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APPROXIMATION OF INDEPENDENTLY CASCADING NUCLEONS IN THE INELASTIC NUCLEUS-NUCLEUS SCATTERING

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1. INTRODUCTION

The problem of finding a theoretical approach which is capable of describing the essential features of multiparticle production processes in inelastic nucleus-nucleus interactions has received much interest recently, because its solution would enable one to "extract" information from the small amounts of interesting experimental data available thus far and also would help in establishing the direction of further investigations. In this paper we take the Glauber approximation to the problem of medium and heavy nucleus interactions as a basis for such an approach.

As it is known, the Glauber approximation [1] allowed to describe satisfactorily both elastic and quasielastic scattering of particles from atomic nuclei [2] and is, essentially, the basic tool for the analysis of such data. That is why natural efforts have been directed towards generalizing it for application to inelastic reactions [3,4]. Due to some progress in this direction interpretations of the leading particle spectra in pA-interactions [5], of the anomalously strong A-dependence of inclusive cross-sections [6] and also some characteristics of the multiparticle production process in hadron-nucleus collisions [7-11] have been obtained. At present the theory of elastic and some inelastic nucleus-nucleus interactions is successfully developing [12-17]. All these facts give one the hope that this approach might be advantageous in the analysis of inelastic nucleus-nucleus reactions as well.

When starting with the theory of hadron-nucleus scattering, one should expect that the first problem to solve will be the representation of the nucleus-nucleus inelastic cross section as a sum of cross sections of different interaction subprocesses of the constituting nucleons. In the second section of the paper we obtain such a representation using, however, some formal considerations. The rigorous theoretical foundation of these would lie far outside the scope of this article, so we omit it without affecting by this the understanding. Besides, our considerations rely heavily on the results of ref. [17], where the structure of the Glauber series for the elastic nucleus-nucleus scattering amplitude is analyed and the computation rules for contributions of the connected diagrams are formulated.

Basing on these results we find in the third section the average multiplicity and the rapidity distribution of secondary particles under the assumption that they all leave the nucleus with no more interactions. Such an assertion, as it has been already shown before [18], is a direct consequence of the application of the standart technique of Regge cuts to the eikonal elastic hadron-nucleus scattering amplitude. It seems that this is also true for the case of nucleus-nucleus scattering. This assumption, at least, considerably simplifies the consideration.

A discussion of the main results, the perspectives and problems of the proposed direction are presented in the concluding section.

2. PARTICLE PRODUCTION CROSS SECTION AND PROBABILITY OF DIFFERENT INELASTIC PROCESSES

The scattering amplitude of two nuclei is given in the Glauber approximation by the following well-known expression:

$$F_{AB}(\vec{q}) = \frac{iP_B}{2\pi} \int d^2b\, e^{i\vec{q}\vec{b}} \langle \Psi_A' \Psi_B' | 1 - \prod_{i=1}^{A} \prod_{j=1}^{B} [1 - \gamma(\vec{b} - \vec{s}_i + \vec{r}_j)] | \Psi_B \Psi_A \rangle, \quad (1)$$

where P_B is the momentum of the projectile nucleus B, $\vec{q}$ is the transverse momentum transferred to it, Ψ_A, Ψ_B and Ψ_A', Ψ_B' are the wave functions of nuclei A and B in the initial and final states, respectively. The averaging with respect to these latter is denoted by the broken brackets:

$$\gamma(\vec{b}) = \frac{B}{2\pi i P_B} \int f_{NN}(\vec{q})\, e^{-i\vec{q}\vec{b}}\, d^2q,$$

$f_{NN}(\vec{q})$ is the amplitude of elastic NN-scattering, $\{\vec{s}_A\}, \{\vec{r}_B\}$ are the coordinates of the nucleons of nuclei A, B within the plane of the impact parameter $\vec{b}$ (in the plane perpendicular to the momentum P_B).

