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ON THE SCOPE OF SUPERSYMMETRIC DIMENSIONAL REGULARIZATION

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The supersymmetric dimensional regularization or regularization by dimensional reduction (RDR) proposed [1] as an invariant regularization for globally supersymmetric models is now known to be inconsistent [2]. However, it works fairly well in practical calculations, preserving the supersymmetric Ward identities (WI) to two loops [3,4,5]. In this connection some natural questions arise: 1) Can one reformulate RDR in a consistent fashion? 2) Would it be invariant under the global supersymmetry transformations? 3) Are the results of the recent RDR calculations reliable, in particular the nullification of β-function in the $N=4$ supersymmetric gauge model [6] up to three loops [5,7]? In this paper we answer yes to the first and third, and no to the second question.

Inconsistency of RDR discovered in [2] stems from the relation

$$\epsilon_{\mu_1\mu_2\mu_3\mu_4}\,\epsilon_{\nu_1\nu_2\nu_3\nu_4} = -\begin{vmatrix} g_{\mu_1\nu_1} & \cdots & g_{\mu_4\nu_1} \\ \vdots & & \vdots \\ g_{\mu_1\nu_4} & \cdots & g_{\mu_4\nu_4} \end{vmatrix} \equiv -\det(\mu_1\mu_2\mu_3\mu_4,\nu_1\nu_2\nu_3\nu_4), \quad (1)$$

where $\epsilon_{\mu_1\ldots\mu_4}$ is the totally antisymmetric tensor and $g_{\mu\nu}$ is the metric tensor of 4-dimensional Minkowski space. This relation turns out to be compatible with the decompisition

$$g_{\mu\nu} = \hat{g}_{\mu\nu} + \tilde{g}_{\mu\nu}\,, \quad (2)$$

where $\hat{g}_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ are symmetric projection operators with the properties

$$\hat{g}_{\mu a}\hat{g}_{a\nu} = \hat{g}_{\mu\nu}\,,\quad \tilde{g}_{\mu a}\tilde{g}_{a\nu} = \tilde{g}_{\mu\nu}\,,\quad \hat{g}_{\mu a}\tilde{g}_{a\nu} = 0,\ \hat{g}_{\mu\mu} = d,\ \tilde{g}_{\mu\mu} = 4-d \quad (3)$$

only for non-negative integral $d \le 4$. RDR of ref. [1] based on the dimensional reduction to $d=4-2\epsilon$ dimensions relies heavily on eqs. (1)-(3) and is therefore inconsistent [2].

In the superfield formalism one cannot eliminate this discrepancy by merely dropping all relations which involve $\epsilon_{\mu_1\ldots\mu_4}$. Really, the supergraph Feynman rules [8] imply the spinor indices α and $\dot\alpha$ being necessarily double-valued. This only allows reducing products of arbitrary number of covariant derivatives to no more than four. Since the combination $(\alpha, \dot\alpha)$ acts effectively as the 4-dimensional Lorentz index, one can construct [2] all ingredients of eqs. (1)-(3) out of the spinor-index quantities.

Although incurable in terms of superfields, RDR can be consistently written in the component-field language. To achieve this goal, we must give up not only eq. (1) with its corollaries but also an ability to count spinor indices, as if they might run through nonintegral number of values.

The regularization is carried out in two steps. At first we change over from Minkowski space with four-component vectors and Majorana spinors to the quasi-four-dimensional space (Q4S) with "nonintegral-valued" vector and spinor indices and without $\epsilon_{\mu_1 \dots \mu_4}$. In this formal space we retain the properties $g_{\mu\mu}=4$ and $\mathrm{tr}\,\mathbf{1}=4$ as relics of four dimensions. Then we perform the dimensional reduction based on (2) and (3) from Q4S to a d-dimensional space. All momenta and coordinates become d-dimensional, the differentiation with respect to $(4-d)$-dimensional coordinates vanishes ($\tilde\partial_\mu = 0$), and the vector field A_μ splits effectively into a d-dimensional vector $\hat A_\mu = \hat g_{\mu\nu} A_\nu$ and a $(4-d)$-dimensional scalar $\tilde A_\mu = \tilde g_{\mu\nu} A_\nu$ [1,3].

