объединенный
институт
ядерных
исследований
дубна



E2-81-235

M.V.Chizhov

# BOGOLUBOV'S SPONTANEOUS-SYMMETRY-BREAKING MECHANISM AND HIGGS PHENOMENON

*Submitted to "Physics Letters"*

1981

## 1. INTRODUCTION

The idea of the dynamic spontaneous-symmetry-breaking in quantum field theory, originating in the papers by Bogolubov [1,2], gains much importance. In the field-theoretical investigations, the Bogolubov method has first been applied [3] to elucidate the dynamic origin of fermion masses *. The possibility for generating composite particles from the fundamental fields of the initial Lagrangian has been mentioned in the subsequent papers [5-7]. Such particles appear in the Hartree-Fock-Bogolubov approximation.

The method for determining anomalous vacuum expectation values for multi-component fields has been developed in ref. [8]. The present paper is aimed at studying the dynamic spontaneous-symmetry-breaking of the four-fermion theory with (V-A)-interaction. We show that in the mean-field approximation besides the collective vector field there arises a collective complex (pseudo) scalar field which plays the role of the Higgs field. Our model has all the features of the Higgs phenomenon in the Abelian case. We would like to stress that the coupling constants in the effective Lagrangian obtained turn out to be known functions of the two parameters: the vacuum expectation value of the Higgs field and the renormalized Yukawa coupling constant.

## 2. ANOMALOUS GREEN FUNCTIONS AND BOGOLUBOV's COMPENSATION EQUATION

We consider the theory with four-fermion (V-A)-interaction

$$\mathcal{L} = \bar{\Psi} i \not{\partial} \Psi - \lambda_0 \bar{\Psi} \frac{1-\gamma^5}{2} \gamma_\mu \Psi \cdot \bar{\Psi} \frac{1-\gamma^5}{2} \gamma^\mu \Psi . \tag{2.1}$$

Using the method described in ref. [8], one can obtain the vacuum generating functional Z for this theory, expressed as a path integral over the collective boson fields $V_\mu$ and B

---

* For the role of the Bogolubov theory of superfluidity [1] in understanding the phenomena related to spontaneous symmetry breaking see, for instance, the review by Higgs [4].

$$Z[\bar{\eta},\eta,J^*,J,\mathcal{J}_\mu] = N\int DV_\mu DB^*DB \exp\{i(S[V_\mu,B^*,B] + $$

$$+ J^*B + B^*J + \mathcal{J}_\mu V^\mu - \bar{\eta} G_0 \eta)\}, \tag{2.2}$$

where

$$S[V_\mu, B^*, B] = \int dx[\frac{M_0^2}{4}(B^*B + V_\mu^2) - \frac{i}{2}\mathrm{Sp}\ln(iG_0^{-1})], \tag{2.3}$$

and $G_0^{-1}$ has the form

$$G_0^{-1}(p) = \begin{pmatrix} g_0 C\frac{1+\gamma^5}{2}B & \tilde{\not p} - e_0 V^\mu \tilde{\gamma}_\mu \frac{1-\gamma^5}{2} \\ \not p + e_0 \frac{1-\gamma^5}{2}\gamma_\mu V^\mu & -g_0 C\frac{1-\gamma^5}{2}B^* \end{pmatrix} \tag{2.4}^*$$

Note, that the action S (2.3) is invariant under the U(1)-transformation

$$B \to B\exp(i\alpha), \tag{2.6a}$$

$$B^* \to B^*\exp(-i\alpha). \tag{2.6b}$$

The condition of stationarity (2.3) is

$$\frac{M_0^2}{2}V_\mu - \frac{i}{2}\mathrm{Sp}(G_0\frac{\delta G_0^{-1}}{\delta V^\mu}) + \mathcal{J}_\mu = 0, \tag{2.7a}$$

$$\frac{M_0^2}{4}B - \frac{i}{2}\mathrm{Sp}(G_0\frac{\delta G_0^{-1}}{\delta B^*}) + J = 0. \tag{2.7b}$$

With the sources off ($J=\mathcal{J}_\mu=0$), the system of equations (2.7) admits the nontrivial solution $B_0 \neq 0$**, $V_0^\mu = 0$. Finally, the Bogolubov compensation equation is (within the dimensional regularization scheme)

$$1 = 8\lambda_0 f(m^2,\mu^2), \tag{2.8}$$

---

*Here,

$$\frac{e_0^2}{M_0^2} = \lambda_0, \quad e_0^2 = \frac{1}{2}g_0^2. \tag{2.5}$$

**The non-zero vacuum expectation value gives rise to pairs in the $\Psi\Psi$ and $\bar{\Psi}\bar{\Psi}$ channels with opposite momenta and spin orientations of particles, that is in complete analogy with the phenomenon in superconductor.

where

$$f(m^2,\mu^2)=\frac{m^2}{(4\pi)^2}\ln\frac{\mu^2}{m^2}\,, \tag{2.9}$$

and

$$m^2=g_0^2 B_0^* B_0\,, \tag{2.10}$$

$\mu$ is the dimensional regularization parameter.

