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ON PROTON-PROTON ELASTIC SCATTERING AT HIGH ENERGIES

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INTRODUCTION

In the present paper the hadron-hadron elastic scattering is proposed to be described as "an effective particle in an effective quasipotential field" [1,2].

The measurements of the hadron-hadron total cross section performed at the accelerators at Serpukhov [3], CERN ISR [4] and FERMILAB [5] have shown that the total cross sections begin to grow at energies 50-70 GeV. The discovery of the Serpukhov effect and the measurement of the proton-proton elastic differential cross sections in FERMILAB [6] and CERN ISR [7] led to revision of the existing ideas of the hadron-hadron interactions. New rigorous consequences based on quantum field theory [8] have been obtained. There appeared a number of new models of hadrons and their interactions (see, e.g., refs. [9-10]).

The aim of the present paper is the construction of the elastic proton-proton scattering amplitude describing the existing experimental data [3-7, 11,12] at energies $\sqrt{s} \geq 10$ GeV, i.e., at FERMILAB and CERN ISR energies.

In section 1 the assumptions allowing the construction of the amplitude are given.

In section 2 the mathematical model is formulated and the inverse problem (i.e., the determination of the number, values and the statistical errors of the unknown parameters from the experimental data) is solved. Thus, an overdetermined system of nonlinear algebraic equations arises, which is solved by the method of the autoregularized iteration processes of the Gauss-Newton type [13] at the computer CDC-6500. The graphical information is obtained on the display by program "HPLOT" [14].

In section 3 the results and predictions, that may be checked at the existing accelerators are given.

In conclusion we would like to note, that the discovery of particles (states) with new quantum numbers gives information mainly about the hadron systematization. The measurement of the elastic differential cross section in the whole allowed kinematical region will shed light on the hadronic structure.

1. MAIN PHYSICAL ASSUMPTIONS

In the present paper we assume the following postulate: The two particle problem of the hadron interactions is reduced to the problem of "an effective particle in an effective quasipotential field, the effective wave function of the system being defined in the Lobachevsky space

$$q = (\sqrt{m^2(s, m_1, m_2) + \vec{q}^{\,2}}, \vec{q}),$$

where the effective mass may depend on the energy and masses.

The basic formulae of the nonrelativistic and relativistic dynamical Fourier analysis /15/, that relate the coordinate and the momentum representation are given below *

$$\Psi(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \int d^3q\, e^{i\vec{q}\vec{r}}\, \Psi(\vec{q}), \quad (1)$$

$$\Psi(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3q}{2\sqrt{m^2+\vec{q}^{\,2}}} \xi(m,\vec{q},\vec{r})\, \Psi(\vec{q}), \quad (2)$$

where

$$\xi(m,\vec{q},\vec{r}) = \left(\frac{\sqrt{m^2+\vec{q}^{\,2}} - \vec{q}\vec{n}}{m}\right)^{-1-imr},$$

and

$$n = \frac{\vec{r}}{|\vec{r}|}.$$

Obviously,

$$\lim_{|\vec{q}| \ll m} \xi(m,\vec{q},\vec{r}) = e^{i\vec{q}\vec{r}}. \quad (3)$$

An analog of the Heisenberg uncertainty relation between the (nonrelativistic) momentum and the coordinate for (2) is the uncertainty relation between the rapidity and the relativistic coordinate**

$$\Delta\chi \; \Delta r \geq \frac{\hbar}{2mc},$$

where

$$\chi = \ln\left(\sqrt{1 + \frac{\vec{q}^{\,2}}{m^2}} + \frac{|\vec{q}|}{m}\right) \quad (4)$$

*Here and further the units $\hbar = c = 1$ are used.
**To make the things clear the dimensional constants are written again.

and the relation

$$\lim_{\frac{|\vec{q}|}{m} \to 0} \chi = \frac{|\vec{q}|}{m} \quad (5)$$

holds.

Using the properties (3) and (5), it is easy to prove that if the amplitude is representated in the form

$$T(s,t) = -\frac{m}{4\pi} \int \overset{+}{\xi}(m,\vec{t},\vec{r})\, T(s,\vec{r})\, d\mu(\vec{r}) \quad (6)$$

and at large $-t$ has power behaviour, then at $\sqrt{-t} \ll m$ this behaviour will be changed into the exponential one, i.e., the exponential-power behaviour of the elastic differential cross section can be explained by the fact the local geometry of the Lobachevsky space is the Euclidean geometry.