In the case of elastic scattering $\Psi_A = \Psi_A'$, $\Psi_B = \Psi_B'$ it follows from the relation (1) that the problem of calculating the amplitude or the phase $\chi^{el}(\vec{b})$ is associated with averaging the double product with respect to the ground states of the nuclei

$$F_{AB}^{el}(\vec{q}) = \frac{iP_B}{2\pi} \int d^2b\, e^{i\vec{q}\vec{b}} \{1 - e^{-\chi^{el}(\vec{b})}\}$$

$$\chi^{el}(\vec{b}) = -\ln \langle \Psi_A \Psi_B | \prod_{i=1}^{A} \prod_{j=1}^{B} [1 - \gamma(\vec{b} - \vec{s}_i + \vec{r}_j)] | \Psi_B \Psi_A \rangle, \quad (2)$$

which is considerably simplified in the optical limit over the mass numbers of the nuclei ($A,B\to\infty$). Here the scattering phase can be presented, with some justified assumptions, as (see refs. [12,13,15-17]):

$$\chi^{el}(\vec{b})=\chi(T_A,T_B,\frac{\tilde{\sigma}}{2})=\frac{2}{\tilde{\sigma}}\int d^2s\sum_{m,n=1}^{\infty}\frac{(-1)^{m+n}}{m!\,n!}m^{n-1}n^{m-1}x^m y^n=$$

$$=\frac{2}{\tilde{\sigma}}\int d^2s\,[u(e^z-1)+z(e^u-1)-uz], \tag{3}$$

$$\begin{cases} u=ye^{-z} \\ z=xe^{-u} \end{cases}, \tag{4}$$

$$x=\frac{\tilde{\sigma}}{2}T_A(\vec{s})\qquad y=\frac{\tilde{\sigma}}{2}T_B(\vec{b}-\vec{s})\qquad \tilde{\sigma}=-i\frac{4\pi B}{P_B}f_{NN}(0),$$

$$T_A(\vec{s})=A\int\rho_A(\vec{s},\xi)d\xi\qquad T_B(\vec{s})=B\int\rho_B(\vec{s},\xi)d\xi,$$

where ρ_A and ρ_B are one-particle densities of the nuclei A and B, respectively. The first term of the series expansion (3) has been found by W.Chyẑ and L.C.Maximon in ref. [19] and is known as the phase in the so-called "optical" limit. The full series expansion has been first obtained by I.V.Andreev [12] and its summation has been performed in ref. [15]. Since computations using the phase (3), as well as the comparison with experimental data have been performed earlier [13-15], we shall not discuss here the accuracy of this approximation but instead turn directly to the case of inelastic processes.

Calculating the squared modulus of the expression (1), summed over the final states of the nuclei and integrated over the transverse momentum, we obviously obtain the cross section of all reactions unaccompanied by particle production. Subtracting it from the total interaction cross section, defined by the amplitude (2), we obtain that the cross section of new particle production is given by the expression:

$$\sigma_{AB}^{prod}=\int d^2b\{1-\langle\Psi_A\Psi_B|\prod_{i=1}^{A}\prod_{j=1}^{B}[1-g(\vec{b}-\vec{s}_i+\vec{r}_j)]|\Psi_B\Psi_A\rangle\approx \tag{5}$$

$$\approx\int d^2b\{1-\exp[-\chi(T_A,T_B,\sigma)]\}$$

$$g(\vec{b})=\gamma(\vec{b})+\gamma^*(\vec{b})-\gamma(\vec{b})\gamma^*(\vec{b})$$

$$\int g(\vec{b})\, d^2 b=\sigma.$$

Let us present it as a sum of cross sections for different inelastic processes. For this purpose we use the identity:

$$1-\prod_{i=1}^{k}(1-h\delta_i)=\sum_{i=1}^{k}\frac{(-h)^i}{i!}\frac{d^i}{dh^i}\prod_{j=1}^{k}(1-h\delta_j)=$$
$$=h\sum_{i=1}^{k}\delta_i\prod_{\substack{j=1\\ j\neq i}}^{k}(1-h\delta_j)+\frac{h^2}{2!}\sum_{\substack{i,j=1\\ i\neq j}}^{k}\delta_i\delta_j\prod_{\substack{\ell=1\\ \ell\neq i\neq j}}^{k}(1-h\delta_\ell)+\dots, \tag{6}$$

