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R.P.Zaikov

HIDDEN SYMMETRIES FOR THE ABELIAN GAUGE THEORIES

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1. INTRODUCTION

Some two-dimensional field theoretical models possess hidden symmetries which give an infinite set of local conserved quantities. Such models are the sine-Gordon model[1], nonlinear sigma models[2] and all other complete-integrable models. For the nonlinear sigma models, it has been shown[3] that there exist also nonlocal conserved quantities generated by nonlinear and nonlocal field transformations[4]. Such nonlocal currents were found in papers[5] also for the supersymmetric sigma-models. In the four-dimensional case such symmetries are found for Yang-Mills fields on contours in paper[6] as well as for a self-dual sector in paper[7].

In the present article the problem of existence of hidden symmetries in the case of field theories invariant with respect to global abelian gauge transformations is investigated. For this purpose a class of "local" gauge transformations with respect to which the action is invariant without including additional compensating fields are considered. In general, invariance of the corresponding equation of motion with respect to these transformations is not required. The problem of finding the explicit form of the transformations under consideration is reduced to the solution of one partial differential equation. In the two-dimensional case the general solution of this equation is found. However, the transformations found in such a manner are symmetry of the action only on the extremals (solutions of the equation of motion). As in the case of chiral models, these transformations are nonlinear and nonlocal. According to the Noether theorem to any one parametric transformation of this class there corresponds one nonlocal conserved current.

In the case of three and four-dimensional space-time the solution of the invariance equation is found only on two-dimensional manifolds.

Our considerations are applicable also to field theoretical models which are invariant with respect to abelian local gauge transformations.

II. THE CONDITION FOR EXISTENCE OF HIDDEN SYMMETRY

Suppose that a set is given of classical fields $\Psi_k(x)$ ($k = 1, \ldots, N$) in a D-dimensional space-time. For these fields the Lagrangian function is given by

$$\mathcal{L}(x) = \mathcal{L}(\Psi_k, \partial_\mu \Psi_k). \tag{2.1}$$

The invariance of (2.1) with respect to the global gauge transformations

$$\begin{aligned} &\Psi'_m(x) = e^{i\alpha}\Psi_m(x), \quad \Psi^{*\prime}_m(x) = e^{-i\alpha}\Psi^*_m(x), \quad (m=1,\ldots,M) \\ &\Psi'_{M+k}(x) = \Psi_{M+k}(x), \quad (k=1,\ldots,N-M) \end{aligned} \tag{2.2}$$

is required. Here the fields Ψ_{M+k} ($k=1, \ldots, N-M$) are gauge invariant, i.e., they are real. As is known, according to the Noether theorem to the transformation (2.2) there corresponds the following current

$$j_\mu(x) = i\left(\frac{\partial \mathcal{L}}{\partial(\partial^\mu \Psi^*_m)}\Psi^*_m - \frac{\partial \mathcal{L}}{\partial \partial^\mu \Psi_m}\Psi_m\right) \tag{2.3}$$

that is conserved $(\partial^\mu j_\mu = 0)$ when the equations of motion

$$\partial^\mu \frac{\partial \mathcal{L}}{\partial \partial^\mu \Psi} - \frac{\partial \mathcal{L}}{\partial \Psi} = 0 \tag{2.4}$$

are satisfied.

Consider "local" gauge transformations

$$\begin{aligned} &\Psi'_m(x) = e^{i\eta(x)}\Psi_m(x), \quad \Psi^{*\prime}_m(x) = e^{-i\eta(x)}\Psi^*_m(x), \\ &\Psi'_{M+k}(x) = \Psi_{M+k}(x), \quad (m=1,\ldots,M), \quad (k=1,\ldots,N-M), \end{aligned} \tag{2.5}$$

where the function $\eta(x)$ is determined by the invariance condition of the Lagrangian (2.2) with respect to the transformations (2.5) without including compensating fields. The invariance of the equations of motion (2.4) is not required.

The invariance condition for the Lagrangian $\mathcal{L}$ with respect to the transformations (2.5) has the following form

$$\Delta\mathcal{L}(x) = j^\mu(x)\partial_\mu \eta(x) = 0, \tag{2.6}$$

where the current $j_\mu(x)$ is given by (2.3). Consequently, the problem of existence of hidden symmetry of type (2.5) is reduced to finding of nontrivial ($\eta \neq$ const) solutions of eq. (2.6). If such a nontrivial solution exists, then according to the Noether theorem it follows that the quantity

$$J^{(\kappa)}_\mu(x) = j_\mu(x)\eta^{(\kappa)}(x) \tag{2.7}$$

is conserved, if the equations of motion (2.4) are satisfied, i.e., $\partial^\mu j_\mu = 0$. In the formula (2.7) to any linear-independent solution of (2.6) there is introduced the corresponding infinitesimal constant parameter ω^κ, i.e.,

$$\eta(x) = \sum_\kappa \eta^{(\kappa)}(x)\omega_\kappa. \tag{2.8}$$

Suppose that $j_0(x)$ decreases on space-infinity so that the charge

$$Q = \int dx^{D-1} j_0(x) \tag{2.9}$$

is conserved, i.e., $\frac{dQ}{dt} = 0$. Then the charges

$$Q^{(\kappa)} = \int dx^{D-1} j_0(x)\eta^{(\kappa)}(x) \tag{2.10}$$

are also conserved if the following boundary conditions

$$|\eta^{(\kappa)}(x)|_{|x|=\infty} \leq M < \infty \tag{2.11}$$

hold.