Assuming the integral representation (6) to be valid, we start to model the amplitude $T(s,\vec{r})$ as a superposition of potential wells and barriers. As a result we obtain a description of the experiment and the dependence of the effective mass on the energy.

2. MATHEMATICAL MODEL AND SOLUTION OF THE INVERSE PROBLEM

Let us assume that the elastic amplitude in r-space has the form*

$$T(s,r,A) = \Sigma \frac{a_k(s)\, e^{-b_k(s)\, r}}{c_k^2(s) \pm r^2 + i d_k^2(s)}, \quad (7)$$

where a_k, b_k, c_k, d_k are unknown functions of the energies, and we denote by A the set of unknown parameters. It is possible the calculate ** $T(s,t,A)$ with the help of formula (6), which in the spherically-symmetrical case (7) is

$$T(s,t,A) = -\frac{1}{\operatorname{sh}\chi} \int_0^\infty r \sin(m\chi r)\, T(s,r,A)\, dr, \quad (8)$$

*Note that an analogous behaviour in r-space has been obtained in /16/ in the framework of quantum field theory.
**In those cases when the integrals (8) cannot be calculated analytically, we have used a program for rapid calculation of the sine-Fourier transform /17/, standard programs of the JINR-C349.

where

$$\chi = \ln\left(\sqrt{-\frac{t}{m^2}} + \sqrt{1-\frac{t}{m^2}}\right),$$

and $-t$ is the momentum transfer squared. To determine the number, values and statistical errors, the following overdetermined system of nonlinear equations was solved

$$\frac{d\sigma}{dt}^{expt}(s,t) = \frac{|T(s,t,A)|^2}{16\pi s(s-s_0)},$$

$$\sigma_{tot}^{expt}(s) = \frac{\mathrm{Im}\,T(s,0,A)}{\sqrt{s(s-s_0)}}, \qquad (9)$$

$$\rho^{expt}(s) = \frac{\mathrm{Re}\,T(s,0,A)}{\mathrm{Im}\,T(s,0,A)}.$$

where $s_0 = 4m_p^2$ by using the autoregularized iteration processes of the Gauss-Newton type [13]. The expression to be minimized by this method is

$$X^2 = \sum_M \left(\frac{y^{expt} - y^{th}(A)}{\Delta}\right)^2,$$

where y^{expt} and $y^{th}(A)$ are left- and right-hand sides of equations (9), and M is the number of all the equations. The solutions of equations (9) have been obtained for the following cases:

a) $\Delta = \Delta^{stat.}$ (X_s^2),

b) $\Delta = \Delta^{stat.}$ (X_{SN}^2),

c) $\Delta = \Delta^{stat.} + \Delta^{syst.}$ (X^2).

In the case b) we found the normalization parameters of the differential cross sections, i.e., the number of the parameters increased by the number of different energies.

The number of the experimental points, squared momentum transfer X_s^2, X_{SN}^2, X^2 and the energies, corresponding to the solution for A of (9), are given in table 1.

The mass m(s) and the amplitude T(s,r,A) for this obtained solution are

$$m(s) = a_1 R(s),$$

$$R(s) = a_2 + a_3/(s/\mathrm{sol})^{a_4} + a_5 \ln(s/\mathrm{sol}),$$

where* $\mathrm{sol} = (1.81 \pm 0.6)\ \mathrm{GeV}^2$

$$T(s,r,A) = \sqrt{s(s-s_0)}\, R(s) \left\{ \frac{U(s)}{(a_6^2+r^2)^2} + \frac{V(s)}{a_7^4+r^4} + \frac{W(s)}{(a_8^2-r^2)^2+a_9^4} + \frac{Z(s)}{(a_{10}^2-r^2)+a_{11}^4} \right\}. \qquad (10)$$