which, when applied to the inelastic hadron-nucleus scattering, yields the well-known effective numbers [3]. In our case we obtain the following expansion*

$$\sigma_{AB}^{prod}=\int d^2b<\Psi_A\Psi_B|\{h\sum_{i=1}^{A}\sum_{j=1}^{B}g_{ij}[\prod_{k=1}^{A}\prod_{\ell=1}^{B}(1-hg_{k\ell})]/(1-hg_{ij})+$$
$$+\frac{h^2}{2!}\sum_{i=1}^{A}\sum_{\substack{j,k=1\\ j\neq k}}^{B}g_{ij}g_{ik}[\prod_{\ell=1}^{A}\prod_{m=1}^{B}(1-hg_{\ell m})]/(1-hg_{ij})(1-hg_{ik})+$$
$$+\frac{h^2}{2!}\sum_{\substack{i,j=1\\ i\neq j}}^{A}\sum_{k=1}^{B}g_{ik}g_{jk}[\prod_{\ell=1}^{A}\prod_{m=1}^{B}(1-hg_{\ell m})]/(1-hg_{ik})(1-hg_{jk})+ \tag{7}$$
$$+\frac{h^2}{2!}\sum_{\substack{i,j=1\\ i\neq j}}^{A}\sum_{\substack{k,\ell=1\\ k\neq\ell}}^{B}g_{ik}g_{j\ell}[\prod_{m=1}^{A}\prod_{n=1}^{B}(1-hg_{mn})]/(1-hg_{ik})(1-hg_{j\ell})+\dots\}|_{h=1}\Psi_B\Psi_A>$$

the first term of which represents the cross section of the processes with one inelastic interaction. Averaging it over the ground states of the nuclei and using the rules formulated in ref. [17], we find that the cross section of the single inelastic interactions is given by the sum of the contributions of all diagrams of the type shown in fig. 1, where the black points correspond to the function hg_{ij}, representing the elastic scattering of nucleons; while the open points, to the same function but without the h factor, which gives the account of inelastic collisions of nucleons. In the third lowest line of fig. 1 the general aspect of these diagrams

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* Here we introduce the notation: $g_{ij}=g(\vec{b}-\vec{s}_i+\vec{r}_j)$.

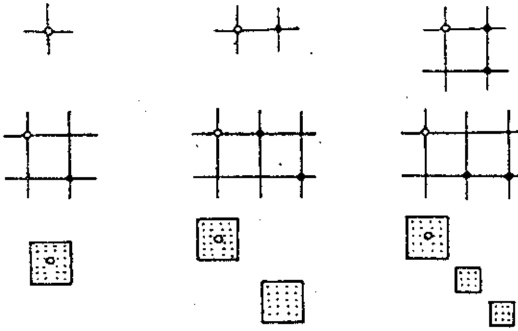

Fig. 1

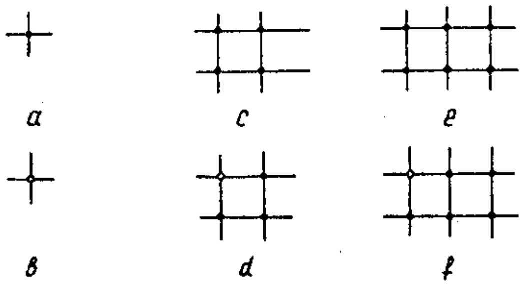

Fig. 2

is presented. Each shadowed square corresponds to some connected diagram with unspecified structure. As it was shown in ref. [17] all connected diagrams can be constructed on the basis of the so-called root diagrams. In particular, the diagrams in fig. 1 can be constructed using roots of the type shown in fig. 2. As it was in the case of elastic nucleus-nucleus scattering, we neglect the contributions of loop diagrams [16], i.e., of diagrams constructed using the roots 2c, 2d, ..., etc. Then, the summation of the contributions of all connected and disconnected diagrams entering the stucture of the diagram in fig. 1 and based on the root 2a leads, in the optical limit over the atomic mass numbers of the nuclei, to the appearance of a factor $e^{-\chi}$, while the sum of all diagrams generated by the root 2b has, in the same limit, the form [17] $\frac{1}{\sigma}\int d^2 s u z$. Thus the cross section of the processes with one inelastic interaction is determined as:

