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THE AMPLITUDE FOR PROCESS $\gamma\gamma \to \pi\pi$ IN CHIRAL QUANTUM FIELD THEORY

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THE AMPLITUDE FOR PROCESS $\gamma\gamma \to \pi\pi$ IN CHIRAL QUANTUM FIELD THEORY

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Волков М.К., Первушин В.Н. **E2 - 8098**

Вычисление амплитуды $\gamma\gamma \to \pi\pi$ в квантовой киральной теории

В квантовой киральной теории вычислена амплитуда $\gamma\gamma \to \pi\pi$. Найдено значение поляризуемости пионов и указывается на возможное аномальное поведение амплитуды вблизи порога рождения двух пионов.

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Pervushin V.N., Volkov M.K. **E2 - 8098**

The Amplitude for Process $\gamma\gamma \to \pi\pi$ in Chiral Quantum Field Theory

The amplitude of process $\gamma\gamma \to \pi\pi$ is calculated in the chiral type quantum field theory for energies $|\sqrt{s}| < 1.2$ GeV. The values for pion polarizabilities are found ($\alpha_{\pi^\pm} = 0.16\,\alpha/m_\pi^3$, $\alpha_{\pi^0} = -0.04\,\alpha/m_\pi^3$).

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Dubna. 1974

1. Introduction

This paper represents another example of the use of chiral type quantum theory for describing the low-energy scattering of elementary particles. The results on such an approach of previous papers devoted to description of the low-energy $\pi\pi$ scattering and calculation of the pion form factor are in a good agreement with the existing experimental data [1-4].

Before to proceed to the subject of this paper we would remind the main advantages of the chiral type quantum theory as compared to the conventional approach of field theory of strong interactions. The first and major advantage of the chiral theory is due to fulfilling of the low-energy theorems and correct description of the scattering amplitude behaviour in the low-energy limit. The next rather important feature of this theory consists in that both the pion-pion interactions and the pion-nucleon interactions are expressed through one and the same pion decay constant F_π = 92 MeV. This latter feature appears to be very useful in constructing perturbation theory as it makes it easy to select the total set of diagrams containing both the pion vertices and the pion-nucleon ones to a given order of perturbation expansion in $1/F_\pi$. The third important feature of the chiral theory is that in describing the low-energy processes in this theory there arises a specific energy scale equal to $4\pi F_\pi \sim$ 1.2 GeV. It is just the scale which allows one to construct the perturbation theory even when strong interactions are involved. It is only necessary that energies under consideration are essentially lower than the above indicated limit. For the $\pi\pi$ scattering this scale has first been discovered by Brown and Goble [5] and also by Lehmann [1]. As will be seen from this paper, in the process $\gamma\gamma \to \pi\pi$ the same energy scale again arises [x]. Finally, the last advantage of the chiral theory consists in that for the case of presence of ultraviolet divergences which cannot be removed by standard methods of the renormalizable field theory the nonpolynomial form of Lagrangian admits of the use of the superpropagator method to regularize diagrams of such a type. We met similar phenomena in describing the $\pi\pi$ scattering and calculating the pion form factor. The process $\gamma\gamma \to \pi\pi$, we are here dealing with, is of interest also because of that for its description it turns out to be sufficient to employ the conventional methods of the renormalizable field theory when calculating the second-order perturbation expansion connected with the one-loop approximation.

In the next section we shall write chiral-type quantum Lagrangian in the Gursey exponential parametrization. The mass term will be taken also in a form suggested by Gursey. An interaction with the electromagnetic field is introduced in the standard gauge-invariant way. As the superpropagator methods

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[x] Note that all calculations have been performed up to now in the two first orders in the constant $1/F_\pi^2$. The second-order contribution was throughout considerably smaller than the first-order one at low energies [1-4].

will not be used here, we shall consider only the lowest orders of the total chiral Lagrangian.

At the end of the section we present the general form of the $\gamma\gamma \to \pi\pi$ scattering amplitude in the two first orders of perturbation theory.

In the third section a contribution is found to the scattering amplitude from the diagrams with the pion loops. It proves to be finite and is an elementary function of $s = (p_1 + p_2)^2$.

In the fourth section a contribution from the pion-nucleon interactions is calculated. Here only the terms proportional to the photon momenta squared are kept as other terms are small (of the type of $(m_\pi/M_N)^{2n}$, $n = 1,2,3$). It is interesting to note that at threshold energies the contribution to the $\gamma\gamma \to \pi^+\pi^-$ amplitude from the pion loops is nearly the same as that from the nucleon loops. For the $\gamma\gamma \to \pi^0\pi^0$ amplitude the contribution from the nucleon loops appears to be zero whereas that from the pion loops is two times larger than for the process $\gamma\gamma \to \pi^+\pi^-$.

In the last sections the hyperon contribution and the pion polarizability are calculated and the obtained results are discussed.

