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NONLOCAL OPERATORS IN LIGHT-CONE EXPANSION OF QCD

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1. For scalar field theory it has been shown[1] that the usual light-cone expansion

$$RT(j(x)\ j(0)S) = \approx \sum_{n_1,n_2} F_{n_1 n_2}(x^2,\mu^2)\,\bar{R}\,(TO_{n_1 n_2}S),$$

$$O_{n_1 n_2} = \int dq_1 dq_2 (\tilde{x}q_1)^{n_1} (\tilde{x}q_2)^{n_2} (n_1!\, n_2!)^{-1} \ :\phi(q_1)\,\phi(q_2): \qquad (1)$$

$$F_{n_1 n_2}(x^2,\mu^2) = \frac{1}{2!}\left(\frac{\partial}{\partial xq_1}\right)^{n_1}\left(\frac{\partial}{\partial xq_2}\right)^{n_2} F_2^{x-prop}(x^2, xq_i, q_i q_j)\Big|_{xq_i=0,\ q_i q_j=\mu_{ij}}$$

(F_ℓ are the coefficient functions of the renormalized product of two local operators $j(x)$: $RT(j(x)\ j(0)S) = 1 + \sum \frac{1}{\ell!} \int dq_1 \ldots dq_\ell \times F_\ell(x^2, q_i, q_i q_j) : \phi(q_1) \ldots \phi(q_\ell):$; R, $\bar{R}$ denote R - operations; S , S - matrix; $\tilde{x}$ light-like vector corresponding to x [1,2]; μ^2, μ_{ij} are subtraction points) exists on a dense set of the Fock space only. This means it is not a true operator identity as it should be. There may exist states for which this identity is not valid. On the other hand, the so-called nonlocal light-cone expansion - proved by S.A.Anikin and O.I.Zavialov[2,3]-exists as an operator identity and is therefore a more fundamental quantity from a theoretical point of view. The simplest form of such an expansion reads

$$RT(j(x)\,j(0)S) = \int_0^1 d\kappa_1 \int_0^1 d\kappa_2\, F(x^2,\kappa_1,\kappa_2)\; RT\,(O(\kappa_1,\kappa_2)S),$$

$$O(\kappa_1,\kappa_2) = \int dq_1 dq_2\, e^{i\kappa_1 \tilde{x}q_1 + i\kappa_2 \tilde{x}q_2} : \phi(q_1)\,\phi(q_2): \qquad (2)$$

$$F(x^2,\kappa_1,\kappa_2) = \frac{1}{8\pi^2}\int d\tilde{x}q_1\, d\tilde{x}q_2\, e^{-i(\kappa_1 \tilde{x}q_1 + \kappa_2 \tilde{x}q_2)} F_2(x^2, \tilde{x}q_i, q_i q_j)\Big|_{q_i q_j=\mu_{ij}} .$$

Here as new interesting objects the nonlocal (light-ray) operators $:\tilde{\phi}(\kappa_1 \tilde{x})\, \tilde{\phi}(\kappa_2 \tilde{x}):$ are introduced. It is important that these operators are very similar to local operators. They need special subtractions and have nontrivial anomalous dimensions which can be related to the anomalous dimensions of the operators $:(\tilde{x}\partial)^{n_1}\tilde{\phi}(\tilde{x}\partial)^{n_2}\tilde{\phi}:$ [8]. This is in contrast to the operators $\tilde{\phi}(x_1)\,\tilde{\phi}(x_2)$ which are well defined operator distributions (without additional subtractions) as long as $(x_1 - x_2)^2 \neq 0$.

2. In gauge field theories there exists up to now no proof of the light-cone expansion of operator products. Nevertheless, N.I.Kartchev has derived a local light-cone expansion and with the help of Slavnov-Taylor identities he was able to sum up this local expansion to get a nonlocal light-cone expansion/4/. The same result can be obtained easier if we use the following representation of gauge-invariant operator products (for axial gauge conditions)

$$RT(j(x)\;j(0)S) \overset{x-\text{prop.}}{=} \int dy_1\, dy_2\, \Sigma_{rs}(x,y_1,y_2) : \bar{\psi}_r(y_1)\, P\exp\Big(-ig\int_{y_2}^{y_1} A_\mu dx^\mu\Big)\psi_s(y_2): + \tag{3}$$

$$+ \int dz_1 dz_2 \Pi_{\mu\nu}(x,z_1,z_2): F_{\mu\rho}(z_1) P\exp\Big(g\int_{y_2}^{y_1} A^a_\mu f^a dx^\mu\Big) F_{\nu\rho}(z_2): + \text{higher terms}$$

