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P-ODD ASYMMETRIES IN THE PROCESSES $\ell^{-}(\ell^{+}) + N \to \ell^{-}(\ell^{+}) + X$ AND A POSSIBLE METHOD OF TESTING OF THE WEINBERG-SALAM THEORY

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1. Present experimental data on charged and neutral currents agree [1] with the gauge SU(2)xU(1) Weinberg-Salam [2] theory (standard theory of the electroweak interaction). It is obvious, however, that a further test of the standard theory with higher accuracy and in a wider range of energies and momenta transferred squared is an important problem of future experiments.

The experiments on high-energy muon beams are being done at present at CERN and Batavia. The q^2 reached there is $\simeq 100\ GeV^2$. It should be quite desirable to continue in muon experiments the program of the measurement of the P-odd asymmetry in deep inelastic scattering of polarized leptons by nucleons (started at SLAC [3] for $q^2 \simeq 1\ GeV^2$). The P-odd asymmetry grows with q^2 and at $q^2 \simeq 100\ GeV^2$ it becomes relatively large ($\simeq 10^{-2}$). It is advantageous to perform the experiments with high-energy muons also for they allow one to measure the P-odd asymmetry in scattering both of polarized μ^- mesons by nucleons (A_-) and of polarized μ^+-mesons by nucleons (A_+).

Based upon the transformation properties of the hadronic neutral current of the standard electroweak theory we obtain here a relation between asymmetries A_- and A_+ and the deep-inelastic neutrino-nucleon and lepton-nucleon cross sections. The test of this relation would enable one to test the validity of the Weinberg-Salam theory without assumptions about the strong interaction dynamics. A relation is also obtained between the parameter $\sin^2\theta$ (θ is the Weinberg angle) and asymmetries A_- and A_+.

2. The effective Hamiltonian of weak interaction of charged leptons and quarks has in the Weinberg-Salam theory the form

$$H = \frac{G}{\sqrt{2}} j^{\ell}_{\alpha} j^{Z}_{\alpha}, \qquad (1)$$

where

$$j^{\ell}_{\alpha} = \sum_{\ell = e, \mu} \bar{\ell} \gamma_{\alpha} (g_V + g_A \gamma_5) \ell, \qquad (2)$$

$$g_V = -\frac{1}{2} + 2\sin^2\theta, \quad g_A = -\frac{1}{2} \qquad (3)$$

is the neutral lepton current. The neutral hadron current of the standard theory is given by the expression

$$j_\alpha^Z = v_\alpha^3 + a_\alpha^3 - 2\sin^2\theta j_\alpha^{em} + \tilde{j}_\alpha^Z. \qquad (4)$$

Here v_α^3 and a_α^3 are the third components of the isovectors

$$v_\alpha^i = \bar{N}\gamma_\alpha \frac{1}{2}\tau_i N,$$
$$a_\alpha^i = \bar{N}\gamma_\alpha\gamma_5 \frac{1}{2}\tau_i N, \qquad (5)$$
$$N = \binom{u}{a}$$

j_α^{em} is the electromagnetic current of hadrons and

$$\tilde{j}_\alpha^Z = -\frac{1}{2}\bar{s}\gamma_\alpha(1+\gamma_5)s + \frac{1}{2}\bar{c}\gamma_\alpha(1+\gamma_5)c + \dots .$$

It follows from neutrino experiments [4] that the number of s quarks in the nucleon represents a few per cent of the number of u and d quarks. The contributions of s, c and other heavier quarks into the cross sections of processes under consideration will therefore be neglected. In this approximation the electromagnetic quark current is given by the expression

$$j_\alpha^{em} = v_\alpha^3 + \frac{1}{3}v_\alpha^S . \qquad (6)$$

Here

$$v_\alpha^S = \bar{N}\gamma_\alpha \frac{1}{2}N \qquad (7)$$

is an isoscalar.