For example, the Lagrangian of the vector multiplet becomes

$$\mathcal{L} = -\frac{1}{4}(\hat F^a_{\mu\nu})^2 + \frac{i}{2}\bar\psi^a \hat\gamma_\mu \hat D_\mu \psi^a - \frac{1}{2}(\hat D_\mu \tilde A^a_\nu)^2 + \frac{ig}{2} f^{abc} \bar\psi^a \tilde\gamma_\mu \tilde A^b_\mu \psi^c - \frac{g^2}{4}(f^{abc} \tilde A^b_\mu \tilde A^c_\nu)^2 \tag{4}$$

with $\hat F_{\mu\nu}$ and the covariant derivatives $\hat D_\mu$ formed out of $\hat A_\mu$. In (4) the matrices γ_μ are decomposed into

$$\gamma_\mu = \hat\gamma_\mu + \tilde\gamma_\mu, \quad \hat\gamma_\mu = \hat g_{\mu\nu}\gamma_\nu, \quad \tilde\gamma_\mu = \tilde g_{\mu\nu}\gamma_\nu . \tag{5}$$

The only properties of $\hat\gamma_\mu$ and $\tilde\gamma_\mu$ we may exploit are the commutation relations obtained from

$$[\gamma_\mu, \gamma_\nu]_+ = 2g_{\mu\nu}\mathbf{1} \tag{6}$$

with the use of projection operators $\hat g_{\mu\nu}$ and $\tilde g_{\mu\nu}$:

$$[\hat\gamma_\mu, \hat\gamma_\nu]_+ = 2\hat g_{\mu\nu}\mathbf{1}, \quad [\tilde\gamma_\mu, \tilde\gamma_\nu]_+ = 2\tilde g_{\mu\nu}\mathbf{1}, \quad [\hat\gamma_\mu, \tilde\gamma_\nu]_+ = 0 . \tag{7}$$

These relations (plus $g_{\mu\mu}=4$, $\mathrm{tr}\,\mathbf{1}=4$, and a cyclic property of traces) suffice for computing diagrams. Assuming the nonintegral-valued spinor indices, we may not use other properties of the 4-dimensional Dirac matrices, like Fierz identities.

We now study invariance features of RDR under the global supersymmetry transformation written for brevity in the Q4S form:

$$\delta A^a_\mu = i\bar\xi\gamma_\mu\psi^a, \quad \delta\psi^a = \frac{1}{2}F^a_{\mu\nu}\gamma_\mu\gamma_\nu\xi , \tag{8}$$

ξ being a constant Majorana spinor. The variation of $\mathcal{L}$ (4) under (8) is (up to a total derivative)

$$\delta\mathcal{L} = \frac{g}{2} f^{abc}(\bar\xi\gamma_\mu\psi^a)(\bar\psi^b\gamma_\mu\psi^c). \tag{9}$$

In Minkowski space $\delta\mathcal{L} = 0$ due to an appropriate Fierz identity. In Q4S this argument fails, and $\delta\mathcal{L}$ may be nonzero, but we cannot check this directly in the form (9). Since the dimensionally regularized momentum integration respects all symmetries, $\delta\mathcal{L} \neq 0$ only can cause RDR to be noninvariant. Hence we may call this regularization invariant if $\delta\mathcal{L}$ (9) does not contribute to any supersymmetric WI. However, it does, as we shall see below, thus rendering RDR noninvariant.

The supersymmetric WI's are normally derived through the change (8) of the path integral variables in the generating functional $Z(J_\mu, \eta)$ followed by differentiating it with respect to sources and ξ. Consider a contribution of $\delta\mathcal{L}$ to WI obtained by differentiating $Z(J_\mu, \eta)$ with respect to three spinor sources $\eta^a_\alpha(x_1), \bar\eta^b_\beta(x_2), \eta^c_\gamma(x_3)$ and to a constant spinor $\bar\xi_\delta$. It reads

$$\frac{g}{2}\langle T\bar\psi^a_\alpha(x_1)\psi^b_\beta(x_2)\bar\psi^c_\gamma(x_3)\int dx_4 f^{deh}(\gamma_\mu\psi^d(x_4))_\delta \bar\psi^e(x_4)\gamma_\mu\psi^h(x_4)e^{iS}\rangle_0 . \tag{10}$$

Let us multiply (10) by $(\hat p_1 A \hat p_2)_{\alpha\beta}(\hat p_3 B)_{\gamma\delta}$ with p_1, p_2, p_3 being the Fourier transforms of x_1, x_2, x_3, $\hat p \equiv p_\mu\hat\gamma_\mu$ and $A = \gamma_{\mu_1}\dots\gamma_{\mu_k}$, $B = \gamma_{\nu_1}\dots\gamma_{\nu_m}$. The nonzero result would imply that RDR breaks supersymmetry. In the tree approximation we come (up to an overall factor) to the following quantity:

$$\Delta(A,B) = \mathrm{tr}(A\gamma_\mu)\,\mathrm{tr}(B\gamma_\mu) + \mathrm{tr}[\gamma_\mu(A-(-)^k A_R)\gamma_\mu B] . \tag{11}$$