2. Interaction Lagrangian and the $\gamma\gamma \to \pi\pi$ amplitude

A chiral - type Lagrangian taken in the Gursey exponential parameterization [7] has the form [x]

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[x] All the notations used hereafter follow the monograph by N.Bogolubov and D.Shirkov [8].

$$\mathcal{L}(\vec{\varphi},\psi) = i\bar{\psi}\hat{\partial}\psi - M\bar{\psi}\exp\left[-\gamma_5\frac{\vec{\tau}\vec{\varphi}}{F_\pi}\right]\psi + \tag{1}$$
$$+\frac{F_\pi^2}{4}Sp\left\{\partial_\mu \exp\left[i\frac{\vec{\tau}\vec{\varphi}}{F_\pi}\right]\partial^\mu \exp\left[-i\frac{\vec{\tau}\vec{\varphi}}{F_\pi}\right]\right\} - \frac{m^2}{2}\vec{\varphi}^2,$$

where ψ is a nucleon field of mass M, $\vec{\varphi}$ is a pion field of mass m, F_π = 92 MeV. The interaction with electromagnetic field A_μ is introduced in the usual gauge-invariant way

$$\partial_\mu\psi \to \partial_\mu\psi + ieA_\mu\frac{1+\tau_3}{2}\psi, \tag{2}$$
$$\partial_\mu\varphi^a \to \partial_\mu\varphi^a + eA_\mu\varepsilon_{3ab}\varphi^b,$$

where ε_{3ab} is the antisymmetric tensor, e is the proton charge.

Since we shall not use the superpropagator methods for calculating the $\gamma\gamma\to\pi\pi$ amplitude in the one-loop approximation, it suffices to know only the lowest orders of perturbation expansion of the chiral Lagrangian in order to perform the required calculations.

Note should be made that in the perturbation expansion order of interest for us – e^2/F_π^2 the only term of Lagrangian (1), dependent on the choice of a form of the chiral Lagrangian is the mass term [x].

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[x] A discussion of possible arbitrariness in a choice of the mass term of Lagrangian (1) can be found in Appendix I.

In deriving the lowest orders from Lagrangian (1) one should remember that the low-energy theorems require a definite renormalization of the pion-nucleon vertices. This problem is discussed in more detail in papers by Lehmann [1b,c] (see also [9]). We write here all the lowest orders of chiral Lagrangian (1), which are of further need for our calculations, taking account of renormalization of the pion-nucleon vertices

$$\mathcal{L}_{\varphi\varphi} = -(2F_\pi)^{-2}:\vec{\varphi}^2[(\partial_\mu\vec{\varphi})^2 - \frac{m^2}{3}\vec{\varphi}^2]:, \tag{3a}$$

$$\mathcal{L}_{\varphi A} = eA_\mu:(\varphi_2\partial^\mu\varphi_1 - \varphi_1\partial^\mu\varphi_2): + \frac{e^2}{2}A_\mu^2:(\varphi_1^2+\varphi_2^2):, \tag{3b}$$

$$\mathcal{L}_{\psi A} = -eA_\mu:\bar{\psi}\gamma^\mu\frac{1+\tau_3}{2}\psi:, \tag{3c}$$

$$\mathcal{L}^{(1)}_{\varphi\psi} = \frac{g_A M}{F_\pi}:\bar{\psi}\gamma_5\vec{\tau}\vec{\varphi}\psi:, \quad \mathcal{L}^{(2)}_{\varphi\psi} = \frac{g_A^2 M}{2F_\pi^2}:\bar{\psi}\psi\vec{\varphi}^2:; \tag{3d}$$

where g_A = 1.25 is the renormalization constant.

Now let us proceed to constructing the $\gamma\gamma\to\pi\pi$ amplitude. This amplitude drawn in Fig.1 is defined as follows

$$\langle\pi^a(p_1)\pi^b(p_2)|S|\gamma_{\lambda_1}(q_1)\gamma_{\lambda_2}(q_2)\rangle = t^{\mu\nu}_{\lambda_1\lambda_2}T^{\mu\nu}_{ab}(p_1,p_2|q_1,q_2), \tag{4}$$
$$t^{\mu\nu}_{\lambda_1\lambda_2} = \frac{i\delta^{(4)}(p_1+p_2-q_1-q_2)}{(2\pi)^2 4\sqrt{q_1^0 q_2^0 p_1^0 p_2^0}}\varepsilon^\mu_{\lambda_1}\varepsilon^\nu_{\lambda_2},$$

where q_1, q_2 are the photon momenta, $\varepsilon^\mu_{\lambda_1}$, $\varepsilon^\nu_{\lambda_2}$ – photon polarizations, p_1, p_2 – the pion momenta, a,b – their isotopic indices. The amplitude $T^{\mu\nu}_{ab}$ breaks into two independent

