сообщения
объединенного
института
ядерных
исследований
дубна

E2-81-331

V.V.Uzhinsky

NUCLEUS-NUCLEUS INELASTIC INTERACTION CROSS SECTIONS CALCULATING PROCEDURE

1981

At present a problem of building the theoretical methods of description of inelastic nucleus-nucleus collisions is very actual in high energy nuclear physics because of the necessity of interpretation of the existing experimental data and a dissatisfaction of the results of known theoretical models most of which contain considerable arbitrariness in their foundation. Thus hopes paid to Glauber eikonal approximation are quite understandable. Below a key problem of eikonal approach-determination of nucleus-nucleus inelastic interaction cross section of different processes will be considered.

According to basis principles of Glauber approximation the cross section of all nucleus-nucleus scattering processes accompanied by new particle production is determinated by the following expression

$$\sigma^{p}_{AB} = \int d^2b \{1 - \int [\prod_{i=1}^{A}\prod_{j=1}^{B}(1-\sigma\cdot g(\vec{b}-\vec{s}_i+\vec{r}_j))]\cdot[\prod_{i=1}^{A}\rho_A(\vec{s}_i)\,d^2s_i]\times$$

$$\times[\prod_{j=1}^{B}\rho_B(\vec{r}_j)d^2r_j]\} = \int d^2b\{1-\exp[-\chi(\vec{b},\sigma)]\} \qquad (1)$$

$$\sigma\cdot g(\vec{b}) = \gamma(\vec{b}) + \gamma^*(\vec{b}) - \gamma(\vec{b})\gamma^*(\vec{b}) \qquad \int g(\vec{b})\,d^2b = 1,$$

where A and B are mass numbers of colliding nuclei; ρ_A and ρ_B are one-particle densities of these nuclei integrated by longitudinal coordinates; $\gamma(\vec{b})$ is a nucleon-nucleon elastic scattering amplitude in the impact parameter representation; σ is an inelastic nucleon-nucleon cross section. $\chi(\vec{b},\sigma)$ is the so-called phase function having different form in different approaches/1/. In particular, in the optical limit on A and B we have/2/

$$\chi(\vec{b},\sigma) = \frac{1}{\sigma}\int d^2s[x\cdot y - \frac{x^2y}{2!} - \frac{xy^2}{2!} + \frac{x^3y}{3!} + \frac{xy^3}{3!} + \frac{4}{2!2!}x^2y^2 + \dots] \qquad (2)$$

$$x = \sigma\cdot A\cdot\rho_A(\vec{s}) \qquad y = \sigma\cdot B\cdot\rho_B(\vec{b}-\vec{s})$$

by this phase function is a sum of the contributions of scattering diagrams of the type shown in fig. 1, where each i-th vertical (j-th horizontal) line corresponds to the function $A\cdot\rho_A(\vec{s}_i)(B\cdot\rho_B(\vec{r}_j))$ and the point $\sigma\cdot g(\vec{b}-\vec{s}_i+\vec{r}_j)$.

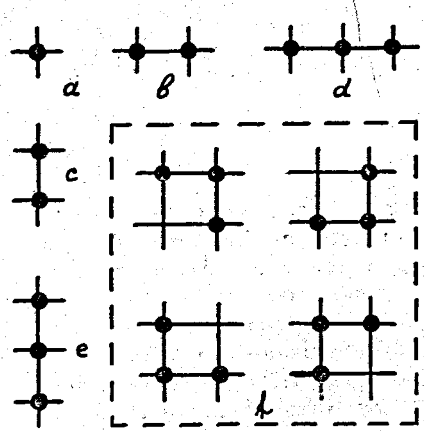

Fig.1. Diagrams representing first (a), second (b) and other terms in eq. (2).

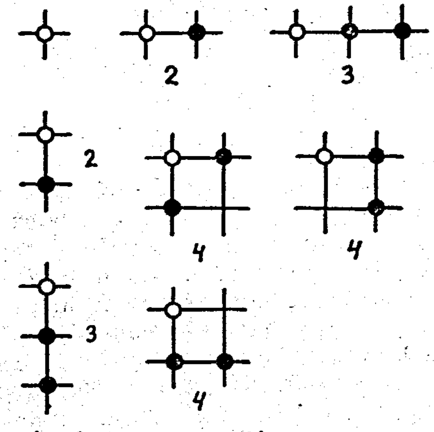

Fig.2. Simplest diagrams corresponding to processes with one inelastic collision of nucleons.