A_R is a product of the same γ_μ-symbols as in A but written in the reverse order. For Dirac matrices the relation $\Delta(A,B)=0$ is proved with arbitrary A and B [9]. In Q4S we have to check it by a direct calculation relying only on (6). This is done straightforwardly if A or B consists of no more than 4 γ_μ -

symbols. However, choosing $A = \gamma_{\mu_1} \dots \gamma_{\mu_5}$, $B = \gamma_{\nu_1} \dots \gamma_{\nu_5}$ yields

$$\Delta(\gamma_{\mu_1} \dots \gamma_{\mu_5}, \gamma_{\nu_1} \dots \gamma_{\nu_5}) = 48 \det(\mu_1 \dots \mu_5, \nu_1 \dots \nu_5). \tag{12}$$

Determinant (12) is the simplest combination of $g_{\mu\nu}$ -tensors which is identical zero in the Minkowski space and nonzero in Q4S because of

$$\hat{g}_{\mu_1\nu_1} \dots \hat{g}_{\mu_5\nu_5} \det(\mu_1 \dots \mu_5, \nu_1 \dots \nu_5) = d(d-1)(d-2)(d-3)(d-4). \tag{13}$$

The non-invariance of RDR is thus shown.

Consider now the propagator-type WI studied in [3,4]. It is derived by differentiating $Z(J_\mu, \eta)$ with respect to η and J_μ. The corresponding diagrams depend on a single momentum p, carry one Lorentz-index μ and two spinor indices. A contribution of $\delta\mathcal{L}$ to this identity is of the form

$$W_\mu(p) = X(p^2)p^2\gamma_\mu + Y(p^2)p_\mu\hat{p}. \tag{14}$$

$W_\mu(p)$ can be calculated, for instance, through the evaluation of $\mathrm{tr}(W_\mu\gamma_\nu)$ which looks like (11) with A and B being determined by particular diagrams and including odd numbers of γ_μ-symbols (γ_ν is absorbed into B). Since the determinant (12) is annihilated by any contraction with $g_{\mu\nu}$, it can produce a nonzero contribution only when contracted with at least 8 momenta (the indices μ and ν remain free). This can occur from the 4-loop level on. Thus the propagator WI is valid without fail through three loops. In papers [3,4] its validity has been verified explicitly at the one-loop level.

Our next example concerns the non-abelian gauge model with the N=4 supersymmetry obtained by dimensional reduction from the 10-dimensional space [6]. It is quite natural to carry out the regularization also by the reduction from the quasi-ten-dimensional space directly to $d = 4 - 2\epsilon$ dimensions. This program is accomplished in ref. [5], where the symbols γ_μ, a^r; β^t can be aggregated into Γ_μ-matrices with the properties

$$[\Gamma_\mu, \Gamma_\nu] = 2G_{\mu\nu} I, \quad \mathrm{tr}\, I = 32, \quad G_{\mu\mu} = 10. \tag{15}$$

Formula (11) is to be substituted by

$$\Delta = \frac{1}{2}\mathrm{tr}(A\Gamma_\mu)\mathrm{tr}(B\Gamma_\mu) + \mathrm{tr}[\Gamma_\mu(A + A_R)\Gamma_\mu B]. \tag{16}$$

Since $\Delta = 0$ in the true ten-dimensional space, the supersymmetry-breaking effects like (12) are controlled by the determinants 11x11 and can display themselves in the propagator WI from 10 loops on, and in the vertex WI obtained by differentiating $Z(J_\mu, \eta)$ with respect to η, J_μ and J_ν from 8 loops on.

To conclude, we note that a detailed investigation of diagrams would probably enlarge the domains of invariance of RDR pointed out above.

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REFERENCES

1. Siegel W. Phys.Lett., 1979, 84B, p.193.
2. Siegel W. Phys.Lett., 1980, 94B, p.37.
3. Capper D.M., Jones D.R.T., van Nieuwenhuizen P. Nucl.Phys., 1980, B167, p.479.
4. Majumdar P., Poggio E.C., Schnitzer H. Phys.Rev., 1980, D21, p.2203.
5. Avdeev L.V., Tarasov O.V., Vladimirov A.A. Phys.Lett., 1980, 96B, p.94.
6. Brink L., Schwarz J.H., Scherk J. Nucl.Phys., 1977, B121, p.77; Gliozzi F., Scherk J., Olive D. Nucl.Phys., 1977, B122, p.253.
7. Grisaru M., Rocek M., Siegel W. Phys.Rev.Lett., 1980, 45, p.1063; Caswell W.E., Zanon D. Phys.Lett., 1981, 100B, p.152.
8. Grisaru M., Siegel W., Rocek M. Nucl.Phys., 1979, B159, p.429.
9. Chisholm J.S.R. Nuovo Cim., 1963, 30, p.426.

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