Table 1

		s GeV²	|−t_min| GeV²	|−t_max| GeV²	Number of points	X_s^2	X_{SN}^2	X^2
FERMILAB DATA	1	95.607	0.0375	0.75	18	165.997	157.456	24.033
	2	133.135	0.0375	0.70	19	233.193	189.912	20.208
	3	189.429	0.0375	0.70	20	70.688	58.659	5.598
	4	264.491	0.0375	0.80	21	180.418	169.085	19.795
	5	330.171	0.0375	0.70	17	575.472	282.552	19.146
CERN ISR DATA	1	547.56	0.825	5.75	57	140.359	134.560	57.184
	2	930.25	0.875	5.75	56	179.301	144.760	103.923
	3	1989.160	0.875	7.25	59	118.737	117.316	50.961
	4	2737.840	0.825	9.75	65	114.552	91.705	45.218
	5	3856.410	0.825	6.25	58	96.387	60.492	54.864

$$X_s^2 = \frac{1875.}{390-23} = 5.11, \qquad X_{SN}^2 = \frac{1047.}{390-33} = 3.94,$$

$$X^2 = \frac{401.}{390-23} = 1.09$$

* The value of the scale sol has been determined in [18].

and

$$U(s) = a_{12} R(s)^{a_{13}} + i a_{14},$$

$$V(s) = a_{15} R(s)^{a_{16}} + i a_{17},$$

$$W(s) = a_{18} R(s)^{a_{19}} + i a_{20},$$

$$Z(s) = a_{21} R(s)^{a_{22}} + i a_{23}.$$

The corresponding amplitude (8) in momentum space has the form

$$\begin{aligned} T(s,t,A) = \frac{\sqrt{s(s-s_0)}}{\mathrm{sh}\,\chi} R(s) \{ & U(s)\, m(s)\, \chi \exp(-RU\chi) + \\ & + V(s) \sin(RV\chi) \exp(-RV\chi) + \\ & + W(s) \sin(RW_+\chi) \exp(-RW_-\chi) + \\ & + Z(s) \sin(RZ_+\chi) \exp(-RZ_-\chi) \}, \end{aligned} \tag{11}$$

where

$$RU = a_6\, m(s), \qquad RV = a_7\, m(s),$$

$$RW_{\pm} = \sqrt{\frac{\sqrt{a_8^4 + a_9^4} \pm a_8^2}{2}}\, m(s), \qquad RZ_{\pm} = \sqrt{\frac{\sqrt{a_{10}^4 + a_{11}^4} \pm a_{10}^2}{2}}\, m(s),$$

$$\chi = \ln\left(\sqrt{-\frac{t}{m^2(s)}} + \sqrt{1 - \frac{t}{m^2(s)}} \right).$$

The solutions for the parameter A and their statistical errors are given in table 2.

3. RESULTS AND PREDICTIONS

The obtained predictions of $\frac{d\sigma}{dt}^{pp\to pp}(s,t)$, $\sigma_{tot}^{pp}(s)$, $\rho(s) = \frac{\mathrm{Re}\,T(s,0)}{\mathrm{Im}\,T(s,0)}$ are given in figs. 1-3. The dependence of the amplitude on

Table 2

N	A	ΔA	N	A	ΔA
1	0.18531	0.00008	13	3.465	0.002
2	0.913	0.009	14	-912.50	0.02
3	2.858	0.006	15	-8730.92	0.002
4	0.324	0.003	16	-3.290	0.002
5	0.1994	0.0008	17	82.170	0.007
6	12.4489	0.0066	18	445.54	0.02
7	7.8602	0.0003	19	0.4073	0.0009
8	8.2110	0.0005	20	-425 .440	0.006
9	15.5527	0.0004	21	-224. 500	0.02
10	7.1927	0.0005	22	1. 402	0.002
11	15.4652	0.0003	23	465. 114	0.006
12	94.3	0.1			

the energy is determined by the function R(s), which is related to the total cross section by

$$\sigma_{tot} = 2\pi R^2(s),$$

i.e., the function R(s) may be interpreted as the effective radius of the hadronic interaction. The effective mass is also proportional to R(s). As the amplitude depends on $-t$ through the rapidity