From (2.5) and (2.8) it follows, that

$$\eta^{(\kappa)}(x) = \frac{\partial T(\omega)}{\partial \omega^\kappa}\Big|_{\omega=0}, \tag{2.12}$$

consequently, $\eta^{(\kappa)}(x)$ are generators of the transformations (2.5).

3. EXPLICIT FORM OF GENERATORS FOR TWO-DIMENSIONAL MODELS

In two-dimensional case eq. (2.6) in general can always be solved. Note that for constructing, in an explicit form, the conserved currents (2.7) it is sufficient to find the solutions of (2.6) only in the case $\partial^\mu j_\mu = 0$, i.e., on the extremals. In the last case the corresponding to (2.6) characteristic differential equation is reduced to an equation for the total differential. The first integral of this equation can be written in the following form

$$\Phi(x) = \int_{-\infty}^{x_1} dy_1\, j_0(x_0, y_1). \tag{3.1}$$

As is known, the general solution of (2.6) is given by

$$\eta(x) = F(\Phi), \tag{3.2}$$

where F is an arbitrary function satisfying the boundary condition (2.11) at space infinity.

One infinite sequence of functions $\eta^{(k)}(x)$ $(k=1, 2, \dots)$ can be selected from (3.2) if

$$\eta^{(k)}(x) = \int_{-\infty}^{x_1} dy_1 j_0(x_0, y_1)\eta^{(k-1)}, \quad (k=2,3,\dots), \tag{3.3}$$

where we start from $\eta^{(1)} = \text{const}$. Corresponding conserved currents are given by

$$J_\mu^{(k)}(x) = j_\mu(x)\eta^{(k)}(x), \quad (k=1,2,\dots). \tag{3.4}$$

Here $J_\mu^{(1)} \sim j_\mu(x)$. This set coincides in form with the known infinite set of nonlocal conserved currents for the chiral models /3/. As in the last case the currents (2.4) are nonlocal and are generated by the nonlocal and nonlinear transformations (3.3).

From (2.9) and (3.1) it follows that

$$\eta^{(k)}(x_0, \infty) = Q^{(k)}(x_0), \quad (k=1,\dots), \tag{3.5}$$

which are conserved charges corresponding to the currents (3.4). Consequently, the boundary condition (2.11) is satisfied if the first conserved charge exists.

It can be checked that the generating function $\eta^{(\kappa)}(x)$ commutes with respect to the Poisson bracket and consequently $\eta^{(\kappa)}(x)$ are generators of the infinite-parametric Abelian group and the corresponding charges are in involution.

4. GENERATOR FUNCTIONS $\eta^{(\kappa)}(x)$ IN THE D-DIMENSIONAL CASE

When $D>2$ eq. (2.6) have no nontrivial solution in all the space-time. As an example, consider the three-dimensional ($D=3$) case. The corresponding characteristic system of eqs. (2.6) can be written in the following form

$$\begin{aligned} j_0 dx_1 + j_1 dx_0 &= 0, \\ j_0 dx_2 + j_2 dx_0 &= 0. \end{aligned} \tag{4.1}$$

One first integral of the system (4.1) can be found if

$$\text{a)}\quad j_\mu = j_\mu(x_0, x_1 + \alpha x_2), \quad (\mu=0,1,2), \tag{4.2}$$

$$\text{b)}\quad j_1 + \alpha j_2 = \frac{j_0}{g} \int_{-\infty}^{x_1} dy_1\, \partial_0 g(x_0, y_1 + \alpha x_2), \tag{4.3}$$

where α is an arbitrary parameter and g an arbitrary function of $x_1 + \alpha x_2$. Then the corresponding first integral of (4.1) is given by

$$\Phi_a = \int_{-\infty}^{x_1} dy_1 j_0(x_0, y_1 + \alpha x_2), \tag{4.4}$$

$$\Phi_b = \int_{-\infty}^{x_1} dy_1 g(x_0, y_1 + \alpha x_2). \tag{4.5}$$

As in the two-dimensional case, any function of Φ_a or Φ_b, when there are satisfied conditions (4.2) or (4.3), obeys eq. (2.6).

In the four-dimensional case the conditions (4.2) and (4.3) have the form

$$\text{a}')\quad j_\mu(x) = j_\mu(x_0, x_1 + \alpha x_2 + \beta x_3) \quad (\mu=0,1,2,3) \tag{4.6}$$

and

$$j_1 + \alpha j_2 + \beta j_3 = \frac{j_0}{g} \int_{-\infty}^{x_1} dy_1 \partial_0 g(x_0, y_1 + \alpha x_2 + \beta x_3). \tag{4.7}$$

The first integrals are given by

$$\Phi_{a'} = \int_{-\infty}^{x_1} dy_1 j_0(x_0, y_1 + \alpha x_2 + \beta x_3), \tag{4.8}$$

$$\Phi_{b'} = \int_{-\infty}^{x_1} dy_1 g(x_0, y_1 + \alpha x_2 + \beta x_3), \tag{4.9}$$

where α, β are arbitrary parameters and g is an arbitrary function.

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