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COVARIANT THREE-DIMENSIONAL EQUATION FOR FERMION-ANTIFERMION SYSTEM.
II. TRANSFORMATION OF INTERACTION AMPLITUDES TO A LOCAL FORM IN THE LOBACHEVSKY MOMENTUM SPACE

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1. Introduction

This paper is a sequal of paper [1] that has started a generalization, to the case of arbitrary coordinate system, of the relativistic three-dimensional two-particle formalism, developed in ref. [2-4] for the case of the c.m.s. of two particles forming a composite system.

In ref. [1] on the basis of the diagram technique arising in the covariant Hamiltonian formulation of quantum field theory [5], a covariant equation is derived of the quasipotential type [6] for the wave function of a system of two particles with equal masses $(m_1=m_2=m)$

$$2\Delta^0_{p,m\lambda_{\mathcal{P}}}\left(M-2\Delta^0_{p,m\lambda_{\mathcal{P}}}\right)\Psi^{\sigma_1\sigma_2}_{J,m_J}\left(p_1,p_2|\mathcal{P},\lambda_{\mathcal{P}}\tau\right)=$$

$$=\frac{1}{(2\pi)^3}\sum_{\lambda_1,\lambda_2=\pm\frac{1}{2}}\int\frac{d^3\vec{\Delta}_{k,m\lambda_{\mathcal{P}}}}{2\Delta^0_{k,m\lambda_{\mathcal{P}}}}V^{\sigma_1\sigma_2}_{\lambda_1\lambda_2}\left(p_1,p_2;\lambda_{\mathcal{P}}\tau|k_1,k_2;\lambda_{\mathcal{P}}\tau'\right)\Psi^{\lambda_1\lambda_2}_{J,m_J}\left(k_1,k_2|\mathcal{P},\lambda_{\mathcal{P}}\tau'\right). \tag{1.1}$$

In equation (1.1) momenta of all particles are on the upper sheet of the mass hyperboloid

$$p_0^2-\vec{p}^{\,2}=m^2 \quad ; \quad k_0^2-\vec{k}^{\,2}=m^2 \tag{1.2}$$

and the spatial component of the 4-vector

$$\left(\Delta_{p_1,m\lambda_{\mathcal{P}}}\right)^\mu \equiv \left(\Lambda^{-1}_{\lambda_{\mathcal{P}}}p_1\right)^\mu, \tag{1.3}$$

where $\Lambda^{-1}_{\lambda_{\mathcal{P}}}$ is the matrix of the pure Lorentz transformation with the system 4-velocity $\lambda^\mu_{\mathcal{P}}=\mathcal{P}^\mu/\sqrt{\mathcal{P}^2}$; $(\Lambda^{-1}_{\lambda_{\mathcal{P}}}\mathcal{P})^\mu=(M,\vec{0})$ (in what follows we use the abbreviation $\Lambda_{\lambda_{\mathcal{P}}}\equiv\Lambda_{\mathcal{P}}$), coincides with the introduced in [9] covariant generalization of the momentum vector in the c.m.s., so that $\vec{\Delta}_{p_1,m\lambda_{\mathcal{P}}}=-\vec{\Delta}_{p_2,m\lambda_{\mathcal{P}}}=\vec{\Delta}_{p,m\lambda_{\mathcal{P}}}$.

Equation (1.1) is a covariant generalization of the earlier obtained [7] equation for the wave function in the c.m.s. The quasipotential $V^{\sigma_1\sigma_2}_{\lambda_1\lambda_2}$ in (1.1) is constructed out of the matrix elements of the relativistic elastic scattering amplitude [5-8].

This paper is aimed at constructing a local quasipotential in terms of variables of the Lobachevsky three-dimensional momentum space in an arbitrary coordinate system (in [2-4] this was done only for the c.m.s.) with further using it in the covariant equation (1.1).

Since in /I/ this problem has been solved for the case of the pseudo-scalar-meson exchange, we will, in §2, concentrate on a more important case, that one of interaction through vector fields, namely on annihilation interaction. In §3 the spin structure is found for the quasipotential for all types of the two-fermion interaction.

2. Three-dimensional covariant form of the interaction kernel for vector boson exchange

In the second approximation in coupling constant the kernel of eq. (1.1), given by rules of the "spurion" diagram technique /5/, is defined, for the choice of the interaction Hamiltonian $\mathcal{H}_{int} = g : \bar{\Psi}(x)\gamma^\mu \Psi(x) A_\mu(x) :$ by the diagrams of the Figure and has the form /7/ $(\lambda \equiv \lambda_{\mathcal{P}})$:

$$\left(V_{(2)}\right)^{\sigma_1\sigma_2}_{\nu_1\nu_2}(p_1,p_2;\lambda\tau|k_1,k_2;\lambda\tau_1) = V^{scatt.}_{(2)} - V^{annih.}_{(2)} \tag{2.1}$$

$$V^{scatt.}_{(2)} = \frac{g^2}{\sqrt{\mu^2 - t + \frac{1}{4}(\tau-\tau_1)^2}} \cdot \frac{\bar{u}^{\sigma_1}(p_1)\gamma^\mu u^{\nu_1}(k_1) \cdot \bar{v}^{\nu_2}(k_2)\gamma_\mu v^{\sigma_2}(p_2)}{\frac{1}{2}(\tau+\tau_1) + \sqrt{\mu^2 - t + \frac{1}{4}(\tau-\tau_1)^2}} \tag{2.2}$$

$$V^{annih.}_{(2)} = \frac{g^2}{2\mu}\left(\frac{1}{\tau_1 + \sqrt{s_p} + \mu - i\epsilon} + \frac{1}{\tau - \sqrt{s_p} + \mu - i\epsilon}\right)\bar{u}^{\sigma_1}(p_1)\gamma^\mu v^{\sigma_2}(p_2) \cdot \bar{v}^{\nu_2}(k_2)\gamma_\mu u^{\nu_1}(k_1), \tag{2.3}$$

where

$$t = (p_1 - k_1)^2 \; ; \; s_p = (p_1+p_2)^2 \equiv 4(\Delta^0_{p,m\lambda_{\mathcal{P}}})^2 \; ; \; s_k = (k_1+k_2)^2 .$$

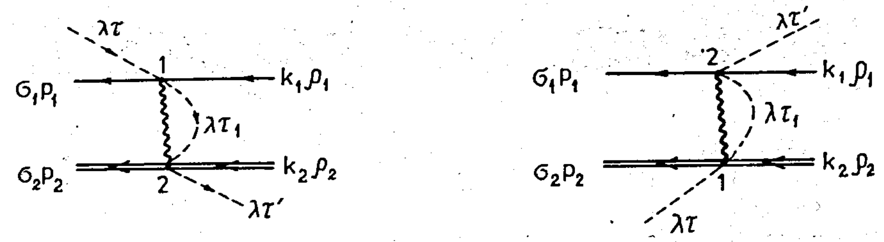
On the "energy" shell $\tau = \tau_1 = 0$ (2.1) transforms into the Feynman matrix element, because $\lambda_{\mathcal{P}} = \lambda_k$ and $\tau - \sqrt{s_p} = \tau_1 - \sqrt{s_k}$

$$T^{\sigma_1\sigma_2}_{(2)\nu_1\nu_2}(p_1,p_2|k_1,k_2) = T^{scatt.}_{(2)} - T^{annih.}_{(2)}, \tag{2.4}$$

where

$$T^{scatt.}_{(2)} = g^2 \frac{\bar{u}^{\sigma_1}(p_1)\gamma^\mu u^{\nu_1}(k_1) \cdot \bar{v}^{\nu_2}(k_2)\gamma_\mu v^{\sigma_2}(p_2)}{\mu^2 - (p_1-k_1)^2} \tag{2.5}$$

$$T^{annih.}_{(2)} = g^2 \frac{\bar{u}^{\sigma_1}(p_1)\gamma^\mu v^{\sigma_2}(p_2) \cdot \bar{v}^{\nu_2}(k_2)\gamma_\mu u^{\nu_1}(k_1)}{\mu^2 - (p_1+p_2)^2}. \tag{2.6}$$



a) b)