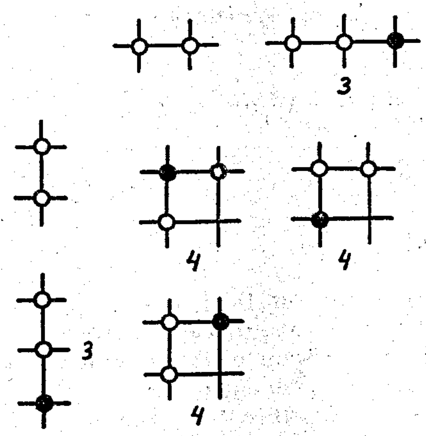

Fig.3. Simplest diagrams representing prosesses with two inelastic collisions of nucleons.

Using the probability interpretation of Glauber approximation, let's represent σ^p_{AB} as a sum of the cross sections of processes with different number ν of inelastic nucleon-nucleon collisions.

$$\sigma^p_{AB} = \sum_{\nu=1}^{AB} \sigma_\nu , \qquad (3)$$

$$\sigma_\nu = \frac{(-\sigma)^\nu}{\nu!} \int d^2b \frac{d^\nu}{d\sigma^\nu} e^{-\chi(\vec{b},\sigma)} . \qquad (4)$$

Let's mark that each term of expansion (3) is, in reality the cross section of the processes with fixed number of inelastic interactions, but with different numbers of elastic rescatterings of constituenting nucleons. For example, σ_1 is defined by the contributions of diagrams contained, as a rule, in their structure one of the diagrams shown in fig. 2, where a black point corresponds to the function $\sigma \cdot g$, representing the elastic interaction of nucleons; while the open point, to the same function but without the σ factor, which gives the account of inelastic collision.

Paying attention to the connection of the diagrams shown in figs. 1 and 2, we formulate a method of building of inelastic process diagrams as a successive replacement of dark points in diagrams of fig. 1 into light ones. One use of this procedure leads to the diagrams of fig. 2, twice, to diagrams of fig. 3. The latter diagrams can be separated into one of three types:

a) Diagrams correspond to the processes in which one of the nucleons of nucleus A undergoes a double inelastic collision;
b) Diagrams correspond to the processes in which one of the nucleons of nucleus B undergoes a double inelastic collision;
c) Diagrams represent the processes in which two inelastic collisions are quasi independent, that means that they are connected by the following elastic rescattering of the nucleons.

In correspondence with this separation σ_2 is determined by the sum of three cross sections. In the case when only contributions of diagrams of fig. 1 are taken into account in the phase function, they are

$$\sigma_2^a = \int d^2b\, e^{-\chi} \frac{1}{\sigma} \int d^2s \left[\frac{xy^2}{2!} - \frac{xy^3}{2!} - x^2y^2\right],$$

$$\sigma_2^b = \int d^2b\, e^{-\chi} \frac{1}{\sigma} \int d^2s \left[\frac{yx^2}{2!} - \frac{yx^3}{2!} - x^2y^2\right], \qquad (5)$$

$$\sigma_2^c = \int d^2b\, e^{-\chi} \left\{\frac{\sigma^2}{2!}\left(\frac{d\chi}{d\sigma}\right)^2 - \frac{1}{\sigma} \int d^2s\, x^2y^2\right\}.$$

Analogous separation can be made for high multiple collision cross sections too.

Summing all what was said above, let's formulate more precisely the proposed method of theoretical describing of inelastic nucleus-nucleus interactions in the eikonal approximation.

It consist of

1) the determination of diagrams yielding the more essential contributions into phase function;
2) the calculation of the quantities σ_ν;
3) the extraction of the cross sections of different subprocesses in the quantities σ_ν.

The following step of nucleus-nucleus collisions modeling - determination of created particle characteristics can be realized with the help of the traditional methods (see ref. [4]).

In the case when expression $e^{-\chi(\vec{b},\sigma)}$ is defined but phase function isn't proposed method can be used with some inessential modifications because eq. (3), (4) stay fair.

In conclusion let's consider some examples.

Example 1.

