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THE NORM INTEGRAL $\int e^{-S}\delta\varphi$ IN TERMS OF PROPAGATOR IN THE $g\varphi^4$ MODEL OF QFT

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1. One considers in quantum field theory the generating functional $w(j)$,

$$\exp(-w(j)) = \int \exp[-S + \int j(x) f(x)\, dx]\, \delta f \tag{1}$$

and connected Green functions

$$G_n(x_1, x_2, \ldots, x_n) = \frac{\delta^n}{\delta j(x_1) \ldots \delta j(x_n)} w(j)\Big|_{j=0} \tag{2}$$

In equation (1) S is the Action

$$S = \int \mathcal{L}(x)\, dx. \tag{3}$$

Here $\mathcal{L}(x)$ is the Lagrangian, defining the model of quantum field theory. One has

$$\mathcal{L} = \mathcal{L}_0 + g\mathcal{L}', \tag{4}$$

where $\mathcal{L}_0$ is the free Lagrangian, quadratic in f, and $g\mathcal{L}'$ is the interaction. In the present paper we take

$$\mathcal{L} = \frac{1}{2} \sum_{a=1}^{d} \left(\frac{\partial f}{\partial x_a}\right)^2 + \frac{1}{2} M^2 f^2 + g f^4 \tag{5}$$

Among the functions (2) the simplest and the most important is the function $G_2(x_1, x_2)$; it depends on the argument $x_2 - x_1$ only:

$$G_2(x_1, x_2) = F(x_2 - x_1). \tag{6}$$

The propagator is defined as a Fourier transform

$$D(p) = \frac{1}{(2\pi)^d} \int F(s)\, e^{ips}\, ds. \tag{6a}$$

In the $g\phi^4$ model one can determine the propagator by the equation (see, e.g., ref. [1])

$$D(p)^{-1} = p^2 + M^2 + 12g \int D(s)\, ds - 96 g^2 \int \Phi(s) D(p-s)\, ds + \tag{7}$$
$$+ 2(12g)^3 \int \Phi^2(s) D(p-s)\, ds + \ldots$$

$$\Phi(s) = \int D(t)D(s-t)dt. \qquad (8)$$

One has to introduce the cutoff into eqs. (7), (8) by limiting all the propagator arguments to be smaller than some large momentum ℓ (c.f. eq. (11)).

1.1. The higher order Green Functions can be expressed via the propagator (cf. [1]). These functions contain information about the physical processes of scattering, creation, and annihilation of particles.

1.2. One has, evidently

$$w(j) = w(0) + \frac{1}{2}\int G_2(x_1,x_2)j(x_1)j(x_2)\,dx_1dx_2 + \frac{1}{4!}\int G_4(x_1x_2x_3x_4)\Big(\prod_1^4 j(x_i)\,dx_i\Big) + \dots$$

where

$$e^{-w(0)} = \int e^{-s}\,\delta f.$$

After the remark at the beginning of subsection 1.1 it is natural to expect that the quantity $e^{-w(0)}$ can also be expressed via the propagator.

This work gives the derivation of the corresponding formula (eq. 26).

1.2.1. The question arises, however: what is the need for deriving eq. (26) if all the necessary information seems to be contained in the Green functions?

1.2.2. The answer is as follows:

The quantity $w(0)$ contains fundamental information about the structure of the ground state. In the limit $M^2 \to +\infty$ the integral $\int e^{-S}\delta f$ is defined by a neighbourhood of the point $f(\ ) = 0$; this fact corresponds to a rather simple structure of the ground state.

In the limit $M^2 \to -\infty$ this integral is determined by a completely different region (see, e.g., ref. [2]) thus giving an entirely different structure of the ground state.

This difference is a base of the opinion wide spread at present that the $g\phi^4$ model undergoes a phase transition when the parameter M^2 varies from $+\infty$ till $-\infty$.