$$\sigma^{1;;}_{1;;} = \int d^2 b\, e^{-\chi} \frac{1}{\sigma} \int d^2 s\, uz. \tag{8}$$

Looking now at the second term of the expansion (7) we can see that it represents the sum of the contributions of all diagrams of the type shown in fig. 3 based on the roots of the types shown in fig 4. Summing up, as above, the contributions of all these diagrams [17], we find that the cross section of the processes in which one of the nucleons of the nucleus A undergoes a double inelastic collision is given by the expression:

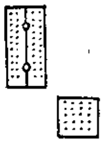

Fig. 3

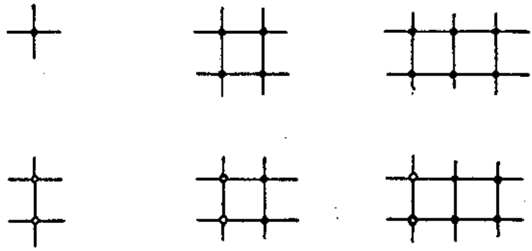

Fig. 4

$$\sigma^{0;1;}_{1;2;} = \frac{1}{2!}\int d^2b\, e^{-\chi}\, \frac{1}{\sigma}\int d^2s u^2 z\,. \qquad (9)$$

The cross section of the process in which one of the nucleons in the nucleus B undergoes a double inelastic collision (the third term of equation (7)) is determined analogously:

$$\sigma^{0;0;1}_{1;2;2} = \frac{1}{2!}\int d^2b\, e^{-\chi}\, \frac{1}{\sigma}\int d^2s u z^2\,. \qquad (10)$$

More complicated is the fourth term of equation (7), presented by the set of giagrams of the type shown in fig. 5. There are, as we see, diagrams of two different structures: a) diagrams in which the single inelastic interactions are by no means interconnected (fig. 5a); b) diagrams in which nucleons from different nuclei participating in inelastic collisions experimence subsequent elastic rescattering on the same band of nucleons (fig. 5b). One can show that diagrams of the latter or similar type are compensated by the higher terms of the expansion (7), which in particular, contains contributions of diagrams of the type shown in fig. 6. As a result, the production cross section appears as a sum of the cross sections of processes in which the nucleons of the colliding nuclei are cascading independently, i.e., it is defined by the contributions of diagrams of figs. 1,3,5a and of similar type. To find the cross sections of all these processes one can follow further the above procedure, however this is not necessary if one uses the following representation for

$$\chi(\vec{b}) = \frac{1}{\sigma}\int [u(e^z - 1) + z(e^u - 1) - uz]\, d^2s = $$
$$= \frac{1}{\sigma}\int [uz + u\sum_{n=2}^{\infty}\frac{z^n}{n!} + z\sum_{n=2}^{\infty}\frac{u^n}{n!}]\, d^2s. \qquad (11)$$

Introducting this into equation (5) we find, after some transformations, that:

a б

Fig. 5

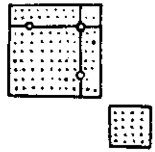

Fig. 6

$$\sigma_{AB}^{prod} = \int d^2b\,\{1-e^{-\chi}\} = \int d^2b\, e^{-\chi}\,\{e^{\chi}-1\} =$$

$$= \sum_{i,j_1,j_2,\dots,k_1,k_2,\dots} \sigma^{\,i;j_1,j_2,\dots;\,k_1,k_2,\dots}_{\,1;m_1,m_2,\dots;\,n_1,n_2,\dots} \tag{12}$$