Then, we use the Laplace method to expand the integral (2.2) in an asymptotic series about the mean-field $B_0$ and $V_0^{\mu}$.

## 3. GREEN FUNCTIONS OF COMPOSITE PARTICLES IN THE LOWEST ORDER OF PERTURBATION THEORY

In the lowest-order of perturbation theory the functional S plays the role of the generating functional of all one-particle-irreducible graphs. Substituting the variables in the functional integral (2.2)

$$B=\rho\exp(i\phi)\,, \tag{3.1a}$$

$$B^*=\rho\exp(-i\phi)\,, \tag{3.1b}$$

we pass to new fields $\rho$ and $\phi$. The functional is stationary with respect to the field $\phi$ and the condition of stationarity with respect to $\rho$ gives eq. (2.8), eq. (2.10) taking the form

$$m=g_0\rho_0\,. \tag{3.2}$$

Now we find the inverse propagator for the field $\phi$

$$\Lambda_{\phi}^{-1}(q^2)=-\frac{\delta^2 S}{\delta\phi^2}=\frac{im^2q^2}{(2\pi)^4}\int dp\int_0^1 dx\,\frac{p^2}{[p^2+q^2x(1-x)-m^2]^3}= \tag{3.3*}$$

$$=\frac{m^2}{g^2}\{-q^2+O(g^2)\}\,.$$

---

* The solution for the vacuum expectation value m is similar to the expression of the energy gap in the theory of superconductivity; here $g=Z^{1/2}g_0$ is the renormalized coupling constant $g_0$ and Z is the renormalization constant of the field $\rho$.

Since

$$\Delta_{\phi}^{-1}(0) = 0 \tag{3.4}$$

the field $\phi$ is massless. It corresponds to the Goldstone boson associated with spontaneous breaking of the continuous U(1) symmetry (3.1) of the action S. The field $\rho$ acquires mass as

$$\Delta_{\rho}^{-1}(q^2) = -\frac{\delta^2 S}{\delta \rho^2} = \frac{2ig_0^2}{(2\pi)^4} \int dp \frac{(p+q)q - 2m^2}{(m^2-p^2)(m^2-(p+q)^2)} =$$
$$= \frac{1}{Z}\{4m^2 - q^2 + O(g^2)\}. \tag{3.5}$$

The inverse propagator for the field $V_\mu$ has the form

$$D_{\mu\nu}^{-1}(q) = -\frac{\delta^2 S}{\delta V^\mu \delta V^\nu} = -\frac{M_0^2}{2} g_{\mu\nu} + \frac{2ie_0^2}{(2\pi)^4} \int dp \frac{p(p+q) g_{\mu\nu} + m^2 g_{\mu\nu} - p_\mu (p+q)_\nu - p_\nu (p+q)_\mu}{(m^2-p^2)(m^2-(p+q)^2)}$$

and the transverse part (3.6)

$$D_{\mu\nu}^{-1}(q) - D_{\mu\nu}^{-1}(0) = \frac{q^2 g_{\mu\nu} - q_\mu q_\nu}{Z_v} + \frac{1}{Z_v} O(e^2), \tag{3.7}$$ *

which under the renormalization of the fields $\bar{V}_\mu = Z_v^{-1/2} V_\mu$ corresponds to the kinetic term $-\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}$ in the Lagrangian.

## IV. EFFECTIVE INTERACTION LAGRANGIAN AND DEFINITION OF COUPLING CONSTANTS

To construct the effective interaction Lagrangian, we note that the coupling constants of the interactions without derivatives are determined by the divergent vertex functions in the momentum space at zero external momenta. The functional derivatives in expanding integral (2.2) are calculated at the saddle point $\rho_0 = m/g_0$, $V_0^\mu = 0$. Therefore, using the formula

$$\frac{dS}{dg} = \int dx \frac{\delta S}{\delta g(x)} \Big|_{g(x) = g = \text{const}}, \tag{4.1}$$

* Here

$$Z_v = 3Z, \quad e^2 = Z_v e_0^2. \tag{3.8}$$

we apply the usual differentiation of expressions (2.7) with respect to the fields $\rho$, $\phi$ and $V^{\mu}$ and get the one-particle-irreducible Green functions at zero external momenta in the lowest-order. Then using the expressions

$$\frac{\delta^2 S}{\delta\rho\,\delta\phi} = \frac{\delta^2 S}{\delta\rho\,\delta V^{\mu}} = 0 \tag{4.2}$$

and

$$\frac{\delta^2 S}{\delta\phi\,\delta V^{\mu}} = -\frac{2e_0 m^2 q_{\mu}}{(2\pi)^4}\int dp\,\frac{1}{(m^2-p^2)(m^2-(p+q)^2)}, \tag{4.3}$$

we find the Lagrangian of composite fields

$$\mathcal{L}^{\text{comp.}} = -\frac{1}{4}(\partial_{\mu}V_{\nu}-\partial_{\nu}V_{\mu})^2 + \frac{1}{2}(\rho+\frac{m}{g})^2(\partial_{\mu}\phi+2eV_{\mu})^2 + \frac{1}{2}(\partial_{\mu}\rho)^2 - \frac{g^2}{2}[(\rho+\frac{m}{g})^2 - \frac{m^2}{g^2}]^2\,. \tag{4.4}$$