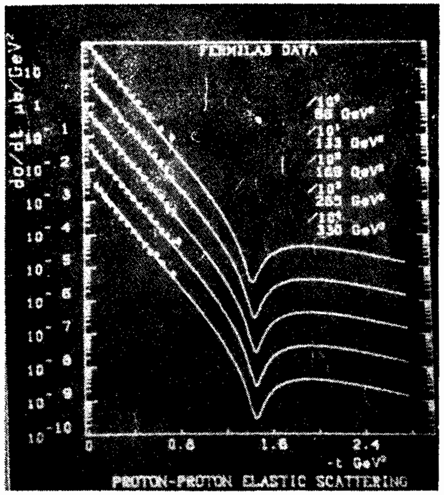

Fig. 1

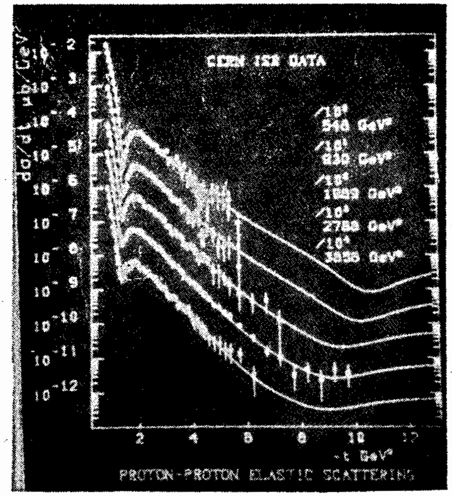

Fig. 2

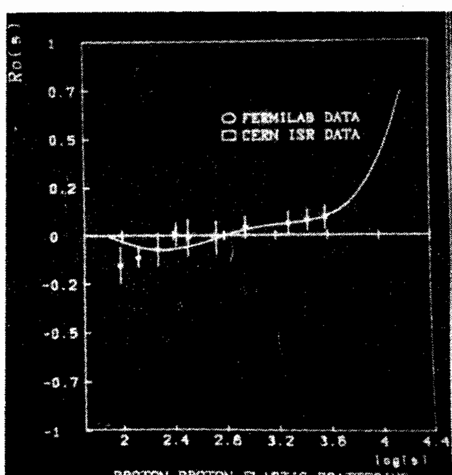

Fig. 3

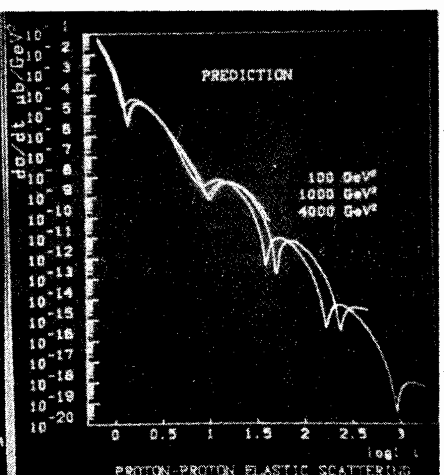

Fig. 4

$$\chi = \ln\left(\sqrt{-\frac{t}{m^2(s)}} + \sqrt{1-\frac{t}{m^2(s)}}\right),$$

it satisfies the property of geometric scaling [19].

If the obtained differential cross section is represented in the form

$$\frac{d\sigma}{dt} = \frac{1}{s^N} f\left(-\frac{t}{s}\right)$$

then for N, when $s \to \infty$ and $-\frac{t}{s}$ is fixed, we find

$$N = 1 + RV(s).$$

Thus, on the basis of the constructed model and the considered experimental data, we conclude that:

1) either the quark counting results [20] are valid at $\sqrt{s} \approx 10^5 - 10^6$ GeV; 2) or they hold for $\sqrt{s} \geq 10^5$ GeV, but the total cross section does not depend on s.

For the function $\rho(s)$ as $s \to \infty$ we obtain

$$\lim_{s\to\infty} \rho(s) = \ln(s/so1)^{35}$$

and for the function $\sigma_{tot}(s)$, we get

$$\lim_{s\to\infty} \sigma_{tot}(s) = c \ln^2(s/so1),$$

i.e., this description agrees with the model of the maximal increase in the total cross section [9]. On fig. 4 the predictions of the diffraction picture of the elastic scattering are given.

In the next paper we shall investigate the agreement with the behaviour of the amplitude in r -space with the QCD.

We note that using the results of paper [18] (where the effective radius is determined as a function of the energy and quantum numbers of the interacting hadrons), we can predict the behaviour of the elastic proton-meson differential cross section.

We hope that in the near future the data for proton-meson elastic scattering at energies $\sqrt{s} > 10$ GeV and squared transfer momentum $-t \approx 50 \div 60$ GeV^2 will also be obtained; this will allow us to find the dependence of the effective mass of the masses of interacting particles and check the predicted diffraction behaviour of the elastic hadron-hadron scattering.

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