Matrix elements (2.2) and (2.5) have been transformed to the three-dimensional form in ref. /10/. Since the method used is analogous to that applied earlier in /3/, we will here only quote the result we will need in the following. So, for $T^{scatt.}_{(2)V}$ in /10/ there was found the representation:

$$T^{\sigma_1\sigma_2}_{\nu_1\nu_2}(p_1,p_2|k_1,k_2) \equiv \langle p_1\sigma_1, p_2\sigma_2 | T | k_1\nu_1, k_2\nu_2\rangle =$$

$$= \sum_{\sigma_i,\nu_i=\pm 1/2} \overset{+}{D}{}^{1/2}_{\sigma_1\sigma_{1p}^{\circ}}\{V^{-1}(\Lambda_{\mathcal{P}},p_1)\} \cdot \overset{+}{D}{}^{1/2}_{\sigma_2\sigma_{2p}^{\circ}}\{V^{-1}(\Lambda_{\mathcal{P}},p_2)\} \cdot \tag{2.7}$$

$$\cdot \langle p_1\sigma^{\circ}_{1p}; p_2\sigma^{\circ}_{2p} | T | k_1\nu^{\circ}_{1p}; k_2\nu^{\circ}_{2p}\rangle \cdot D^{1/2}_{\nu^{\circ}_{1p},\nu^{\circ}_{1k}}\{V^{-1}(\Lambda_{k},\overset{\circ}{p}_1)\}$$

$$\cdot D^{1/2}_{\nu^{\circ}_{1k},\nu_1}\{V^{-1}(\Lambda_{\mathcal{P}},k_1)\} \cdot D^{1/2}_{\nu^{\circ}_{2p},\nu^{\circ}_{2k}}\{V^{-1}(\Lambda_{k_2}^{\circ},\overset{\circ}{p}_2)\} D^{1/2}_{\nu^{\circ}_{2k},\nu_2}\{V^{-1}(\Lambda_{\mathcal{P}},k_2)\},$$

where the matrix element $T^{scatt}_{(2)V}$, remaining after the separation of Wigner rotations, has the form

$$\langle p_1\sigma^{\circ}_{1p}; p_2\sigma^{\circ}_{2p} | T_{(2)V} | k_1\nu^{\circ}_{1p}; k_2\nu^{\circ}_{2p}\rangle = \overset{*}{\xi}{}^{\sigma^{\circ}_{1p}}\overset{*}{\xi}{}^{\sigma^{\circ}_{2p}}\, \hat{T}_{(2)V}(\overset{\circ}{\vec{\varkappa}}, \vec{\Delta}_{p,m\lambda_{\mathcal{P}}})\, \xi_{\nu^{\circ}_{1p}}\xi_{\nu^{\circ}_{2p}} \tag{2.8}$$

$$\hat{T}^{scatt}_{(2)V}(\overset{\circ}{\vec{\varkappa}}, \vec{\Delta}_{p,m\lambda_{\mathcal{P}}}) = \frac{g^2}{\mu^2 + 4\overset{\circ}{\vec{\varkappa}}{}^2}\Big\{4m^2 + 4(\vec{\sigma}_1\overset{\circ}{\vec{\varkappa}})(\vec{\sigma}_2\overset{\circ}{\vec{\varkappa}}) -$$

$$- 4(\vec{\sigma}_1\vec{\sigma}_2)\overset{\circ}{\vec{\varkappa}}{}^2 + 8\overset{\circ}{\varkappa}_0 \cdot \Delta^0_{p,m\lambda_{\mathcal{P}}} \cdot m^{-2} \cdot i(\vec{\sigma}_1+\vec{\sigma}_2)\left[\vec{\Delta}_{p,m\lambda_{\mathcal{P}}} \times \overset{\circ}{\vec{\varkappa}}\right] +$$

$$+ 8m^{-2}(\Delta^0_{p,m\lambda_{\mathcal{P}}})^2 \cdot (\overset{\circ}{\varkappa}_0)^2 + 2\Delta^0_{p,m\lambda_{\mathcal{P}}} \cdot \overset{\circ}{\varkappa}_0 \cdot (\overset{\circ}{\vec{\varkappa}} \cdot \vec{\Delta}_{p,m\lambda_{\mathcal{P}}}) -$$

$$-m^4 + 8m^{-2}(\vec\sigma_1 \vec\Delta_{p,m\lambda_{\mathcal P}})(\vec\sigma_1 \overset{\circ}{\vec\varkappa})\cdot(\vec\sigma_2\vec\Delta_{p,m\lambda_{\mathcal P}})(\vec\sigma_2\overset{\circ}{\vec\varkappa})\Big\},$$

$$\mu^2 - (p-k)^2 = \mu^2 + 4\overset{\circ}{\vec\varkappa}{}^2. \qquad (2.9)$$

Vector $\overset{\circ}{\varkappa}{}^\mu$ was introduced in /10/ : x)

$$\overset{\circ}{\vec\varkappa} = \vec\Delta_{\overset{\circ}{k},\overset{\circ}{p}}\cdot\sqrt{m(2m+\Delta^0_{\overset{\circ}{k},\overset{\circ}{p}})}\ ;\quad \overset{\circ}{\varkappa}_0 = \sqrt{m^2+\overset{\circ}{\vec\varkappa}{}^2} \qquad (2.10)$$

$$\Delta^\mu_{\overset{\circ}{k},\overset{\circ}{p}} \equiv [\overset{\circ}{k}(-)\overset{\circ}{p}]^\mu = (\Lambda_{\overset{\circ}{p}}\overset{\circ}{k})^\mu \equiv (\Lambda^{-1}_{\Delta_{p,m\lambda_{\mathcal P}}}\cdot\Delta_{k,m\lambda_{\mathcal P}})^\mu = [\Delta_{k,m\lambda_{\mathcal P}}(-)\Delta_{p,m\lambda_{\mathcal P}}]^\mu \qquad (2.11)$$