($\Sigma_{rs}, \Pi_{\mu\nu}$ are arbitrary coefficient functions $f^a = (f^{bac})$, structure constants of SU(3) ; t^a, matrices of the fundamental representation; ψ_r, $A_\mu = A^a_\mu t^a$, $F_{\mu\nu}$ are fields of QCD). For simplicity we choose a straight-line integration path. As subtraction operator we use the light-cone subtraction operator $\mathfrak{M}^a$ ($a \geq 0$, integer)/2,3/:

$$\mathfrak{M}^a \sum_{\ell=2}^{\infty} \int dy_1 \ldots dy_\ell\, F_\ell(x,y_1,\ldots,y_\ell) : \phi(y_1)\ldots\phi(y_\ell): =$$

$$= \sum_{\ell=2}^{\infty} M_\sigma^{a-\ell+s}\, \sigma^{-4\ell+s} \int dy_1 \ldots dy_\ell\, F_\ell\Big(\frac{x_\sigma}{\sigma}, \frac{y_i}{\sigma}\Big) : \phi(y_1)\ldots\phi(y_\ell): \quad s \geq 0 \text{ integer}$$

$$M_\sigma^d f(\sigma) = \sum_{k=0}^{d} \frac{1}{k!}\Big(\frac{\partial}{\partial\sigma}\Big)^k f(\sigma)\Big|_{\sigma=0}, \quad x_\sigma = \begin{cases} \tilde{x} & \text{for } \sigma = 0 \\ x & \text{for } \sigma = 1 \end{cases}, \quad \text{see ref./2,3/.}$$

By an application of $\mathfrak{M}^a$ to objects of the type (3) this operator acts in fact on the input-coefficient functions $\Sigma_{rs}, \Pi_{\mu\nu}, \ldots$ only

$$\mathfrak{M}^a \int dy_1\, dy_2\, \Sigma_{rs}(x,y_1,y_2) : \bar{\psi}_r(y_1)\, P\exp\Big(ig\int_{y_2}^{y_1} A_\mu dx^\mu\Big)\psi_s(y_2): =$$

$$= M_\sigma^{a-2d+s} \int dy_1 dy_2\, \sigma^{s-8}\, \Sigma_{rs}\Big(\frac{x_\sigma}{\sigma}, \frac{y_i}{\sigma}\Big) : \bar{\psi}(y_1)\, P\exp\Big(-ig\int_{y_2}^{y_1} A_\mu dx^\mu\Big)\psi_s(y_2):$$

($d_\psi = \frac{3}{2}$ is the canonical dimension of the spinor field). This can be seen easily by expanding the exponentials and by an explicit treatment of each operator expression. To avoid infrared difficulties we use this subtraction operator directly in momentum space at shifted subtraction points

$$\int dy_1\, dy_2\, e^{-i(q_1 y_1 + q_2 y_2)}\, M_\sigma^{a-2d_\psi+s}\, \sigma^{s-8}\, \Sigma_{rs}\Big(\frac{x_\sigma}{\sigma}, \frac{y_i}{\sigma}\Big) =$$

$$M_\sigma^{a-2d_\psi+s}\, \sigma^s\, \tilde{\Sigma}_{rs}\Big(\frac{x_\sigma}{\sigma}, \sigma q_i\Big) \rightarrow M_\sigma^{a-2d_\psi+s}\, \sigma^s\, \tilde{\Sigma}_{rs}\Big(\frac{x_\sigma}{\sigma}, \sigma q_i\Big)\Big|_{\sigma^2 q_i q_j \rightarrow \sigma^2 (q_i q_j - \mu_{ij}) + \mu_{ij}}$$

This can be done without complications because the Slavnov-Taylor identities have been taken into account from the very beginning and the coefficient functions $\Sigma_{rs}, \Pi_{\mu\nu}$ are really not restricted by such conditions. Using the well-known technique/2/ the following nonlocal light-cone expansion can be obtained ($\alpha = 4$)

$$RT\,(j(x)\; j(0)\; S) \approx \sum_{i=1,2} \int d\kappa_1 d\kappa_2\, F_i(x^2, \kappa_j, \mu^2,\; RT(O_i(\kappa_j) S) \tag{4}$$