For the treatment of the P-odd asymmetries in deep inelastic scattering of polarized leptons by nucleons it appears convenient to single out the isoscalar term v_α^S out of the neutral current j_α^Z. From eqs. (4) and (6) we get

$$j_\alpha^Z = (1-2\sin^2\theta)j_\alpha^{em} - \frac{1}{3}v_\alpha^S + a_\alpha^3 . \qquad (8)$$

The cross section of deep inelastic scattering of longitudinally polarized leptons (antileptons) on unpolarized nucleons

$$\ell^-(\ell^+) + N \to \ell^-(\ell^+) + N \qquad (9)$$

has the following general form

$$\left(\frac{d\sigma_{\mp}}{dq^2 d\nu}\right)_{\lambda} = \frac{d\sigma^{em}}{dq^2 d\nu}(1 + \lambda A_{\mp}). \qquad (10)$$

Here λ is the longitudinal polarization of leptons (anti-leptons), $\frac{d\sigma^{em}}{dq^2 d\nu}$ is the cross section of scattering of unpolarized particles. P-odd asymmetries A_- and A_+ are given by the following general expressions, respectively [5,6,7]

$$A_{\mp} = \eta(g_V \alpha_A \pm g_A \alpha_V). \qquad (11)$$

Here

$$\eta = \frac{G}{\sqrt{2}} \frac{q^2}{2\pi\alpha} \simeq 1{,}5 \cdot 10^{-4} \frac{q^2}{M^2}, \qquad (12)$$

M is the nucleon mass, and $\alpha_A(\alpha_V)$ characterizes the contribution of the interference between electromagnetic and axial-vector (vector) part of the hadronic neutral current into the asymmetry. We have

$$\alpha_A(q^2, \nu, y) = \frac{e_{\alpha\beta\rho\sigma} k_\rho k'_\sigma W^I_{\alpha\beta}(p,q)}{L_{\alpha\beta}(k,k') W^{em}_{\alpha\beta}(p,q)}, \qquad (13)$$

$$\alpha_V(q^2, \nu, y) = \frac{L_{\alpha\beta}(k,k') W^I_{\alpha\beta}(p,q)}{L_{\alpha\beta}(k,k') W^{em}_{\alpha\beta}(p,q)}. \qquad (14)$$

Here

$$W^I_{\alpha\beta}(p,q) = -(2\pi)^2 \frac{p_0}{M} \int e^{-iqx} \langle p|(J^Z_\beta(x) J^{em}_\alpha(0) + J^{em}_\beta(x) J^Z_\alpha(0))|p\rangle dx, \qquad (15)$$

$$W^{em}_{\alpha\beta}(p,q) = -(2\pi)^2 \frac{p_0}{M} \int e^{-iqx} \langle p|J^{em}_\beta(x) J^{em}_\alpha(0)|p\rangle dx, \qquad (16)$$

and

$$L_{\alpha\beta}(k,k') = k_\alpha k'_\beta - \delta_{\alpha\beta} kk' + k'_\alpha k_\beta \qquad (17)$$

(p is the momentum of an initial nucleon, k and k' are the momenta of initial and final leptons, respectively, $q = k - k'$, $\nu = -\frac{pq}{M}$, $y = \frac{pq}{pk}$).

We will consider deep inelastic scattering of longitudinally polarized leptons on isoscalar targets. Obviously, the interference between the isovectors (A^3 and V^3) and the isoscalar V^S does not contribute into α_A and α_V in this case. The quantity α_A is now connected with the cross sections of deep inelastic neutrino processes

$$\nu_\mu + N \to \mu^- + X, \qquad \bar{\nu}_\mu + N \to \mu^+ + X \tag{18}$$

$\left(\frac{d\sigma_\nu^{CC}}{dq^2 d\nu}\right.$ and $\left.\frac{d\sigma_{\bar{\nu}}^{CC}}{dq^2 d\nu}\right)$ and with the cross section of deep inelastic scattering of unpolarized leptons by nucleons $\left(\frac{d\sigma^{em}}{dq^2 d\nu}\right)$. We have [5]

$$\alpha_A = \frac{2\pi^2\alpha^2}{G^2 q^4} \; \frac{\left(\frac{d\sigma_\nu^{CC}}{dq^2 d\nu} - \frac{d\sigma_{\bar{\nu}}^{CC}}{dq^2 d\nu}\right)}{\frac{d\sigma^{em}}{dq^2 d\nu}} \; . \tag{19}$$