For deriving (2.9) we used the relations

$$\Lambda^\alpha_\mu(\mathcal P)(\overset{\circ}{p}_1)_\alpha\, \Lambda^\mu_\beta(\mathcal P)(\overset{\circ}{p}_2)^\beta = (\overset{\circ}{p}_1)_\mu(\overset{\circ}{p}_2)^\mu = m\,(\overset{\circ}{p}_2(-)\overset{\circ}{p}_1)_0$$

$$\Lambda^\alpha_\mu(\mathcal P)(\overset{\circ}{p}_1)_\alpha\cdot\Lambda^\mu_\beta(\mathcal P)W_2^\beta(\overset{\circ}{p}_2) = (\overset{\circ}{p}_1)_\mu\cdot W_2^\mu(\overset{\circ}{p}_2) = \qquad (2.12)$$

$$= (\overset{\circ}{p})_0\cdot(\vec\sigma_2\overset{\circ}{\vec p}) \equiv (\Delta^0_{p,m\lambda_{\mathcal P}})\cdot(\vec\sigma_2\cdot\vec\Delta_{p,m\lambda_{\mathcal P}});$$

$$\Lambda^\mu_\alpha(\mathcal P)W_1^\alpha(\overset{\circ}{p}_1)\cdot\Lambda^\beta_\mu(\mathcal P)W_{2\beta}(\overset{\circ}{p}_2) = W_1^\mu(\overset{\circ}{p}_1)\cdot W_{2\mu}(\overset{\circ}{p}_2) =$$

$$= -\overrightarrow{(\Lambda^{-1}(p_2)\cdot W_1(p_1))}\cdot\frac{m\vec\sigma_2}{2} = -\frac{m^2}{4}(\vec\sigma_1\vec\sigma_2) - \frac12(\vec\sigma_1\overset{\circ}{\vec p})(\vec\sigma_2\overset{\circ}{\vec p});$$

$$\bar u^{\sigma_1}(p_1)\gamma_\mu u^{\nu_1}(k_1) = (\Lambda_{\mathcal P})^\nu_\mu\cdot \overset{+}{D}{}^{1/2}_{\sigma_1\sigma_{1\overset{\circ}{p}}}\{V^{-1}(\Lambda_{\mathcal P},p)\}\times$$

$$\times\overset{*}{\xi}{}^{\sigma_{1\overset{\circ}{p}}}\{2(\Delta_{p_1,m\lambda_{\mathcal P}})_\nu\cdot\overset{\circ}{\varkappa}_0 + 4W_\nu(\Delta_{p_1,m\lambda_{\mathcal P}})(\vec\sigma_1\overset{\circ}{\vec\varkappa})\}\,\xi_{\nu_{1\overset{\circ}{p}}}\times$$

$$\times D^{1/2}_{\nu_{1\overset{\circ}{p}},\nu_{1\overset{\circ}{k}}}\{V^{-1}(\Lambda_{\Delta_{k_1,m\lambda_{\mathcal P}}},\Delta_{p_1,m\lambda_{\mathcal P}})\}\cdot D^{1/2}_{\nu_{1\overset{\circ}{k}},\nu_1}\{V^{-1}(\Lambda_{\mathcal P},k_1)\} \qquad (2.13)$$

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x) Sign (-) denotes the difference of two momenta in the Lobachevsky space realized on the mass hyperboloid (1.2) /11/ .

and the obtained in /2,3/ formula

$$(p-k)^\mu W_\mu(p) = m\cdot\frac{\vec\sigma}{2}\,(\vec k(-)\vec p) \qquad (2.14).$$

Here $W^\mu(p)$ is the four-vector of the relativistic spin of Pauli-Lubansky-Bargman-Shirokov /13/

$$W^0(p) = \frac{\vec\sigma\vec p}{2};\quad \vec W(p) = \frac{m\vec\sigma}{2} + \frac{\vec p\,(\frac{\vec\sigma\vec p}{2})}{p_0+m};\quad p^\mu W_\mu(p)=0,$$

In the rest frame $W_\mu(p)$ has three components only:

$$W^\mu(p) = (\Lambda_p)^\mu_\nu\, W^\nu(0);\quad W^0(0)=0;\quad \vec W(0)=\frac{m\vec\sigma}{2}.$$

The expression (2.9) differs from its analog obtained in the c.m.s. in /3/ by the change of the momentum vector of a particle in the c.m.s. $\vec p$ by its covariant generalization, vector $\overset{\circ}{\vec p} \to \overset{\circ}{\vec p} \equiv \vec\Delta_{p,m\lambda_{\mathcal P}}$.

The application to $V^{annih}_{(2)}$ and $T^{annih}_{(2)}$ the Firtz theorem and transition to the charge-conjugated bispinors $v = C\,\bar u^c$; $\bar v = u^c(C)^{-1}$ ($C=\gamma_0\gamma_2$) allow us to represent them in the form (cf. / /):

$$(T^{annih}_{(2)})^{\sigma_1\sigma_2}_{\nu_1\nu_2}(p_1p_2|k_1k_2) \equiv g^2 \frac{\bar u^{\sigma_1}(p_1)\gamma^\mu v^{\sigma_2}(p_2)\cdot\bar v^{\nu_2}(k_2)\gamma_\mu u^{\nu_1}(k_1)}{\mu^2-(p_1+p_2)^2} =$$

$$= -g^2\frac{\left[JJ_S - JJ_{PS} + \frac12 JJ_V + \frac12 JJ_A\right]^{\sigma_1\sigma_2}_{\nu_1\nu_2}}{\mu^2-(p_1+p_2)^2}, \qquad (2.15)$$

where for $\alpha = S, PS, V, A$

$$(JJ_\alpha)^{\sigma_1\sigma_2}_{\nu_1\nu_2} = \bar u^{\sigma_1}(p_1)\,\hat O_\alpha\, u^{\sigma_2}(k_1)\cdot\bar u^{c\nu_2}(p_2)\,\hat O^\alpha\, u^{c\nu_1}(k_2) \qquad (2.16)$$

$$\hat O_S = I;\quad \hat O_{PS}=\gamma_5;\quad \hat O_V=\gamma_\mu;\quad \hat O_A=\gamma_5\gamma_\mu.$$

The term JJ_V is already transformed to the form (2.7) by relations (2.13) and (2.9) in /10/, JJ_{PS} in /I/ (see /I/, §2).

Let us now perform in JJ_A transformations which allow the transition from bispinors to two-component Pauli spinors:

$$\bar u^{\sigma_1}(p_1)\gamma_5\gamma_\mu u^{\nu_1}(k_1) = \bar u^{\sigma_1}(0)\, S^{-1}(p_1)\, S(\mathcal{P})\cdot S^{-1}(\mathcal{P})\gamma_5\gamma_\mu S(\mathcal{P})\cdot S^{-1}(\mathcal{P})\cdot S(k_1)\, u^{\nu_1}(0). \qquad (2.17a)$$

The application to (2.17a) of the obtained in /3/ formula

$$S^{-1}(\mathring p)\gamma_\mu S(\mathring p) = (\Lambda_{\mathcal P})_\mu^{\nu} \cdot \frac{\gamma^0}{m}\cdot\left\{\mathring p^{\mu} + 2\gamma_5 W^{\mu}(\mathring p)\right\},$$

allows us to get the relation

$$\bar u^{\sigma_1}(p_1)\gamma_5\gamma_\mu u^{\nu_1}(k_1) = -i\,(\Lambda_{\mathcal P})_\mu^{\nu}\cdot D^{1/2}_{\sigma_1\sigma_{1\mathring p}}\left\{V^{-1}(\Lambda_{\mathcal P}, p_1)\right\}\cdot \qquad (2.17b)$$

$$\cdot\overset{*}{\xi}{}^{\sigma_{1\mathring p}}\left\{2(\Delta_{p_1,m\lambda_{\mathcal P}})_\nu (\vec\sigma_1\mathring{\vec\varkappa}) + 4\left[W_1(\Delta_{p,m\lambda_{\mathcal P}})\right]_\nu \mathring\varkappa_0\right\}\xi_{\nu_{1\mathring p}}\cdot$$

$$\cdot D^{1/2}_{\nu_{1\mathring p},\nu_{1\mathring k}}\left\{V^{-1}(\Delta_{k_1,m\lambda_{\mathcal P}},\Delta_{p_1,m\lambda_{\mathcal P}})\right\}\cdot D^{1/2}_{\nu_{1\mathring k},\nu_1}\left\{V^{-1}(\Lambda_{\mathcal P},k_1)\right\}.$$

AS a result, the term $\mathcal{J}\mathcal{J}_A$ also takes the form (2.7). The transformation of $\mathcal{J}\mathcal{J}_S$ to (2.7) is readily achieved.