$$O_1(\kappa_i) = : \bar{\psi}_r(\kappa_1 \tilde{x})\, P\exp\Big(-ig\int_{\kappa_2}^{\kappa_1} A_\mu \tilde{x}^\mu d\tau\Big)\psi_s(\kappa_2\tilde{x}) : \tag{5}$$

$$O_2(\kappa_i) = : F_{\mu\rho}(\kappa_1\tilde{x})\, P\exp\Big(g\int_{\kappa_2}^{\kappa_1} A^a_\mu f^a \tilde{x}^\mu d\tau\Big) F_{\nu\rho}(\kappa_2\tilde{x}) : \tag{6}$$

whereby the light-cone coefficients are determined by the coefficient functions of the functional (3)

$$F_1 = \frac{1}{4\pi^2}\int d\tilde{x}q_1 d\tilde{x}q_2\, e^{-i(\kappa_1 \tilde{x}q_1 + \kappa_2 \tilde{x}q_2)} M_\sigma^{4-3}\tilde{\Sigma}_{rs}\Big(\frac{x_\sigma}{\sigma}, \sigma q_i\Big)\Big|_{\sigma^2 q_i q_j = \mu_{ij}}$$

$$F_2 = \frac{1}{4\pi^2}\int d\tilde{x}q_1 d\tilde{x}q_2\, e^{-i(\tilde{x}q_1\kappa_1 + \tilde{x}q_2\kappa_2)} M_\sigma^{4-4}\, \Pi_{\mu\nu}\Big(\frac{x_\sigma}{\sigma}, \sigma q_i\Big)\Big|_{\sigma^2 q_i q_j = \mu_{ij}}.$$

The corresponding local light-cone expansion takes the expected form. It contains the operators: $(\tilde{x}D^*)^{n_1}\bar{\psi}(\tilde{x}D)^{n_2}\psi:,\ldots$.

3. As fundamental operators there apper generalized nonlocal operators of the type (5,6). They have to be understood in the sense

$$O_1(\kappa_1, \kappa_2) = \sum_{n=0}^{\infty}\int_{\kappa_2}^{\kappa_1} d\tau_1 \int_{\kappa_2}^{\tau_1} d\tau_2 \ldots \int_{\kappa_2}^{\tau_{n-1}} d\tau_n\, t^{a_1}_{\alpha\alpha_1} t^{a_2}_{\alpha_1\alpha_2} \ldots t^{a_n}_{\alpha_{n-1}\alpha_n}\,.$$

$$\times : \bar{\psi}_\alpha(\kappa_1\tilde{x})(-ig A^{a_1}_\mu \tilde{x}^{\mu_1})(\tau_1\tilde{x}) \ldots (-ig A^a_\mu \tilde{x}^\mu)(\tau_n\tilde{x})\, \psi_{a_n}(\kappa_2\tilde{x}) :$$

This means the P-ordering acts on the variables and the group-theoretical factors only. The quantum field operators are orderd by the Wick product. Of course vacuum expectation values of such operators are zero by definition (This is very different for Wilson-type operators which have not the character of external operators). These operators can be directly related to the usual local operators

$$:(\tilde{x}D^*)^{n_1}\bar{\psi}(\tilde{x}D)^{n_2}\psi: = \Big(\frac{d}{d\kappa_1}\Big)^{n_1}\Big(\frac{d}{d\kappa_2}\Big)^{n_2} : \bar{\psi}(\kappa_1\tilde{x})\, P\exp\Big(-ig\int_{\kappa_2}^{\kappa_1} A_\mu \tilde{x}^\mu d\tau\Big)\bar{\psi}(\kappa_2\tilde{x}):\Big|_{\kappa_i = 0}$$

$$D \,\hat{=}\, D_\mu = \frac{\partial}{\partial x^\mu} + igA_\mu\,.$$

They are direct generalizations of the nonlocal operators of the scalar case, they have nontrivial renormalization properties and have additional anomalous dimensions.