$$\sigma^{\,i;j_1,j_2,\dots;\,k_1,k_2,\dots}_{\,1;m_1,m_2,\dots;\,n_1,n_2,\dots} = \int d^2b\, e^{-\chi(b)}\frac{(d_1)^i}{i!}\cdot\frac{(a_{m_1})^{j_1}}{j_1!}\times$$

$$\times\frac{(a_{m_2})^{j_2}}{j_2!}\cdot\dots\cdot\frac{(b_{n_1})^{k_1}}{k_1!}\cdot\frac{(b_{n_2})^{k_2}}{k_2!}\cdot\dots$$

$$a_m = \frac{1}{\sigma}\int\frac{u^m z}{m!}\,d^2s \qquad b_n = \frac{1}{\sigma}\int\frac{uz^n}{n!}\,d^2s$$

$$d_1 = \frac{1}{\sigma}\int u\,z\,d^2s \tag{13}$$

$$W^{\,i;j,\dots;\,k,\dots}_{\,1;m,\dots;\,n,\dots} = \sigma^{\,i;j,\dots;\,k,\dots}_{\,1;m,\dots;\,n,\dots}/\sigma_{AB}^{prod}\ ,$$

Here $W^{\,i;j,\dots;k,\dots}_{\,1;m,\dots;n,\dots}$ and $\sigma^{\,i;j,\dots;k,\dots}_{\,1;m,\dots;n,\dots}$ are the probabilities and cross sections of different inelastic interactions, respectively, i represents the number of independent single interactions, j is the number of nucleons of the nucleus A subjected to m-fold inelastic collisions; k, the number of nucleons in B subjected to n-fold collisions, etc. For instance, the cross section of two independent single collisions (fig. 5a) is given by the expression

$$\sigma^{2;;}_{1;;} = \frac{1}{2!}\int d^2b\, e^{-\chi(b)} d_1^2 = \frac{1}{2!}\int d^2b\, e^{-\chi(b)} [\frac{1}{\sigma}\int u z d^2 s]^2. \tag{14}$$

All other quantities are transcribed in the same way.

Note, that given above relations (12) and (13) do not define the cross sections of the processes of the type presented in fig. 6, which is due to some incorrectness in the decomposing equation (7). Leaving aside a more correct consideration up to section 4 it seems reasonable now to extract the practical outcome from the results already obtained.

3. APPROXIMATION OF INDEPENDENTLY CASCADING NUCLEONS. CHARACTERISTICS OF THE SHOWER PARTICLES PRODUCED IN INELASTIC NUCLEUS-NUCLEUS INTERACTIONS

Let us suppose that the inelastic nucleus-nucleus interaction process is essentially determined by the subprocesses of independent cascading of nucleons, where the probabilities of the different subprocesses are given by equations (12) and (13) and the produced particles leave the nuclei undergoing no other interactions. Then, using the elaborated apparatus of the leading hadron cascade model /9-11/, it is not difficult to determine various characteristics of the produced particles.

1) The Average Multiplicity of Secondary Particles

It is obvious, that under our assumption, the average multiplicity of the particles produced in a certain process of inelastic interaction is determined by the sum of the average multiplicities of its subprocesses, i.e.,

$$\bar{n}^{i;j,\ldots;k,\ldots}_{1;m,\ldots;n,\ldots} = i\cdot\bar{n}_1 + j\cdot\bar{n}_{A,m} + \ldots + k\cdot\bar{n}_{B,n} + \ldots . \tag{15}$$

For instance, the average multiplicity in i independent single collisions is $i\cdot\bar{n}_1$ and in $j(k)$ independent double collisions of the nucleons of the nucleus A(B) it equals to $j\cdot\bar{n}_{A,2}(k\cdot\bar{n}_{B,2})$. Multiplying the sum of these three quantities by the corresponding probability of the process and performing the summation over i,j,k, we find the contribution of these subprocesses to the average multiplicity:

$$\bar{n}_{1,2} = \sum_{\substack{i,j,k \\ i+j+k\neq 0}} (i\cdot\bar{n}_1 + j\cdot\bar{n}_{A,2} + k\cdot\bar{n}_{B,2})\cdot W^{i;j;k}_{1;2;2} =$$

