R^n)$ has a quasiasymptotics in x_0 at ∞ (resp. at 0) with respect to an automodel function ρ of an order a if there exist limits respectively[12,14].

$$\lim_{k\to\infty} \frac{f(kx_0,x)}{\rho(k)} = f_{a+1}(x_0) \times g_\infty(x) \quad \text{in } \mathcal{S}'_+ \otimes \mathcal{S}'(R^n), \quad g_\infty \neq 0;$$

$$\lim_{k\to\infty} \frac{f(x_0/k,x)}{\rho(k)} = f_{-a+1}(x_0) \times g_0(x) \quad \text{in } \mathcal{S}'_+ \otimes \mathcal{S}'(R^n), \quad g_0 \neq 0,$$

where f_a is the canonical kernel of fractional differentiation (integration).

Let $\tilde{F} \in \mathcal{S}'(\bar{V})$, i.e. $\tilde{F} \in \mathcal{S}'(R^4)$ and $\operatorname{supp} \tilde{F} \subset \bar{V} = [x : x^2 \ge 0]$. We split $\tilde{F}(x)$ into odd $\tilde{F}_a(x)$ and even $\tilde{F}_s(x)$ parts in x_0, $\tilde{F} = \tilde{F}_a + \tilde{F}_s$, and introduce distributions $\Phi_j(\kappa, x)$, $j = a, s$ from $\mathcal{S}'_+ \otimes \mathcal{S}'(R^3)$ by Eqs.: for any $\phi \in \mathcal{S}_+ \otimes \mathcal{S}(R^3)$

$$(\Phi_a, \phi) = (\tilde{F}_a(x), x_0 \phi(x^2, x)), \quad (\Phi_s, \phi) = (\tilde{F}_s(x), \phi(x^2, x)).$$

Formally:

$$F(x) = \epsilon(x_0)\Phi_a(x^2, x) + |x_0|\Phi_s(x^2, x). \qquad (5)$$

The transformation $\tilde{F} \to (\Phi_a, \Phi_s)$ is an isomorphism of $\mathcal{S}'(\bar{V})$ onto $(\mathcal{S}'_+ \otimes \mathcal{S}'(R^3)) \oplus (\mathcal{S}'_+ \otimes \mathcal{S}'(R^3))$.

Definition 3. We say that $\tilde{F}(x)$ has a quasiasymptotics in a neighbourhood of the light cone with respect to an automodel function ρ of an order a if $\Phi_j(\kappa, x)$ have a quasiasymptotics in κ at 0 with respect to ρ:

$$\frac{\Phi_j(\kappa/k, x)}{\rho(k)} \to f_{-a+1}(\kappa) \times g_j(x), \quad k \to \infty \quad \text{in } \mathcal{S}'_+ \otimes \mathcal{S}'(R^3), \quad j = a, s \qquad (6)$$

provided $(g_a, g_s) \neq 0$. Formally we will write (6) so (cf. (5)):

$$\tilde{F}(x) \sim \frac{\rho[(x_+^2)^{-1}]}{\Gamma(1-a)} [\epsilon(x_0) g_a(x) + |x_0| g_s(x)], \quad x^2 \sim 0, \qquad (7)$$

where $x_+^2 = \theta(x^2) x^2$.

5. Let a distribution $F(q)$ from $\mathcal{S}'(R^4)$ satisfies the conditions 1)-2) of §2 and ρ be an automodel function of an order a. The following results hold.

Theorem 1. If $F(q)$ has an automodel asymptotics with respect to ρ along the directions n_0 and $-n_0$ then there exists such a neighbourhood $U \subset S^2$ of n_0 such that distributions $\Phi_j(\kappa, \sigma n)$, $j = a, s$, $n \in U$, in the variables (κ, σ) have a quasiasymptotics in κ at 0 with respect to $\kappa^2 \rho(\kappa)$ uniformly in $n \in U$.

Remark. Distributions $\Phi_j(\kappa, n)$ from $(\mathbb{S}'_+ \otimes \mathbb{S}'(R^3)$ are entire functions in x /14/ so distributions $\Phi_j(\kappa, \sigma n)$ are well defined.

Theorem 2. F(q) has an automodel asymptotics (2) with respect to $\rho(k)$ if and only if $\tilde{F}(x)$ has a quasiasymptotics in a neighbourhood of the light cone with respect to $\kappa^2 \rho(\kappa)$,

$$\tilde{F}(x) \sim \frac{(x_+^2)^{-2}\rho[(x_+^2)^{-1}]}{\Gamma(-a-1)}[\epsilon(x_0)g_a(x) + |x_0| g_s(x)], \quad x^2 \sim 0. \tag{8}$$

In addition the distributions $F_\pm(\xi, n)$, $g_a(x)$ and $g_s(x)$ are connected by the following Eqs.