4. Light-cone expansions are primarily applied to forward scattering. Here all expressions are simplified considerably[8]

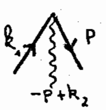
$$<p\,|\,RT\,(j(x)\,j(0)\,S)\,|\,p> \approx \int d\kappa_- \sum_i C_i\,(x^2,\kappa_-)\,<p\,|\,\tilde{R}T(O_i\,(\kappa_1,\kappa_2)S)\,|\,p>, \tag{7}$$

$$C_i\,(x^2,\kappa_-) = 2\int d\kappa_+ F\,(x^2,\kappa_1,\kappa_2), \quad \kappa_\pm = \kappa_1 \pm \kappa_2 .$$

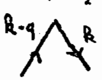
Because of translation-invariance the matrix elements $<p\,|\,\tilde{R}TO_i\,(\kappa_1,\kappa_2)\,S\,|\,p>$ do not depend on κ_+ so that this integration can be performed separately. Correspondingly in local light-cone expansion it is sufficient to take into account operators with $n_1=0$, $n_2\neq 0$. If we consider the light-cone expansion of the product of two electromagnetic currents, then the expansions given above are kinematically more involved and as operators there appear for example: $\bar{\psi}\,\lambda^a\,\gamma_\mu\,\tilde{x}^\mu\,(\tilde{x}D)^{n-1}\psi$: or : $\bar{\psi}\,(\kappa_1\,\tilde{x})\lambda^a\,\gamma_\mu\,\tilde{x}^\mu\,\mathrm{Pexp}(-ig\int A_\mu \tilde{x}^\mu d\tau)\,\psi\,(\kappa_2\,\tilde{x})$: , λ^a flavour matrix. The anomalous dimensions of the local operators are well known for forward scattering[6], flavour octet fermion operator:

$$\gamma^F_n = \frac{g^2}{8\pi^2} C_N\,\left(1 - \frac{2}{n\,(n+1)} + 4\sum_{j=2}^{n}\frac{1}{j}\right), \quad \delta_{rs}\,C_N = \sum_a t^a t^a . \tag{8}$$

We want to calculate the anomalous dimensions of the nonlocal operators $O_i(\kappa_i)$ using standard methods of QFT. For simplicity we substitute the correct action of the operators $\mathfrak{M}^a$ and $\tilde{R}$ in the course of the renormalization procedure by the treatment of the divergent quantities only. This means we apply dimensional regularization and look for the pole terms only. Furthermore our calculation is performed in the Feynman gauge whereas the important representation (3) is proved for the axial gauge. For the leading operators this brings no complications. If nonleading operators are taken into account one must be more carefully. For the explicit calculation of the Feynman graphs, we have to take into account the operator vertices (Fermion octet operator)



$$-igt^d\,\lambda^a\,\gamma_\mu\,\tilde{x}^\mu\,\frac{e^{-i(p-k_2)\tilde{x}\kappa_2} - e^{-i(p-k_2)\tilde{x}\kappa_1}}{(k_2-p)\tilde{x}}\,e^{-ik_1\tilde{x}\kappa_1 + ip\kappa_2\tilde{x}}$$



$$\tilde{x}^\mu\gamma_\mu\,\lambda^a\,e^{-ik(\kappa_1-\kappa_2)\tilde{x} + iq\kappa_1\tilde{x}}$$

additionally to the usual Feynman rules. Then we get expressions of the type $\int dq\, e^{-i\kappa_-\tilde{x}q}\,[q^2+M^2]^{-2}\,q_\mu\,\tilde{x}^\mu, \ldots$. Because of the light-like character of $\tilde{x}$ such integrals can be calculated using dimensional regularization. In this way we get the following result for the anomalous dimension of the flavour octet Fermion operator (for forward scattering)

$$\tilde{\gamma}^F(z) = -\frac{C_N g}{4\pi^2}\left[\,\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\,\delta(z-1)\right], \quad \gamma^F(\kappa_-,\kappa_-') = \frac{1}{\kappa_-}\tilde{\gamma}^F\left(\frac{\kappa_-'}{\kappa_-}\right). \tag{9}$$

The anomalous dimensions of nonlocal operators are defined quite similar to the corresponding definitions in the local case $Z_2^{-1}\int d\kappa_-'\,Z(\kappa_-,\kappa_-')\,(\bar{\psi}(p)\,O(\kappa_i)\,\psi(p))^{ren,1PJ}(\kappa_-') = (\bar{\psi}(p)O(\kappa_i)\psi(p))^{un\,1PJ}(\kappa_-)$, $\int d\kappa'\,Z(\kappa,\kappa')\,\gamma(\kappa',\tau) = \mu\frac{\partial}{\partial\mu}Z(\kappa,\tau)$. Eq. (9) coincides with the expression given by G.A.Altarelli and G.Parisi[7]. Their results are obtained by physical intuition about transition processes mainly. Here it comes out that these anomalous dimensions belong to nonlocal operators. In fact all results of ref.[7] can be understood clearly on the basis of nonlocal light-cone expansions. Explicitly this has been shown for scalar field theories[8].

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