Substituting of variables

$$W_{\mu} = V_{\mu} + \frac{1}{2e}\partial_{\mu}\phi \tag{4.5}$$

makes the quadratic part of the Lagrangian (4.4) diagonal. Finally, we have

$$\mathcal{L}^{\text{comp.}} = -\frac{1}{4}(\partial_{\mu}W_{\nu}-\partial_{\nu}W_{\mu})^2 + 2e^2(\rho+\frac{m}{g})^2 W_{\mu}^2 + \frac{1}{2}(\partial_{\mu}\rho)^2 - \frac{g^2}{2}[(\rho+\frac{m}{g})^2 - \frac{m^2}{g^2}]^2. \tag{4.6}$$

The Lagrangian (4.6) is just the Higgs Lagrangian in the Abelian case. Moreover, as is seen from eqs. (2.5), (3.8) and (4.4), we get the relation between the gauge interaction constant e and the renormalized $\rho$ self-coupling

$$e^2 = \frac{3}{2}g^2. \qquad (4.7)^*$$

---

*In this case at $\frac{e^2}{\pi^2} < 1$ the expansion over the loops in this model gives just corrections to the tree approximations and does not change the structure of spontaneous symmetry breaking. Cf., the lectures by Coleman /9/.

## 5. CONCLUSION

In this section we briefly comment on the fermion sector of the theory. Spontaneous symmetry breaking leads to the generation of massive spinor particles; their description requires both left and right spinors. Therefore, one should add to the Lagrangian (2.1) the extra four-fermion (V+A)-interaction. Performing the Pauli-Gürsey transformation /10/

$$\Psi' = a\Psi + b\gamma^5 C\bar{\Psi}, \tag{5.1a}$$

$$\bar{\Psi}' = a^*\bar{\Psi} - b^*\gamma^5 C\Psi, \tag{5.1b}$$

where

$$|a|^2 + |b|^2 = 1, \tag{5.2}$$

and the chiral transformation $\exp(i\frac{\pi}{4}\gamma^5)$, we get the theory with massive fermions. Note, that the transformation (5.1) is similar to the Bogolubov canonical transformation in the theory of superconductivity /2/.

It is rempting to interpret the fundamental fermion field as the neutrino field. As it is seen from eq. (2.4), as a result of the dynamic spontaneous symmetry breaking, the Weyl spinor $\Psi_R$ acquires Majorana mass /11/, $\Psi_L$ being massless.

The given formalism can easily be extended to the non-Abelian case in particular to the U(2) group, that leads to the standard SU(2)xU(1) unified theory of weak and electromagnetic interactions. However, it is a subject of the subsequent paper.

The author is grateful to V.G.Kadyshevsky and M.D.Mateev for interest in this work and fruitful discussions.

## REFERENCES

1. Bogolubov N.N. Izv.Akad.Nauk SSSR, ser.fiz., 1947, 11, p.77.
2. Bogolubov N.N. Zh.Eksp.Teor.Fiz., 1958, 34, p.58; Bogolubov N.N. JINR, D-781, Dubna, 1961.
3. Nambu Y., Jona-Lasinio G. Phys.Rev., 1961, 122, p.345; Vaks V.G., Larkin A.I. Zh.Eksp.Teor.Fiz., 1961, 40, p.282; Arbuzov B.A., Tavkhelidze A.N., Faustov R.N. Doklady Akad.Nauk SSSR, 1961, 139, p.345.
4. Higgs P.W. In: Phenomenology of Particles at High Energies, ed. R.L.Crawford and R.Jennings, Academic Press, 1974, p.529.

5. Eguchi T. Phys.Rev., 1976, D14, p.2755.
6. Bender C.M., Cooper F., Guralnik G.S. Ann.Phys., 1977, 109, p.165.
7. Tamvakis K., Guralnik G.S. Phys.Rev., 1978, D18, p.4551; Cooper F., Guralnik G.S., Snyderman N. Phys.Rev.Lett., 1978, 40, p.1620; Kadyshevsky V.G., Mateev M.D., Chizhov M.V. JINR, P2-80-27, Dubna, 1980.
8. Chizhov M.V. JINR, P2-81-36, Dubna, 1981.
9. Coleman S. In: Laws of Hadromic Matter, ed. A.Zichichi, Academic Press, 1975, p.139.
10. Pauli W. Nuovo Cim., 1957, 6, p.204; Gürsey F. Nuovo Cim., 1958, 7, p.411.
11. Mannheim P.D. Dept. of Phys.Univ. of Connecticut, Storrs, CT 06268, June, 1980.

Received by Publishing Department
on April 6 1981.