$$g_a(\lambda n) = -\Delta^{1-a} \pi^2 4^{a+2} e^{i\frac{\pi a}{2}} (\lambda - i0)^{a+1} \frac{\tilde{F}_+(\Delta\lambda, n) - \tilde{F}_-(\Delta\lambda, n)}{2},$$

$$g_s(\lambda n) = \Delta^{1-a} \pi^2 4^{a+2} e^{i\frac{\pi a}{2}} (\lambda - i0)^{a} \frac{\tilde{F}_+(\Delta\lambda, n) + \tilde{F}_-(\Delta\lambda, n)}{2}, \tag{9}$$

where $\tilde{F}_\pm(t, n)$ is the Fourier-transform of $F_\pm(\xi, n)$ in ξ. The distributions

$$F_a(\xi, n) = \frac{1}{2}[{}^{(a+1)}F_+(\xi, n) - {}^{(a+1)}F_-(\xi, n)],$$

$$F_s(\xi, n) = \frac{1}{2}[{}^{(a)}F_+(\xi, n) + {}^{(a)}F_-(\xi, n)]$$

have to be even, $F_j(\xi, n) = F_j(-\xi, -n)$, and have to satisfy the following Eqs. ("sum rules")

$$\int_{-\infty}^{\infty} F_j(\xi, n)\xi^k d\xi = \sum_{|\beta|=k} a_\beta^{(j)} n^\beta, \quad k = 0, 1, \dots, \tag{10}$$

where $\{a_\beta^{(j)}, |\beta| \geq 0\}$ is a moment sequence of a distribution $\psi_j(u)$ with a support in the closed unit ball. In particular from (10) the Eqs. follow that

$$\int_{-\infty}^{\infty} \int_{|n|=1} F_j(\xi, n)\xi^k \mathcal{P}_m(n)\, dn\, d\xi = 0, \quad m > k, \quad k = 0, 1, \dots, \tag{11}$$

where $\mathcal{P}_m(x)$ is any harmonic polynomial of degree m (j = a, s). Here ${}^{(a)}F$ is the left (fractional) derivative (primitive) of

$$F: {}^{(a)}F = F * \hat{f}_{-a} = \widehat{\hat{F} * f_{-a}} = \widehat{\hat{F}^{(a)}}, \quad \hat{F}(\xi) = F(-\xi).$$

Remark. The distributions $F_j(\xi, n)$ are the Radon-transforms /15/ of the distributions $\psi_j(u)$ resp.

6. For proofs of these results the methods developed in the paper /14/ as well as a refined version of the Jost-Lehmann-Dyson (J-L-D) integral representation for symmetric in q_0 domains of spectrality /14, 18/ are used. Let

$$\psi(\lambda, u) = \psi_a(\lambda, u) \times \delta(u_0) + \psi_s(\lambda, u) \times \delta'(u_0)$$

be a spectral function for F involving in the J-L-D-representation with weight functions ψ_a and ψ_s from $\mathbb{S}'_+ \otimes \mathbb{S}'(R^3)$. Then the following inversion formulas for J-L-D-representation /14/ hold

$$\psi_a(\lambda, u) = \frac{1}{4\pi^2 i} F_x^{-1}[B_\kappa[\Phi_a]], \quad \psi_s(\lambda, u) = \frac{1}{4\pi^2} F_x^{-1}[B_\kappa[\Phi_s]] \tag{12}$$

and also the transformation $\tilde{F} \to (\psi_a, \psi_s)$ is an isomorphism of $\mathbb{S}'(V)$ onto $(\mathbb{S}'_+ \otimes \mathbb{S}'(R^3)) \oplus (\mathbb{S}'_+ \otimes \mathbb{S}'(R^3))$.

In addition for any $\phi \in \mathbb{S}(R^1)$ satisfying conditions

$$\int_{-\infty}^{\infty} \phi(\lambda)\lambda^k d\lambda = 0, \quad k = 0, 1, \dots, N$$

for sufficiently large N which depends only on the order of F the following Eqs. hold

$$(\psi_a(\lambda, u), \phi(-\lambda + (a, u))) = \frac{1}{\pi^{3/2}}(F(q), q_0 \phi^{(3/2)}(-\frac{|a|^2}{4} - q_0^2 + (a, q))),$$

$$(\psi_s(\lambda, u), \phi(-\lambda + (a, u))) = \frac{1}{2\pi^{3/2}}(F(q), \phi^{(5/2)}(-\frac{|a|^2}{4} - q_0^2 + (a, q))) \tag{13}$$

for all $a \in R^3$.

We also used essentially the following Lemma about the existence of a quasiasymptotics. This Lemma has an independent significance.

Lemma. Let ρ be an automodel function and let $f(x_0, x)$ be from $\mathbb{S}'(T_0)$ where $T_0 = [(x_0, x): x_0 \geq 0, |x| \leq 1]$ and $f(x_0, x)$ possesses the following properties: for all $\phi \in \mathbb{S}(R^1)$, $n \in S^{n-1}$

$$\left|\left(\frac{f(kx_0, x)}{\rho(k)}, \phi(-x_0 + (n, x))\right)\right| \leq C_\phi, \quad k \to \infty$$

and there exists a limit

$$\lim_{k \to \infty} \left(\frac{f(kx_0, x)}{\rho(k)}, \phi(-x_0 + (n, x))\right) \neq 0.$$

Then f has a quasiasymptotics in x_0 at ∞ with respect to ρ.

Corollary. If $f \in \mathbb{S}'_+$ such that for some even $h \in \mathcal{E}'(R^1)$ and for all $\phi \in \mathbb{S}(R^1)$ there exists a limit

$$\lim_{k\to\infty} \left(\frac{f(k\lambda)}{\rho(k)}, h * \phi \right) \neq 0,$$

then $f(\lambda)$ has a quasiasymptotics in λ at ∞ with respect to ρ.

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