$$= \frac{1}{\sigma_{AB}^{prod}} \sum_{\substack{i,j,k\\ i+j+k\neq 0}} \int d^2b\, e^{-\chi} (i\cdot\bar{n}_1 + j\cdot\bar{n}_{A,2} + k\cdot\bar{n}_{B,2}) \cdot \frac{(d_1)^i}{i!} \cdot \frac{(a_2)^j}{j!} \cdot \frac{(b_2)^k}{k!} =$$

$$= \frac{1}{\sigma_{AB}^{prod}} \int d^2b\, e^{-\chi + d_1 + a_2 + b_2} (\bar{n}_1 \cdot d_1 + \bar{n}_{A,2} \cdot a_2 + \bar{n}_{B,2} \cdot b_2). \qquad (16)$$

In a similar way we find that the average multiplicity of the produced particles in inelastic nucleus-nucleus collision is given by the expression:

$$\bar{n}_{AB} = \frac{1}{\sigma_{AB}^{prod}} \int d^2b [\bar{n}_1 \cdot d_1 + \sum_{j=2}^{\infty} (\bar{n}_{A,j} \cdot a_j + \bar{n}_{B,j} \cdot b_j)], \qquad (17)$$

where $\bar{n}_{A,j}$ ($\bar{n}_{B,k}$) is the average multiplicity of particles produced in inelastic collision of a nucleon of the nucleus A(B) with j(k) nucleons of the nucleus B(A), which we determine following ref. [9].

If the average multiplicity of secondary particles in free NN-interactions has a power law dependence on the squared total energy in the centre-of-mass system ($\bar{n}_1(s_{NN}) \sim s_{NN}^{\alpha}$, $s_{NN} \approx 2\frac{P_B}{B} m_N$), then:

$$\bar{n}_{A,j} = \bar{n}_1 \frac{1-f^j}{1-f} = \bar{n}_{B,j}. \qquad (18)$$

Here $f = (1+\beta)/(1+\alpha+\beta)$ and β is a parameter characterizing the spectrum of leading particles in free NN-interactions ($\frac{d\sigma_{NN}}{dx} = (1+\beta)x^{\beta}$, $x = E/E_0$). Using these relations we obtain that the quantity

$$R_{AB} = \bar{n}_{AB}/\bar{n}_1 = \frac{1}{\sigma_{AB}^{prod}(1-f)} \int d^2b \{\chi(\vec{b}) - \frac{1}{\sigma} \int d^2s [u(e^{fz} - 1) + z(e^{fu} - 1) - fuz]\} \qquad (19)$$

is energy independent in this case and is determined only by the mass numbers of the colliding nuclei. Fig. 7 shows the quantity R_{AB} as a function of the average number of nucleon-nucleon collisions $\bar{\nu} = A \cdot B \cdot \sigma / \sigma_{AB}^{prod}$ [20], calculated with the following values of parameters $\sigma = 30$ mb; $\alpha = 0.33$; $\beta = 0$; B = =12,16; A =12,16,24, 28,32,40; the nuclear parameters are taken from ref.[21]. As it is seen, computations demonstrate a scaling behaviour of the quantities R_{AB}, analogous to the

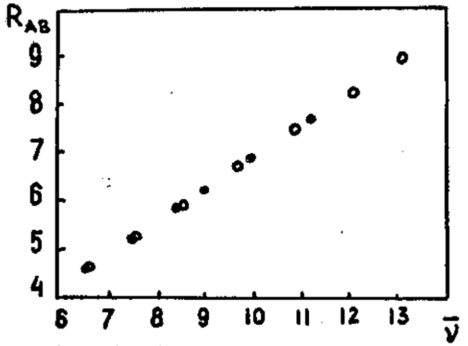

Fig.7. R_{AB} from eq. (19) as a function of the average number of collisions $\bar{\nu}$. Closed and opened cycles correspond to ^{12}C and ^{16}O induced reactions on nuclear target, respectively.

known behaviour of the average multiplicities in hadron-nucleus collisions. It is difficult to show that a similar statement is true for heavier colliding nuclei as well, since for $A,B \geq 24$ there are domains where the system (4) has nonunique solution[16]. Finding all the solutions of the system (4), however, significantly increases the computation time.