Cross section of new particle production processes in deuteron-deuteron collisions according to eq. (1) is determined by the following expression

$$\sigma^{p}_{dd} = \int d^2b\{1-\langle\Psi_B\Psi_T|\prod_{i=1}^{2}\prod_{j=1}^{2}(1-\sigma\cdot g(\vec{b}-\vec{s}_i+\vec{r}_j))|\Psi_T\Psi_B\rangle \ , \qquad (6)$$

where Ψ_B and Ψ_T are the ground state wave functions of beam and target deuterons respectively. From it we have

$$\exp[-\chi(\vec{b},\sigma)] = \langle\Psi_B\Psi_T|\prod_{i=1}^{2}\prod_{j=1}^{2}(1-\sigma\cdot g(\vec{b}-\vec{s}_i+\vec{r}_j))|\Psi_T\Psi_B\rangle. \qquad (7)$$

Now using eq. (4), we find

$$\sigma_1 = \sigma\int d^2b\langle\Psi_B\Psi_T|\sum_{i,j=1}^{2} g(\vec{b}-\vec{s}_i+\vec{r}_j)\prod_{\substack{k=1\\k\neq i}}^{2}\prod_{\substack{\ell=1\\ \ell\neq j}}^{2}(1-\sigma\cdot g(\vec{b}-\vec{s}_k+\vec{r}_\ell))|\Psi_T\Psi_B\rangle$$

$$\sigma_2^a = \sigma^2\int d^2b\langle\Psi_B\Psi_T|\sum_{i=1}^{2} g(\vec{b}-\vec{s}_i+\vec{r}_1)\cdot g(\vec{b}-\vec{s}_i+\vec{r}_2)\times$$

$$\times(1-\sigma\cdot g(\vec{b}-\vec{s}_{3-i}+\vec{r}_1))(1-\sigma\cdot g(\vec{b}-\vec{s}_{3-i}+\vec{r}_2))|\Psi_T\Psi_B\rangle$$

$$\sigma_2^b = \sigma^2\int d^2b\langle\Psi_B\Psi_T|\sum_{i=1}^{2} g(\vec{b}-\vec{s}_1+\vec{r}_i)\cdot g(\vec{b}-\vec{s}_2+\vec{r}_i)\times$$

$$\times(1-\sigma\cdot g(\vec{b}-\vec{s}_1+\vec{r}_{3-i}))(1-\sigma\cdot g(\vec{b}-\vec{s}_2+\vec{r}_{3-i}))||\Psi_T\Psi_B\rangle$$

$$\sigma_2^c = \sigma^2\int d^2b\langle\Psi_T\Psi_B|\sum_{i=1}^{2} g(\vec{b}-\vec{s}_i+\vec{r}_1)\cdot g(\vec{b}-\vec{s}_{3-i}+\vec{r}_2)\times$$

$$\times(1-\sigma\cdot g(\vec{b}-\vec{s}_i+\vec{r}_2))\cdot(1-\sigma\cdot g(\vec{b}-\vec{s}_{3-i}+\vec{r}_1))|\Psi_B\Psi_T\rangle$$

$$\sigma_3 = \sigma^3\int d^2b\langle\Psi_B\Psi_T|\sum_{i=1}^{2}\sum_{j=1}^{2} g(\vec{b}-\vec{s}_i+\vec{r}_j)\cdot g(\vec{b}-\vec{s}_{3-i}+\vec{r}_j)\times$$

$$\times g(\vec{b}-\vec{s}_i+\vec{r}_{3-j})(1-\sigma\cdot g(\vec{b}-\vec{s}_{3-i}+\vec{r}_{3-j}))|\Psi_T\Psi_B\rangle$$

$$\sigma_4 = \sigma^4\int d^2b\langle\Psi_B\Psi_T|\prod_{i=1}^{2}\prod_{j=1}^{2} g(\vec{b}-\vec{s}_i+\vec{r}_j)|\Psi_T\Psi_B\rangle.$$

Example 2.

If we suppose that in the optical limit the phase function is defined only by first term

$$\chi(\vec{b},\sigma) = \sigma\cdot A\cdot B\cdot\int\rho_A(\vec{s})\cdot\rho_B(\vec{b}-\vec{s})d^2s, \qquad (8)$$

then, using eq. (4), we arrive at the results of ref. /5/.

$$\sigma_\nu = \int d^2b\, e^{-\chi}\frac{\chi^\nu}{\nu!}$$

$$\sigma_2^a = 0 \quad \sigma_2^b = 0 \quad \sigma_2^c = \sigma_2 . \qquad (9)$$

Example 3.