2. We introduce, instead of $f(x)$, its Fourier transform $\phi(p)$:

$$\phi(p) = (2\pi)^{-d/2}\int e^{ipx} f(x)\,dx \qquad (9)$$

and transform all the integrals into the sums:

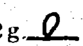
$$\int K(p)\,dp \approx h\sum_p K(p), \quad p = hi, \ i = 0, \pm 1, \dots \qquad (10)$$

Then the action (3) takes the form (for the $g\phi^4$ model)

$$S = \frac{h}{2}\sum_{|p|<\ell} A(p)\phi(p)\phi(-p) + gh^3 \sum_{\substack{\sum_i^4 p_i = 0 \\ |p|<\ell}} \phi(p_1)\phi(p_2)\phi(p_3)\phi(p_4) \qquad (11)$$

$$A(p) = p^2 + M^2. \qquad (12)$$

According to (7), one can express the function $A(p)$ in terms of $D(\ )$:

$A(p) = D(p)^{-1} - 12g$  $+\ 96g^2$ 

$-2(12g)^3$  $+ \dots$ (13)

to each internal line here corresponds a factor $\int D(s)\,ds$; to each vertex except one, a factor $\delta(\sum_i^4 s_i)$.

2.1. Let us consider the quantity $A(p)$ as an arbitrary one, $A(-p) = A(p)$. Then one has

$$D(p) = \frac{\partial w(0)}{\partial A(p)}\epsilon(p), \qquad \epsilon(p) = \begin{cases} 2 & p = 0 \\ 1 & p \neq 0. \end{cases} \qquad (14)$$

The latter equation allows one to express $w(0)$ in terms of $D(\cdot)$. Let us rewrite eq. (14) in the form

$$D(p) = \epsilon(p)\sum_{q\geq 0}\frac{\partial w(0)}{\partial D(q)}\frac{\partial D(q)}{\partial A(p)} \qquad (15)$$

or

$$\frac{\partial w(0)}{\partial D(q)} = \sum_{p\geq 0} D(p)\frac{\partial A(p)}{\partial D(p)}\frac{1}{\epsilon(p)} = \frac{1}{2}\frac{\partial}{\partial D(q)}\sum_{p \lessgtr 0} D(p)A(p) - \frac{A(q)}{\epsilon(q)}. \qquad (16)$$

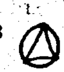
Equation (16) implies

$$2w(0) = \sum_{p\lessgtr 0} D(p)A(p) - F + C(g), \qquad (17)$$

where the functional F is defined through the equation

$$\frac{\partial F}{\partial D(q)} = A(q)\frac{2}{\epsilon(q)} \qquad (18)$$

Equation (13) gives

$\sum D(p)A(p) = \sum_p 1 - 12g$  $+\ 96g^2$  $-\ 2(12g)^3$  $+ \dots,$ (19)

$$F = \frac{1}{h} \int \ln D(s)\, ds - 6g$$  $$+ 24 g^2$$  $$- \frac{1}{3}(12g)^3$$  $$+ \dots \qquad (20)$$

For the determination of the constant $C(g)$ consider the limit

$$M^2 \to \infty. \qquad (21)$$

Then

$$e^{-w(0)} \sim \prod_p \sqrt{\frac{2\pi}{h(p^2 + M^2)}} \qquad (22)$$

and

$$\Sigma\, D(p) A(p) - F \cong \sum_p 1 + \sum_p \ln(p^2 + M^2), \qquad (23)$$

$$w(0) \cong \frac{1}{2} \{ \sum_p [1 + \ln(p^2 + M^2)] + C(g) \}$$
$$= \frac{1}{2} \sum_p [\ln \frac{h}{2\pi} + \ln(p^2 + M^2)] \qquad (24)$$

so that

$$C(g) = \sum_p \ln \frac{h}{2\pi e}. \qquad (25)$$

Thus, we get the equation

$$w(0) = -\ln[\int e^{-S} \delta\phi] = \frac{1}{2} \sum_p \ln \frac{h}{2\pi}$$

$$- \frac{1}{2h} \int \ln D(s)\, ds - 3g$$  $$+ 36 g^2$$  $$- \frac{5}{6}(12g)^3$$  $$+ \dots \qquad (26)$$

expressing the quantity $w(0)$ in terms of the propagator; the expansion (26) contains no self-energy parts.

2.2. Note, that eq. (14) implies

$$\frac{\partial D(p)}{\partial A(q)} = \epsilon(p) \frac{\partial^2 w(0)}{\partial A(q)\, \partial A(p)}\ ; \quad \frac{\partial D(q)}{\partial A(p)} = \epsilon(q) \frac{\partial^2 w(0)}{\partial A(q)\, \partial A(p)} \qquad (27)$$

and

$$\frac{1}{\epsilon(q)} \frac{\partial A(q)}{\partial D(p)} = \frac{1}{\epsilon(p)} \frac{\partial A(p)}{\partial D(q)}$$

thus the functional F, defined by eq. (18), does exist.

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