2) The Rapidity Distribution of Secondary Particles

According to the basic assumptions of the initial approximation the rapidity distributions of secondaries obeys the same summation rules as those for the average multiplicities. Therefore, the summed distribution is determined as

$$F_{AB}(y) = \frac{dn_{AB}}{dy} = \frac{1}{\sigma^{prod}_{AB}} \int d^2b\,[\,d_1 \cdot F_1(y) + \\ + \sum_{j=2}^{\infty} (a_j \cdot F_{A,j}(y) + b_j \cdot F_{B,j}(y))], \tag{20}$$

where $y = \frac{1}{2}\ln\frac{E+P}{E-P}$, E and P are the energy and longitudinal momentum of the produced particle in the laboratory frame, respectively; $F_1(y) = \frac{dn_1}{dy}$ is the rapidity distribution in NN-interaction, $F_{B,k}(y)(F_{A,j}(y))$ is the rapidity distribution in the inelastic collision of a nucleon of the nucleus $B(A)$ with $k(j)$ nucleons of the nucleus $A(B)$, which is determined, following the recipe of the leading particle cascade model[10] as:

$$F_{B,k}(y) = \sum_{i=1}^{k} F_1(y, s_{NN} \cdot f^{(i-1)/\alpha}), \\ F_{A,k}(y) = F_{B,k}(Y_0 - y) \tag{21}$$

$Y_0 \approx \ln\frac{2P_B}{B \cdot m_N}$ represents the maximal rapidity of the fragmentation nucleon of the nucleus B.

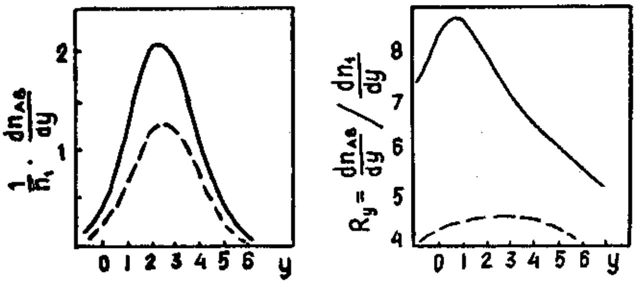

Fig. 8. Rapidity distributions of secondary particles in $^{12}C + ^{40}Ca$ and $^{12}C + ^{12}C$ reactions (solid and dashed lines, respectively) in the lab. system at energy 100 GeV/nucleon calculated from eq. (20).

Computations of the rapidity distributions according to equation (20), performed using a gaussian parametrization for $F_1(y)$ /10/ are presented in fig. 8. This figure shows that the model predicts a strong dependence of the rapidity distribution on the mass number of the colliding objects.

Here upon we end our considerations of the characteristics of generated particles, though the possibilities of the approach, as we shall comment below, are far from being exhausted.

4. PROBLEMS AND PERSPECTIVES

We shall briefly discuss the unsolved problems and the possibilities of the proposed theoretical scheme.

1. To have a more rigorous theoretical foundation it is necessary to prove the applicability of the identity (6) in the theory of nucleus-nucleus reactions, i.e., it is necessary to show that under certain assumption the quantities $\sigma^{i;j,\dots;k,\dots}_{1;m,\dots;n,\dots}$ or their analogues have indeed the meaning given to them above. The similar problem of isolating different reactions and determining their cross sections is encountered in every approach really pretending to describe the inelastic nucleus-nucleus interactions.