With the help of (8) and (14) we have

$$\alpha_V = 2(1 - 2\sin^2\theta) - \frac{2}{9}\alpha_V^S . \tag{20}$$

Here

$$\alpha_V^S = \frac{L_{\alpha\beta} W_{\alpha\beta}^S}{L_{\alpha\beta} W_{\alpha\beta}^{em}} , \tag{21}$$

$$W_{\alpha\beta}^S = -(2\pi)^2 \frac{p_0}{M} \int e^{-iqx} \langle p | V_\beta^S(x) V_\alpha^S(0) | p \rangle \, dx . \tag{22}$$

The quantity α_V^S that characterizes the relative contribution of the isoscalar enters into the expression for α_V with the coefficient

$$2Y_q^2 ,$$

where $y_q = 1/3$ is the hypercharge of a quark with the fractional charge. It is easy to see that exactly the smallness of this coefficient justifies the applicability [8,9] of the parton model (when calculating α_V). Indeed, we have

$$\alpha_V^S = \frac{\frac{\langle SS \rangle}{\langle VV \rangle}}{1 + \frac{1}{9}\frac{\langle SS \rangle}{\langle VV \rangle}} , \tag{23}$$

where

$$<SS> = L_{\alpha\beta} W^{S}_{\alpha\beta},$$

$$<VV> = L_{\alpha\beta} W^{V}_{\alpha\beta}, \tag{24}$$

$$W^{V}_{\alpha\beta} = -(2\pi)^2 \frac{p_0}{M} \int e^{-iqx} <p| V^3_\beta (x) V^3_\alpha (0) |p> dx.$$

It is obvious, that in the parton approximation

$$<SS> = <VV> \tag{25}$$

and from eq. (23) we get

$$\frac{2}{9}(\alpha^S_V)_0 = \frac{1}{5}. \tag{26}$$

We introduce

$$\delta = \frac{<SS>}{<VV>} - 1. \tag{27}$$

This quantity depends generally upon q^2, ν and y and characterizes the deviation of the ratio $\frac{<SS>}{<VV>}$ from its parton value. Substituting (27) into (23), we have

$$\frac{2}{9}\alpha^S_V = \frac{1}{5}\ \frac{1+\delta}{1+\frac{1}{10}\delta} \simeq \frac{1}{5}(1+\frac{9}{10}\delta). \tag{28}$$

If $\delta < 0.2 - 0.3$, then it follows from (20) and (28) that the contribution into α_V of the term proportional to δ does not exceed 0.04-0.06. At the value of the parameter $\sin^2\theta \simeq \frac{1}{4}$ (in accordance with the experimental data) the first term in (20) is approximately equal to 1. Thus, neglecting the term proportional to δ, which contributes to α_V at most a few per cent of the contribution of the leading terms, we have

$$\alpha_V = 2(1 - 2\sin^2\theta) - \frac{1}{5}. \tag{29}$$

From (11) and (29) we get

$$\sin^2\theta = \frac{A_- - A_+}{4\eta} + \frac{9}{20}. \tag{30}$$

Thus the measurement of the asymmetries A_- and A_+ would determine the parameter $\sin^2\theta$ directly from the experimental data.

With the help of eq. (11) at q^2, ν and y fixed we obtain the relation

$$\frac{1}{\eta a_A}[A_+(a_A+1) - A_-(a_A-1)] = 1 - \frac{2}{9}a_V^S, \tag{31}$$

that is valid independently of the value of the parameter $\sin^2\theta$. The contribution of the isoscalar, which amounts to ~20 per cent of the isovector contribution, was neglected in ref. [10]. From the above arguments it follows that a much more accurate relation is obtained provided the a_V^S is replaced by its parton value. From eqs. (26) and (31) we find

$$\frac{1}{\eta a_A}[A_+(a_A+1) - A_-(a_- -1)] = \frac{4}{5}. \tag{32}$$

Only experimentally measurable quantities enter into the relation (32)- The test of this relation would be a direct test of the Weinberg-Salam theory.