Thus, the V_2^{annih} and T_1^{annih} assume the form (2.7) and the part remaining after separating 6 Wigner rotations has the form

$$\left(T_{(2)}^{annih}\right)^{\sigma_{1\mathring p}\sigma_{2\mathring p}}_{\nu_{1\mathring p}\nu_{2\mathring p}} = -g^2\,\frac{\left(\mathcal{J}\mathcal{J}_S - \mathcal{J}\mathcal{J}_P + \frac12\mathcal{J}\mathcal{J}_V + \frac12\mathcal{J}\mathcal{J}_A\right)^{\sigma_{1\mathring p}\sigma_{2\mathring p}}_{\nu_{1\mathring p}\nu_{2\mathring p}}}{\mu^2 - (2\mathring p_0)^2}, \qquad (2.18)$$

where

$$\left(\mathcal{J}\mathcal{J}_S\right)^{\sigma_{1\mathring p}\sigma_{2\mathring p}}_{\nu_{1\mathring p}\nu_{2\mathring p}} = 4\,(\mathring\varkappa_0)^2\cdot\delta_{\sigma_{1\mathring p}\nu_{1\mathring p}}\cdot\delta_{\sigma_{2\mathring p}\nu_{2\mathring p}} \qquad (2.19)$$

$$\left(\mathcal{J}\mathcal{J}_{PS}\right)^{\sigma_{1\mathring p}\sigma_{2\mathring p}}_{\nu_{1\mathring p}\nu_{2\mathring p}} = -4\,(\vec\sigma_1\mathring{\vec\varkappa})_{\sigma_{1\mathring p},\nu_{1\mathring p}}\cdot(\vec\sigma_2\mathring{\vec\varkappa})_{\sigma_{2\mathring p},\nu_{2\mathring p}}. \qquad (2.20)$$

$$\left(\mathcal{J}\mathcal{J}_A\right)^{\sigma_{1\mathring p}\sigma_{2\mathring p}}_{\nu_{1\mathring p}\nu_{2\mathring p}} = m^2\Big\{4\left[2(\Delta^0_{p,m\lambda_{\mathcal P}})^2 - m^2\right](\vec\sigma_1\mathring{\vec\varkappa})(\vec\sigma_2\mathring{\vec\varkappa}) +$$

$$+ 8\Delta^0_{p,m\lambda_{\mathcal P}}\cdot\mathring\varkappa_0\left[(\vec\sigma_1\vec\Delta_{p,m\lambda_{\mathcal P}})(\vec\sigma_2\mathring{\vec\varkappa}) + (\vec\sigma_1\mathring{\vec\varkappa})(\vec\sigma_2\vec\Delta_{p,m\lambda_{\mathcal P}})\right] +$$

$$+ 2m(\Delta^0_{p,m\lambda_{\mathcal P}}+m)\left[2(\vec\sigma_1\vec\Delta_{p,m\lambda_{\mathcal P}})\cdot(\vec\sigma_2\vec\Delta_{p,m\lambda_{\mathcal P}}) + m^2(\vec\sigma_1\vec\sigma_2)\right]\Big\}^{\sigma_{1\mathring p}\sigma_{2\mathring p}}_{\nu_{1\mathring p}\nu_{2\mathring p}} \qquad (2.21)$$

$$\left(\mathcal{J}\mathcal{J}_V\right)^{\sigma_{1\mathring p}\sigma_{2\mathring p}}_{\nu_{1\mathring p}\nu_{2\mathring p}} = g_V^{-2}\left(\mu^2 + 4\mathring{\vec\varkappa}^2\right)^{-1}\left\{T_{(2)V}(\mathring{\vec\varkappa},\mathring{\vec p})\right\}^{\sigma_{1\mathring p}\sigma_{2\mathring p}}_{\nu_{1\mathring p}\nu_{2\mathring p}} \qquad (2.22)$$

with $\mathring p_0$ defined $(\mathring p) = (\Lambda^{-1}_{\mathcal P}p)_0$. Inserting (2.19)-(2.22) into (2.18) we arrive at the following expression for $\hat T^{annih}_{(2)V}$:

$$\left(\hat T^{annih}_{(2)V}\right)^{\sigma_{1\mathring p}\sigma_{2\mathring p}}_{\nu_{1\mathring p}\nu_{2\mathring p}}(p_1p_2;\lambda\tau|k_1k_2;\lambda\tau_1) = \overset{*}{\xi}{}^{\sigma_{1\mathring p}}\overset{*}{\xi}_{\nu_{1\mathring p}}\,\hat T^{annih}_{(2)V}\,\xi^{\sigma_{2\mathring p}}\xi_{\nu_{2\mathring p}} \qquad (2.23)$$

$$\hat T^{annih}_{(2)V} = -g^2\left[\mu^2 - (2\Delta^0_{p,m\lambda_{\mathcal P}})^2\right]^{-1}\Big\{4(\Delta^0_{p,m\lambda_{\mathcal P}})^2 - 6m^2 +$$

$$+ 8m^{-2}(\mathring\varkappa_0)^2(\Delta^0_{p,m\lambda_{\mathcal P}})^2 + 8m^{-2}\Delta^0_{p,m\lambda_{\mathcal P}}\mathring\varkappa_0(\vec\Delta_{p,m\lambda_{\mathcal P}}\cdot\mathring{\vec\varkappa}) +$$

$$+(8 + 4m^{-2}\vec\Delta^2_{p,m\lambda_{\mathcal P}})(\vec\sigma_1\mathring{\vec\varkappa})(\vec\sigma_2\mathring{\vec\varkappa}) + 2m^2(\vec\sigma_1\vec\sigma_2) +$$

$$+ m^{-2}(2\Delta^0_{p,m\lambda_{\mathcal P}}\mathring\varkappa_0 + \vec\Delta_{p,m\lambda_{\mathcal P}}\cdot\mathring{\vec\varkappa})(\vec\sigma_1+\vec\sigma_2)\cdot i\cdot\left[\vec\Delta_{p,m\lambda_{\mathcal P}}\times\mathring{\vec\varkappa}\right] +$$

$$+ 2m^{-2}\left(i(\vec\sigma_1+\vec\sigma_2)\left[\vec\Delta_{p,m\lambda_{\mathcal P}}\times\mathring{\vec\varkappa}\right]\right)^2 +$$

$$+ 4\Delta^0_{p,m\lambda_{\mathcal P}}\cdot\mathring\varkappa_0\cdot m^{-2}\times\left[(\vec\sigma_1\vec\Delta_{p,m\lambda_{\mathcal P}})(\vec\sigma_2\mathring{\vec\varkappa}) + (\vec\sigma_1\mathring{\vec\varkappa})(\vec\sigma_2\vec\Delta_{p,m\lambda_{\mathcal P}})\right] +$$

$$+ 4(\mathring\varkappa_0)^2\cdot m^{-2}\left[(\vec\sigma_1\vec\Delta_{p,m\lambda_{\mathcal P}})(\vec\sigma_2\vec\Delta_{p,m\lambda_{\mathcal P}})\right]\Big\} \qquad (2.24)$$

In the nonrelativistic limit and for $\mu^2=0$

$$\hat{T}_2^{\,annih.} \xrightarrow[c\to\infty]{} \frac{g^2}{2c^2}\left(3+\vec{\sigma}_1\vec{\sigma}_2\right) \qquad (2.25)$$

and we obtain the operator used in the Breit equation for describing the positronium and charmonium.