2. To simplify the proposed theoretical scheme it would be desirable to exclude from consideration the contributions of multiloop diagrams, containing a dimensionless parameter of order 0.7, instead of 0.25 as it was in the case of elastic

nucleus-nucleus scattering, where their contribution to the phase function was not significant [15,16]. The smallness of these contributions in the theory of inelastic nucleus-nucleus reactions cannot be presupposed without proper estimates. Thus the question arises to determine the influence of the contributions of multiloop diagrams upon the final results. It is believed that the question becomes urgent when considering "central" nucleus-nucleus collisions, because the contribution of the multiloop giagrams grows at small impact parameters [16].

3. It should be noted that the representation of the production cross section in the form (12), obtained following the idea of maximal simplification of the computing apparatus of the proposed scheme, is not entirely correct, since it does not contain the cross section of processes of the type presented in fig. 5b and 6. The correct application of the identity (6) leads, for example, to the following expression for the cross section of double inelastic collision:

$$\sigma^{(2)} = \int d^2b \frac{(-\sigma)^2}{2!} \frac{d^2}{d\sigma^2} e^{-\chi(T_A, T_B, \sigma)} =$$

$$= \int d^2b e^{-\chi} \left(\frac{1}{\sigma} \int d^2s \frac{uz^2}{2}\right) + \int d^2b\, e^{-\chi} \left(\frac{1}{\sigma} \int d^2s \frac{u^2 z}{2}\right) + \tag{22}$$

$$+ \frac{1}{2} \int d^2b\, e^{-\chi} \left[\left(\frac{1}{\sigma} \int d^2s uz\right)^2 - \frac{1}{\sigma} \int d^2s \frac{u^2 z^2}{1-uz}(2+u+z)\right].$$

Here the first and the similar second term, as well as the third one define the cross sections of the processes schematically presented in figs. 3 and 5, respectively. Since the expressions for the cross sections of other processes are no less complicated, we are faced here with still much work in determining and systematizing them.

4. For determining the characteristics of secondary particles it is necessary to make assumptions concerning their production mechanism and behaviour in the collision process. The question of the mechanism of multiparticle production is far from being solved at present even in hadron-hadron interactions. Therefore, this point opens the way to different theoretical constructions. However, it seems that the dependence of the final results and of their degree of agreement with experimental data, on the initial assumptions allows to eliminate some of them. Note, that in most cases the compa-

rison of theoretical predictions with experimental data is complicated by the fact that the domain of applicabillity of one or another theoretical scheme is unknown.

In the framework of the discussed problem one can foresee that the premises of different hadron-nucleus interaction models will come into use. When estimating the final results one has to keep in mind that most of the models pretend to describe the high momentum part of the hadron-nucleus reactions, leaving out of attentive consideration the domain of the target nucleus fragmentation and its dependence on the character of the process. Experimental investigations of the nucleus-nucleus collisions at high energies from the begining goes towards establishing the dependence of the characteristics of secondaries on the degree of the colliding nuclei fragmentation. Therefore, the above utilization requires further elaboration of the models of hadron-nucleus interactions. Probably, the solution to this problem can be found on the way of creating a symbiosis of the mentioned models with the cascade model [22].

It is seen from above discussion, that the enumerated questions are sufficiently comprehensive and their analysis and careful consideration cannot be completed within one publication. That is why this paper presents the simplest version of the problem (approximation of independently cascading nucleons), which, however, has sufficiently large possibilities. Let us enumerate some of them:

a) On the basis of cross sections (12) one can determine the cross sections of the production of rapid nuclear fragments.

b) Using the basic assumptions of section 3 and the elaborated apparatus of the leading hadron cascade model different characteristics of generated particles can be computed, such as: multiplicity distribution, inclusive one-particle distributions, correlation functions, etc.

c) The relatively simple computing apparatus of the model allows one to analyze the characteristics of different subprocesses, thereby isolating some of them as more promising for further experimental investigation.

Apparently, due to many unsolved problems, the predictions of the model have and will have a qualitative character. Nevertheless, the character of disagreement with experimental data can give a hint about the direction of further development of the proposed theoretical scheme.

In conclusion, I would like to remark that this paper presents a preliminary analysis aiming to clarify the possibilities and difficulties of the theoretical description of high-energy nucleus-nucleus interactions.

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