The authors of paper /6/ supposed that when one of the nuclei (B) is a light one

$$e^{-\chi(\vec{b},\sigma)} \approx [\int\rho_B(\vec{s})\cdot e^{-\sigma\cdot A\cdot\rho_A(\vec{b}-\vec{s})}d^2s]^B = [I(\vec{b})]^B . \qquad (10)$$

Inserting this relation into eq. (4) we have

$$\sigma_\nu = \int d^2b\sum_{i=1}^{\nu} C_B^i[I(\vec{b})]^{B-i}\sum_{\substack{j_1,j_2,\dots,j_i\\ j_1+j_2+\dots+j_i=\nu\\ j_1\cdot j_2\cdot\dots\cdot j_i\neq 0}}\{\int\rho_B(\vec{s})\times$$

$$\times\frac{[\sigma\cdot A\cdot\rho_A(\vec{b}-\vec{s})]^{j_1}}{j_1!}\cdot e^{-\sigma\cdot A\cdot\rho_A(\vec{b}-\vec{s})}d^2s\}\cdot\{\int\rho_B(\vec{s})\cdot\frac{[\sigma\cdot A\cdot\rho_A(\vec{b}-\vec{s})]^{j_2}}{j_2!}\times \qquad (11)$$

$$\times e^{-\sigma\cdot A\cdot\rho_A(\vec{b}-\vec{s})}d^2s\}\cdot\ldots\cdot\{\int\rho_B(\vec{s})\cdot\frac{[\sigma\cdot A\cdot\rho_A(\vec{b}-\vec{s})]^{j_i}}{j_i!}\cdot e^{-\sigma\cdot A\cdot\rho_A(\vec{b}-\vec{s})}d^2s\}$$

$$\sigma_2^a = \int d^2b[I(\vec{b})]^{B-1}\cdot B\cdot\int\rho_B(\vec{s})\frac{[\sigma\cdot A\cdot\rho_A(\vec{b}-\vec{s})]^2}{2!}e^{-\sigma\cdot A\cdot\rho_A(\vec{b}-\vec{s})}d^2s$$

$$\sigma_2^b = 0$$

$$\sigma_2^c = \int d^2b\, C_B^2[I(\vec{b})]^{B-2}\{\sigma\cdot A\cdot\int\rho_B(\vec{s})\rho_A(\vec{b}-\vec{s})\cdot e^{-\sigma\cdot A\cdot\rho_A(\vec{b}-\vec{s})}d^2s\}^2$$

It isn't surprising that σ_2^b is equal to zero, because in this approach cascading of nucleons of heavy nucleus into light one is neglected.

In this way the other approaches can be considered. The proposed method, as it was shown above, can be used effectively for theoretical analysis of inelastic nucleus-nucleus interactions.

I'm thankful to V.Sh.Navotnov and K.G.Gulamov for stimulating discussions.

REFERENCES

1. Uzhinsky V.V., Tseren Ch. JINR, P2-12079, Dubna, 1979.
2. Andreev I.V., Chernov A.V. Yad.Fiz., 1979, 28, p. 447.
   Andreev I.V., Hein L.A. Yad.Fiz., 1978, 28, p. 1499.
   Pak A.S.et al. JETP Lett., 1978, 28, p. 314; Pak A.S. et al. Yad.Fiz., 1979, 30, p. 102.
3. Uzhinsky V.V. JINR, P2-13054, Dubna, 1980.
4. Barashenkov V.S., Toneev V.D. Interaction of High-Energy Particles and Nuclei with Nuclei, Atomizdat, M., 1972.
   Azimov S.A. et. al. Z.Phys., 1979, A291, p. 189; Alaverdian G.B. et. al. JINR, P2-12536, Dubna, 1979.
5. Vary J.P. Phys.Rev.Lett., 1978, 40, p. 295.
6. Gasparian A.P., Cheplakov A.P., Shabelski Yu.M. JINR, 1-80-853, Dubna, 1980.

Received by Publishing Department
on May 18 1981.