The expressions (2.9) and (2.24) substituted into (2.1), (2.4) give us the three-dimensional quasipotential which describes the interaction in a fermion-antifermion system through the exchange by a vector boson with mass μ . Since the spin structures of expressions (2.2), (2.3) and (2.5), (2.6) coincide, the corresponding (2.1) quasipotential comes from (2.4) by the change of propagators $\left[\mu^2-(p_1-k_1)^2\right]^{-1}$ and $\left[\mu^2-(p_1+p_2)^2\right]^{-1}$ by the spurion propagators, entries of (2.2) and (2.3).

3. Covariant three-dimensional spin structures in the Lobachevsky momentum space and their connection with standard spin structures

It is known (see, e.g., /13/) that the amplitude of elastic scattering of spin 1/2 particles can be expanded over 5 independent spin structures, formed by 4 structures (2.16) added with the structure x)

$$\mathcal{J}\mathcal{J}_T = \bar{u}^{\sigma_1}(p_1)\sigma_{\mu\nu}u^{\nu_1}(k_1)\cdot\bar{v}^{\nu_2}(k_2)\sigma^{\mu\nu}v^{\sigma_2}(p_2) \qquad (3.1)$$

$$\sigma_{\mu\nu} = \frac{1}{2}\left(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu\right)$$

from (2.17a) to (2.17b). Namely, let us first represent the currents in the form

$$\bar{u}^{\sigma_1}(p_1)\sigma_{\mu\nu}u^{\nu_1}(k_1) = \qquad (3.2)$$
$$= \bar{u}^{\sigma_1}(0)\, S^{-1}(p_1)\, S(\mathcal{P})\, S^{-1}(\mathcal{P})\,\sigma_{\mu\nu}\, S(\mathcal{P})\, S^{-1}(\mathcal{P})\, S(k_1)\, u^{\nu_1}(0)$$

and with the use of the equalities

$$S^{-1}(\mathcal{P})\,\sigma_{\mu\nu}\,S(\mathcal{P}) = (\Lambda_{\mathcal{P}})^{\rho}_{\mu}(\Lambda_{\mathcal{P}})^{\alpha}_{\nu}\,\sigma_{\rho\alpha} \qquad (3.3)$$

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x) Off the energy shell one must introduce a 6-th structure of the form $\gamma_5^{(1)}\gamma_\mu^{(1)}(p_1-p_2)^\mu\cdot\gamma_5^{(2)} + \gamma_5^{(1)}\cdot\gamma_5^{(2)}\gamma_\mu^{(2)}(k_1-k_2)^\mu$ [13]

$$S^{-1}(\mathcal{P})\, S(k_1) = S(\Delta_{k_1,m\lambda_{\mathcal{P}}})\cdot D^{1/2}\{V^{-1}(\Lambda_{\mathcal{P}},k)\} \qquad (3.4)$$

$$S(\Delta_{k_1,m\lambda_{\mathcal{P}}}) = S(\Delta_{p_1,m\lambda_{\mathcal{P}}})\cdot S(\Lambda^{-1}_{\Delta_{p_1,m\lambda_{\mathcal{P}}}}\cdot\Delta_{k_1,m\lambda_{\mathcal{P}}})\cdot$$
$$\cdot D^{1/2}\{V^{-1}(\Lambda_{\Delta_{p_1,m\lambda_{\mathcal{P}}}},\Delta_{k_1,m\lambda_{\mathcal{P}}})\}, \qquad (3.5)$$

rewrite them in the following way

$$\bar{u}^{\sigma_1}(p_1)\sigma_{\mu\nu}u^{\nu_1}(k_1) = (\Lambda_{\mathcal{P}})^{\rho}_{\mu}\cdot(\Lambda_{\mathcal{P}})^{\alpha}_{\nu}\cdot\bar{u}^{\sigma_1}(0)\cdot D^{+1/2}\{V^{-1}(\Lambda_{\mathcal{P}},p_1)\}$$
$$\cdot S^{-1}(\Delta_{p_1,m\lambda_{\mathcal{P}}})\cdot\sigma_{\rho\alpha}\cdot S(\Delta_{p,m\lambda_{\mathcal{P}}})\cdot S(\Delta_{k,m\lambda_{\mathcal{P}}}(-)\Delta_{p,m\lambda_{\mathcal{P}}})\cdot \qquad (3.6)$$
$$\cdot D^{1/2}\{V^{-1}(\Lambda_{\Delta_{p_1,m\lambda_{\mathcal{P}}}},\Delta_{k_1,m\lambda_{\mathcal{P}}})\}\cdot D^{1/2}\{V^{-1}(\Lambda_{\mathcal{P}},k_1)\}\, u^{\nu_1}(0)$$

Then we apply to (3.6) the obtained earlier in [3] relation

$$S^{-1}(\Delta_{p,m\lambda_{\mathcal{P}}})\cdot\sigma_{\mu\nu}\cdot S(\Delta_{p,m\lambda_{\mathcal{P}}}) = -4m^{-2}\Sigma_{\mu\nu}(\Delta_{p,m\lambda_{\mathcal{P}}}) - \qquad (3.7)$$
$$-2m^{-2}\gamma_5\cdot\left[W_\mu(\Delta_{p,m\lambda_{\mathcal{P}}})\cdot(\Delta_{p,m\lambda_{\mathcal{P}}})_\nu - W_\nu(\Delta_{p,m\lambda_{\mathcal{P}}})\cdot(\Delta_{p,m\lambda_{\mathcal{P}}})_\mu\right],$$

where the quantity $\left(\overset{\circ}{p}{}^{\mu}\equiv\Delta^{\mu}_{p,m\lambda_{\mathcal{P}}}\right)$

$$\Sigma_{\mu\nu}(\overset{\circ}{p}) = \frac{1}{2}\left\{W_\mu(\overset{\circ}{p})W_\nu(\overset{\circ}{p}) - W_\nu(\overset{\circ}{p})W_\mu(\overset{\circ}{p})\right\} \qquad (3.8)$$

is constructed by analogy with $\sigma_{\mu\nu}$ with the use of the relativistic spin vector $W_{\mu\nu}(\overset{\circ}{p})$ (2.14) instead of γ_μ matrices.