3. It is possible to get the exact formula (in the u, d approximation) relating a_V^S to experimentally measurable quantities. Let us consider the processes

$$\nu_\mu(\bar\nu_\mu) + N \to \mu^-(\mu^+) + X, \tag{33}$$

$$\nu_\mu(\bar\nu_\mu) + N \to \nu_\mu(\bar\nu_\mu) + X, \tag{34}$$

$$\ell + N \to \ell + X. \tag{35}$$

For the isoscalar targets, we are interested in, we have

$$L_{\alpha\beta}W_{\alpha\beta}^{CC} = 2[\langle VV\rangle + \langle AA\rangle] = \frac{2\pi}{G^2}\frac{(pk)^2}{M^2}\frac{d\sigma^{CC}_{\nu+\bar\nu}}{dq^2 d\nu}, \tag{36}$$

$$L_{\alpha\beta}W_{\alpha\beta}^{NC} = (1-2\sin^2\theta)^2\langle VV\rangle + \langle AA\rangle + \frac{4}{9}\sin^4\theta\langle SS\rangle = \frac{2\pi}{G^2}\frac{(pk)^2}{M^2}\frac{d\sigma^{NC}_{\nu+\bar\nu}}{dq^2 d\nu},$$

$$L_{\alpha\beta}W_{\alpha\beta}^{em} = \langle VV\rangle + \frac{1}{9}\langle SS\rangle = \frac{q^4}{2\pi a^2}\frac{(pk)^2}{M^2}\frac{d\sigma^{em}}{dq^2 d\nu}. \tag{37}$$

Here

$$\langle AA\rangle = L_{\alpha\beta}W_{\alpha\beta}^A,$$

$$W_{\alpha\beta}^A = -(2\pi)^2\frac{p_0}{M}\int e^{-iqx}\langle p|A^3_\beta(x)A^3_\alpha(0)|p\rangle dx,$$

$$\frac{d\sigma^{CC}_{\nu+\bar{\nu}}}{dq^2 d\nu} \left(\frac{d\sigma^{NC}_{\nu+\bar{\nu}}}{dq^2 d\nu}\right)$$

is the sum of the cross sections of the charged current processes (33) (of the neutral processes (34)). From eq. (36) it is easy to get a relation, which couples a^S_V with the cross sections of the processes (33)-(35). Taking into account that a^S_V is independent of $\sin^2\theta$, we have *

$$\frac{2}{9} a^S_V = \frac{2\pi^2 a^2}{\sin^2\theta \, G^2 q^4} \, \frac{1}{d\sigma^{em}/dq^2 d\nu} \left[\frac{d\sigma^{NC}_{\nu+\bar{\nu}}}{dq^2 d\nu} - \frac{1}{2} \frac{d\sigma^{CC}_{\nu+\bar{\nu}}}{dq^2 d\nu}\right] + 2(1 - \sin^2\theta), \qquad (38)$$

where the parameter $\sin^2\theta$ is given as [12]

$$\frac{1}{2}(1 - 2\sin^2\theta) = \frac{\sigma^{NC}_{\nu} - \sigma^{NC}_{\bar{\nu}}}{\sigma^{CC}_{\nu} - \sigma^{CC}_{\bar{\nu}}} .$$

The relation (31) contains the quantities A_- and A_+ and also a_A and $\frac{2}{9} a^S_V$. These last two quantities are related by eqs. (19) and (38) with the cross sections of deep inelastic processes (33)-(35). Thus, the measurement of the P-odd asymmetries A_- and A_+ would enable us to test the validity of the Weinberg-Salam theory without assumptions about the strong interaction dynamics.

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* Combining eqs. (19), (20) and (38) the relation for the asymmetry A_- found in ref. [11] is readily obtained.

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