From the comparison of formulae (3.6) and (2.13), (2.17b) we see that (3.6) contains the same matrices of Wigner rotations $D^{1/2}\{V^{-1}\}$ that allowed us to obtain the representation (2.7) for the spin structures (2.16). For the part $\mathcal{J}\mathcal{J}_T$ remaining after separating the Wigner rotations we get with the use of (2.10), (2.12) and (2.14) the expression

$$(\mathcal{J}\mathcal{J}_T)^{\sigma_{1\overset{\circ}{p}}\sigma_{2\overset{\circ}{p}}}_{\nu_{1\overset{\circ}{p}}\nu_{2\overset{\circ}{p}}} = 8m^{-6}\overset{*}{\xi}{}^{\sigma_{1\overset{\circ}{p}}}\overset{*}{\xi}_{\nu_{1\overset{\circ}{p}}}\Big\{2m^2\left[(\Delta^{0}_{p,m\lambda_{\mathcal{P}}})^2-m^2\right](\vec{\sigma}_1\vec{\sigma}_2)(\vec{\sigma}_1\overset{\circ}{\vec{\varkappa}})(\vec{\sigma}_2\overset{\circ}{\vec{\varkappa}})+$$

$$+\left[(2\Delta^c_{p,m\lambda_p})^2-m^2\right](\vec{\sigma}_1\vec{\Delta}_{p,m\lambda_p})(\vec{\sigma}_2\vec{\Delta}_{p,m\lambda_p})(\vec{\sigma}_1\mathring{\vec{\varkappa}})(\vec{\sigma}_2\mathring{\vec{\varkappa}})+$$

$$+4\mathring{\varkappa}_0\,\Delta^c_{p,m\lambda_p}\,m^2\, i\vec{\sigma}_1\left[\vec{\sigma}_2\times\vec{\Delta}_{p,m\lambda_p}\right](\vec{\sigma}_1\mathring{\vec{\varkappa}})(\vec{\sigma}_2\mathring{\vec{\varkappa}})-m^2\mathring{\varkappa}_i^2(\vec{\sigma}_1\vec{\sigma}_2)$$

$$-2\left[(p\Delta^c_{p,m\lambda_p})^2-m^2\right](\vec{\sigma}_1\vec{\Delta}_{p,m\lambda_p})(\vec{\sigma}_2\vec{\Delta}_{p,m\lambda_p})\Big\}\,\xi^{\sigma_{2\mathring{p}}}\,\xi_{\vartheta_{2\mathring{p}}}. \qquad (3.9)$$

Thus, we are convinced that all the 5 standard spin invariant structures (2.16), (3.1) can be transformed to the form (2.7) and the remaining part can be expanded over the set of scalar and vector products of the 3 - vectors $\vec{\sigma}_1$, $\vec{\sigma}_2$, $\vec{\Delta}_{p,m\lambda_p}\equiv\mathring{\vec{p}}$ and $\mathring{\vec{\varkappa}}$. And the parts of spin structures (2.19)-(2.22) that have appeared after separating the kinematic Wigner rotations have the same form as the spin structures considered earlier in the c.m.s. /2-4/, but with the change of all the momentum variables $\vec{p}=\vec{p}_1=-\vec{p}_2$ and $\vec{\Delta}=\vec{k}(-)\vec{p}$ and $\vec{\varkappa}$ by their covariantly generalized analogs: $\mathring{\vec{p}}\equiv\vec{\Delta}_{p,m\lambda_p}=\vec{\Delta}_{p_1,m\lambda_p}=$

$$=-\vec{\Delta}_{p_2,m\lambda_p}\ ;\quad \mathring{\Delta}_{\mathring{p},\mathring{k}}=\vec{\Delta}_{p,m\lambda_p}(-)\vec{\Delta}_{k,m\lambda_p}\quad \text{and}\ \mathring{\vec{\varkappa}}.$$

Taking this into account and the fact that earlier in /4/ for the c.m.s. with the use of the basis of unit vectors $(\vec{p}=\vec{p}_1=-\vec{p}_2;\ \vec{k}=\vec{k}_1=-\vec{k}_2)$

$$\vec{\ell}=\frac{\vec{k}(-)\vec{p}}{|\vec{k}(-)\vec{p}|}\ ;\quad \vec{n}=\frac{[\vec{k}\times\vec{p}]}{|[\vec{k}\times\vec{p}]|}\ ;\quad \vec{m}=[\vec{\ell}\times\vec{n}], \qquad (3.10)$$

the P and T-invariant parametrization of scattering amplitude x)

$$T^{\sigma_1\sigma_2}_{\vartheta_1\vartheta_2}(\vec{p},\vec{k})=\Big\{V_1+V_2(\vec{\sigma}_1\vec{n}+\vec{\sigma}_2\vec{n})+V_3(\vec{\sigma}_1\vec{n})(\vec{\sigma}_2\vec{n})+$$

x) On the energy shell $V_6=0$ [7,13]. The index p in a row with the polarization index means /1/, e.g.,for ϑ_{1p} that this index ϑ_1 "sits" on the momentum $\vec{p}$ like σ_1 and σ_2.

$$+V_4(\vec{\sigma}_1\vec{\ell})(\vec{\sigma}_2\vec{\ell})+V_5(\vec{\sigma}_1\vec{m})(\vec{\sigma}_2\vec{m})+V_6\left[(\vec{\sigma}_1\vec{\ell})(\vec{\sigma}_2\vec{m})+\right.$$

$$\left.+(\vec{\sigma}_1\vec{m})(\vec{\sigma}_2\vec{\ell})\right]\Big\}^{\sigma_1\sigma_2}_{\vartheta_{1p}\vartheta_{2p}}\cdot\mathcal{D}^{1/2}_{\vartheta_{1p},\vartheta_1}\{V^{-1}(\Lambda_p,k)\}\,\mathcal{D}^{1/2}_{\vartheta_{2p},\vartheta_2}\{V^{-1}(\Lambda_p,k)\} \qquad (3.11)$$

has been found,we may assume a new generalization to an arbitrary coordinate system of another popular basis used for the expansion of elastic scattering amplitude of two particles in the c.m.s. $\vec{p}=\vec{p}_1=-\vec{p}_2;\ \vec{k}=\vec{k}_1=-\vec{k}_2$ obtained in /14/ within the nonrelativistic formalism:

$$T(\vec{p},\vec{k})=V_1+V_2(\vec{\sigma}_1\vec{n}+\vec{\sigma}_2\vec{n})+V_3(\vec{\sigma}_1\vec{n})(\vec{\sigma}_2\vec{n})+ \qquad (3.12)$$

$$+V_4(\vec{\sigma}_1\vec{\ell})(\vec{\sigma}_2\vec{\ell})+V_5(\vec{\sigma}_1\vec{m})(\vec{\sigma}_2\vec{m})+V_6\left[(\vec{\sigma}_1\vec{\ell})(\vec{\sigma}_2\vec{m})+(\vec{\sigma}_1\vec{m})(\vec{\sigma}_2\vec{\ell})\right]$$

with the Euclidean basis (index "E"):

$$\vec{\ell}_E=\frac{\vec{k}-\vec{p}}{|\vec{k}-\vec{p}|}\ ;\quad \vec{n}_E=\frac{[\vec{p}\times\vec{k}]}{|[\vec{p}\times\vec{k}]|}\ ;\quad \vec{m}_E=[\vec{\ell}\times\vec{n}]. \qquad (3.13)$$

Just the covariant expansion of amplitude over spin structures can be defined if all its spin indices are first "removed" on the same vector $\mathring{p}$ with the help of the formula (2.7) obtained in /10/ and then the separated amplitude is represented in the form

$$T^{\sigma_{1\mathring{p}}\sigma_{2\mathring{p}}}_{\vartheta_{1\mathring{p}}\vartheta_{2\mathring{p}}}(\mathring{\vec{p}},\mathring{\vec{k}})=\langle\vec{p}_1\sigma_{1\mathring{p}};\vec{p}_2\sigma_{2\mathring{p}}|\,T\,|\vec{k}_1\vartheta_{1\mathring{p}};\vec{k}_2\vartheta_{2\mathring{p}}\rangle=$$

$$=\Big\{T_1+T_2(\vec{\sigma}_1\mathring{\vec{n}}+\vec{\sigma}_2\mathring{\vec{n}})+T_3(\vec{\sigma}_1\mathring{\vec{n}})(\vec{\sigma}_2\mathring{\vec{n}})+T_4(\vec{\sigma}_1\mathring{\vec{\ell}})(\vec{\sigma}_2\mathring{\vec{\ell}})+ \qquad (3.14)$$

$$+T_5(\vec{\sigma}_1\mathring{\vec{m}})(\vec{\sigma}_2\mathring{\vec{m}})+T_6\left[(\vec{\sigma}_1\mathring{\vec{\ell}})(\vec{\sigma}_2\mathring{\vec{m}})+(\vec{\sigma}_1\mathring{\vec{m}})(\vec{\sigma}_2\mathring{\vec{\ell}})\right]\Big\}^{\sigma_{1\mathring{p}}\sigma_{2\mathring{p}}}_{\vartheta_{1\mathring{p}}\vartheta_{2\mathring{p}}}$$

with the following choice of the basis of unit vectors

$$\mathring{\vec{n}}=\frac{[\mathring{\vec{p}}\times\mathring{\vec{k}}]}{|[\mathring{\vec{p}}\times\mathring{\vec{k}}]|}\equiv\frac{[\vec{\Delta}_{p,m\lambda_p}\times\vec{\Delta}_{k,m\lambda_p}]}{|[\vec{\Delta}_{p,m\lambda_p}\times\vec{\Delta}_{k,m\lambda_p}]|}\ ; \qquad (3.15)$$

$$\overset{\circ}{\vec{l}} = \frac{\overset{\circ}{\vec k}(-)\overset{\circ}{\vec p}}{|\overset{\circ}{\vec k}(-)\overset{\circ}{\vec p}|} \equiv \frac{\vec\Delta_{k,m\lambda_P}(-)\vec\Delta_{p,m\lambda_P}}{|\vec\Delta_{k,m\lambda_P}(-)\vec\Delta_{p,m\lambda_P}|} = \frac{\overset{\circ}{\vec\varkappa}}{|\overset{\circ}{\vec\varkappa}|}; \qquad \overset{\circ}{\vec m} = [\overset{\circ}{\vec l}\times\overset{\circ}{\vec n}]; \qquad (3.16)$$

which is a clear covariant generalization of the basis (3.10). Under the transformations from the Lorentz group $p'_\mu = (L p)_\mu$ the vectors $\Delta^\mu_{p,m\lambda_P}$ and $\Delta^\mu_{k,m\lambda_P}$ transform by the law $(i,j=1,2,3)$

$$\overset{\circ}{\Delta}_{p,m\lambda_P} = inv; \quad (\Delta'_{p',m\lambda_{P'}})_i \equiv (\overset{\circ}{p}{}')_i = R_{ij}\{V^{-1}(L,\overset{\circ}{p})\}\overset{\circ}{p}_j$$

so that vectors $\overset{\circ}{\vec l}, \overset{\circ}{\vec n}, \overset{\circ}{\vec m}$ undergo only the Wigner rotation.

Further separation, from the amplitude, of 4 Wigner rotations $\mathcal{D}^{1/2}\{V^{-1}(\Lambda_P, p_i)\}, \mathcal{D}^{1/2}\{V^{-1}(\Lambda_P,k_i)\}$ besides 2 Wigner rotations allowing for (3.11), the spin rotations connected with the change of particle momenta due to interaction, is caused by the following fact.

In the relativistic theory each polarization spin index "sits" (in the language of /12/) on its own momentum. In particular, indices σ_i of the matrix element of amplitude $\langle \vec p_1\sigma_1, \vec p_2\sigma_2|T|\vec k_1\nu_1, \vec k_2\nu_2\rangle$ "sit" on momenta $\vec p_i$ and ν_i on $\vec k_i$ (see also the discussion in /3,10/ and, in more detail, our subsequent paper). Therefore, if we would like to find an adequate relativistic generalization of the nonrelativistic parametrization (3.12) thourgh the matrices $\vec\sigma_1, \vec\sigma_2$, we should attach them to one quantization axis, i.e., make all spin polarization indices "sitting" on one momentum, e.g., $\overset{\circ}{\vec p} = \vec\Delta_{p,m\lambda_P}$ the covariant momentum vector in the c.m.s. before interaction.

In (2.7) two matrices $\mathcal D^{1/2}\{V^{-1}(\Lambda_P,p_i)\}$ makes indices $\sigma_i\ (i=1,2)$ sitting on vector $\overset{\circ}{\vec p}$, two other matrices $\mathcal D^{1/2}\{V^{-1}(\Lambda_P,k_i)\}$ transfer indices ν_i on vector $\overset{\circ}{\vec k}$ (see also /10/). So, to complete the transition to a quantization on one axis we should further transfer indices ν_i from momentum $\overset{\circ}{\vec k}$ on $\overset{\circ}{\vec p}$ what requires two more matrices $\mathcal D^{1/2}\{V^{-1}(\Lambda_{\overset{\circ}{k}},\overset{\circ}{p})\}$ one for ν_1 and the other for ν_2.

In paper /15/ it was proposed (without demonstrating in formulae) that the expansion of the relativistic amplitude of two spin 1/2 particles elastic scattering can be obtained from the nonrelativistic one by changin in (3.12) the nonrelativistic momenta to relativistic ones and by further multiplaying (3.12) by Wigner rotations $\mathcal D^{1/2}\{V(\Lambda_P,p_i)\}$; $\mathcal D^{1/2}\{V^{-1}(\Lambda_P,k_i)\}$ $(i=1,2)$. Due to /13/ this Wigner rotations arise under the transformation $p_i' = \Lambda_P p_i;\ k_i' = \Lambda_P k_i\ (m_i = m)$ in the matrix element $\langle \vec p_1\sigma_1,\vec p_2\sigma_2|T|\vec k_1\nu_1,\vec k_2\nu_2\rangle$.

The expansion, we propose here, of the two spin 1/2 particles elastic scattering amplitude defined by formulae (2.7) and (3.14)-(3.16) contains not 4, but 6 Wigner rotations, i.e., differs from the expansion proposed in /15/ by two further Wigner rotations which represent the spin rotations caused by the change, due to interaction, of the particle momenta in the c.m.s. from $\overset{\circ}{\vec p}$ to $\overset{\circ}{\vec k}$. As a result, under transformations of the Lorentz group all the spin variables in (3.14) are transformed by the same Wigner rotation defined by a small group of the chosen vector $\overset{\circ}{\vec p}$; and the expression (3.14) will not change in form.

The parametrization of spin structures of the interaction amplitude on the basis of (3.14) and (2.7) obtained in /10/ is more suitable for our aim - construction of the interaction kernel local in the Lobachevsky space. As is shown in /2-4/ in the c.m.s. where 4 functions $\mathcal D^{1/2}\{V^{-1}(\Lambda_P,k_i)\}$ and $\mathcal D^{1/2}\{V^{-1}(\Lambda_P,p_i)\}$ (i=1,2) becomes equal to unity, just the separation of 2 functions $\mathcal D^{1/2}\{V^{-1}(\Lambda_{\overset{\circ}{p}},\overset{\circ}{k})\}$ transforms one-boson-exchange amplitudes into local expressions in the Lobachevsky momentum space and makes effective the transition in eq. (1.1) to the relativistic configurational representation for solving this equation.

To illustrate this and to write in a more compact form the spin structure of matrix elements in terms of variables of the Lobachevsky space, we consider the spin-dependent part $\vec j(\vec\sigma,\vec p,\vec k)$ of the three-dimensional part /16/ $(\vec\alpha = \gamma_0\vec\gamma)$

$$\bar u(p)\vec\gamma u(k) = 2\sqrt{\frac{k_0+m}{p_0+m}}\ \vec p + \vec j(\vec\sigma,\vec p,\vec k) \qquad (3.17)$$

$$\vec j(\vec\sigma,\vec p,\vec k) = \vec\alpha(\vec\alpha\vec k)\sqrt{\frac{p_0+m}{k_0+m}} - \vec\alpha(\vec\alpha\vec p)\sqrt{\frac{k_0+m}{p_0+m}}, \qquad (3.18)$$

widely used for describing NN-interactions /17/. Off the energy shell, where $\sqrt{(p_0+m)/(k_0+m)}\neq 1$ (3.18) can be written as

$$\vec\alpha(\vec\alpha\vec k)\sqrt{\frac{p_0+m}{k_0+m}} - \vec\alpha(\vec\alpha\vec p)\sqrt{\frac{k_0+m}{p_0+m}} = 2\vec\alpha(\vec\alpha\vec\varkappa)\,\mathcal D^{1/2}\{V^{-1}(\Lambda_p,k)\}. \qquad (3.19)$$

The relation (3.19) is important in that it shows that upon separating the kinematic Wigner rotation of the spin the spin part of current (3.19) is a local function in the Lobachevsky space as it depends on the momentum half-transfer $\vec\varkappa = \vec k(-)\vec p\sqrt{\frac{m}{2(m+(k(-)p)_0)}}$ in this space $(q^2 = (p-k)^2 = -4\vec\varkappa^2$ /3,4/).

Another merit of the expansion by formulae (2.7),(3.14) is related to the parametrization of the one-boson-exchange matrix element (2.16) with $\alpha = PS$; $\overset{\circ}{O}_{PS} = \gamma_5$. As is shown in /7/ in expanding (2.16) over the basis (3.13) this matrix element contributes to 3 spin structures, whereas in expanding over our basis (3.15) the matrix element (2.16), according to (2.20), contributes only to one spin structure T_4 .

Since the matrices of Wigner rotations $D^{1/2}\{V^{-1}(\Lambda_{\overset{\circ}{p}}, \overset{\circ}{k})\}$ in the explicit form contain $\vec{\sigma}$ -matrices, the nonseparation from the amplitude of these, pure kinematic in natural matrices, may lead to the increase of spin structures or their rearrangement in expanding the amplitude.

Note also that the parametrization of spin structures of the amplitude on the basis of formulae (2.7) and (3.14) was obtained by applying the transformation of standard invariant spin structures (2.16).

Our parametrization has an easily traced, through formulae (2.7), (2.12), (2.13) and (2.17b), (3.9), connection with the covariant parametrization of matrix elements in terms of the relativistic spin-4-vector, given by Shirokov and Cheshkov /12/ and developed for the two-particle case namely for deuteron in /18/ . So, by using the relation (2.14) that relates the three-dimensional formalism, we use, to the four-dimensional one, the structure $\vec{\sigma}\overset{\circ}{\vec{\ell}}$ can be written as

$$\vec{\sigma}\overset{\circ}{\vec{\ell}} = 2\,\frac{\overset{\circ}{k}{}_1^{\mu}\cdot W_{\mu}(\overset{\circ}{p}_1)}{\sqrt{(k_1 p_1)^2 - m^4}} \qquad (3.20)$$

The structure $(\vec{\sigma}\overset{\circ}{\vec{n}})$ can be expressed in terms of the vector W^{μ} by the formula $(\overset{\circ}{\vec{n}} = [\overset{\circ}{\vec{k}}\times\overset{\circ}{\vec{p}}]/|[\overset{\circ}{\vec{k}}\times\overset{\circ}{\vec{p}}]|)$:

$$\vec{\sigma}_1\overset{\circ}{\vec{n}} \sim \vec{\sigma}_1\cdot[\overset{\circ}{\vec{k}}\times\overset{\circ}{\vec{p}}] \equiv \vec{\sigma}_1[\vec{\Delta}_{k_1,m\lambda_p}\times\vec{\Delta}_{p_1,m\lambda_p}] =$$

$$= \frac{\sqrt{(p_1+p_2)^2}}{2m\,p_1(p_1+p_2)}\cdot \varepsilon^{\mu\nu\rho\sigma}(\Delta_{p_1,m\lambda_p})_{\mu}\cdot W_{1\nu}(\Delta_{p_1,m\lambda_p})\cdot$$

$$\cdot(\Delta_{k_1,m\lambda_p})_{\rho}\cdot(\Delta_{p_2,m\lambda_p})_{\sigma} \qquad (3.21)$$

The parametrization of the structure $\vec{\sigma}_1\overset{\circ}{\vec{m}} = \vec{\sigma}_1[\overset{\circ}{\vec{\ell}}\times\overset{\circ}{\vec{n}}]$ in terms of vector W^{μ} can readily be found if one represents it in the form

$$\vec{\sigma}_1\overset{\circ}{\vec{m}} = \frac{(\vec{\sigma}_1\overset{\circ}{\vec{k}})(\overset{\circ}{\vec{\ell}}\overset{\circ}{\vec{p}}) - (\vec{\sigma}_1\overset{\circ}{\vec{p}})(\overset{\circ}{\vec{\ell}}\overset{\circ}{\vec{k}})}{|[\overset{\circ}{\vec{k}}\times\overset{\circ}{\vec{p}}]|} \qquad (3.22)$$

and make use of the equalities:

$$\frac{\vec{\sigma}_1\overset{\circ}{\vec{p}}}{2} = \frac{W^{\mu}(\overset{\circ}{p}_1)\cdot(\overset{\circ}{p}_2)_{\mu}}{(\overset{\circ}{p}_1)_0} \;;\; (\overset{\circ}{p}_1)_0 = \frac{p_1(p_1+p_2)}{\sqrt{(p_1+p_2)^2}} \;;$$

$$\overset{\circ}{\vec{\ell}}\overset{\circ}{\vec{p}} = \frac{-\overset{\circ}{\varkappa}{}^{\mu}\overset{\circ}{p}_{\mu} + \overset{\circ}{\varkappa}_0\cdot\overset{\circ}{p}_0}{|\overset{\circ}{\vec{\varkappa}}|} = \frac{-\overset{\circ}{\varkappa}{}^{\mu}\overset{\circ}{p}_{\mu} + \frac{p_1(p_1+p_2)}{\sqrt{(p_1+p_2)^2}}\cdot\sqrt{m^2 - t/4}}{m^{-1}\cdot\sqrt{(kp)^2 - m^4}}$$

and analogous equalities with the change of $\overset{\circ}{\vec{p}}$ to $\overset{\circ}{\vec{k}}$.

4. Conclusion

The main result of the paper is the construction of the covariant quasipotential local in the Lobachevsky momentum space via matrix elements of the relativistic amplitude of vector-meson exchange between two spin 1/2 particles. The obtained quantities $\hat{T}_{2V}^{scatt.}$ (2.9) and $\hat{T}_{2V}^{annih.}$ (2.24) together represent the relativistic covariant three-dimensional generalization of the Breit-Fermi potential widely used in the quark model, but in contrast to the latter, the quantities $\hat{T}_{2V}^{scatt.}$ (2.9) and $\hat{T}_{2V}^{annih.}$ (2.24) are obtained without expanding in powers of v^2/c^2 , i.e.,they keep all the relativistic information contained in the matrix elements of quantum field theory. In § 3 it is shown that the obtained representation of matrix elements can be used to introduce a new relativistic generalization of the widely used in nuclear physics, expansion over spin structures of NN-interaction.

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