Fig. 1

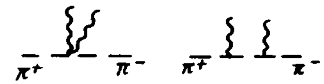

Fig. 2

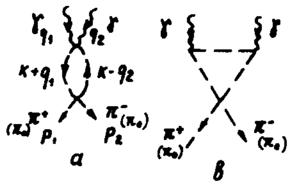

Fig. 3

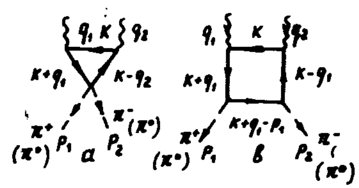

Fig. 4

Fig. 5

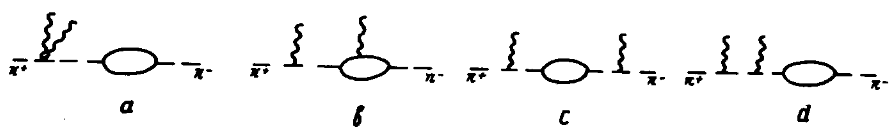

Fig 6

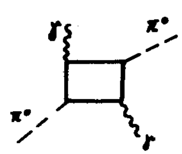

Fig 7

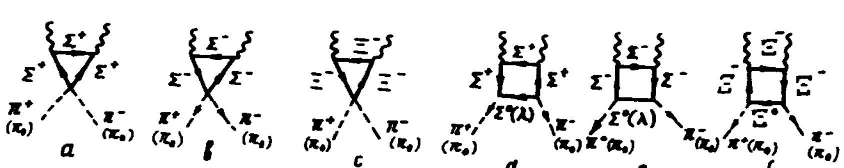

Fig. 8

gauge-invariant parts (see e.g. [10]). Leaving aside the general form of this covariant amplitude we only write down that form which is obtained in the two first orders of perturbation theory $(e^2, e^2/F_\pi^2)$

$$T^{\mu\nu}_{ab} = 2e^2(\delta_{ab}-\delta_{3a}\delta_{3b})\Big\{g^{\mu\nu} - \frac{p_1^\mu p_2^\nu}{p_1 q_1} - \frac{p_1^\nu p_2^\mu}{p_1 q_2} + \gamma(g^{\mu\nu} q_1 q_2 - q_1^\nu q_2^\mu)\times \tag{5}$$

$$\times\big[\beta^{(\pi)}(q_1 q_2) + \beta^{(N)}\big]\Big\} + 4e^2\delta_{3a}\delta_{3b}\gamma(g^{\mu\nu}q_1q_2 - q_1^\nu q_2^\mu)\beta^{(\pi)}(q_1q_2).$$

Here $\gamma = (4\pi F_\pi)^{-2}$ is the inverse square of that energy scale which was said about in the Introduction, three first terms in braces are the Born terms (see Fig.2), $\beta^{(\pi)}(q_1 q_2)$ is the pion loop contribution (Fig.3), $\beta^{(N)}$ is the nucleon-loop contribution (Fig.4). Only the constant terms are kept in $\beta^{(N)}$ because other terms are small in the region under consideration (of the type $O\left(\frac{m^2}{M^2}\right)$ or $O\left(\frac{q_1 q_2}{M^2}\right)$). Moreover, in (5) there are omitted the terms proportional to q_1^μ and q_2^ν as $(q_1\varepsilon_1) = (q_2\varepsilon_2) = 0$. The equalities $q_1^2 = q_2^2 = 0$, $p_1^2 = p_2^2 = m^2$ are taken into account here, as well. Bearing in mind all the aforesaid we obtain for $\beta^{(\pi)}(q_1 q_2)$ and $\beta^{(N)}$ the following expressions

$$\beta^{(\pi)}(q_1 q_2) = \left(1 - \frac{2}{3}\frac{m^2}{q_1 q_2}\right)\left\{\frac{2m^2}{q_1 q_2}\left[\operatorname{arctg}\left(\frac{2m^2}{q_1 q_2} - 1\right)^{-1/2}\right]^2 - 1\right\}, \tag{6}$$

$$\beta^{(N)} = \frac{2}{3} g_A^2 \approx 1{,}04 . \tag{7}$$

Now we will demonstrate in what manner these quantities are calculated.

3. The pion-loop contribution (e^2/F_π^2).

The contribution to the amplitude $T_{ab}^{\mu\nu}$ from the pion-loop diagrams drawn in Fig.3 is calculated as follows. The quadratic ultraviolet divergences entering into the integrals can be eliminated by transfering derivatives of the integrands to the exponents dependent on external momenta. The "contracted" diagrams arising in this procedure, of the type drawn in Fig.5 can be ruled out of the consideration as these can give only the constant contributions to the amplitude and the latter ones are required to be zero due to gauge invariance [x].
Finally it turns out that the remaining in the integrands logarithmic divergences cancel if the contributions from two-vertex and three-vertex diagrams are calculated together.

Let us now demonstrate the way of this procedure. We begin with considering the two-vertex pion diagram. Using interaction Lagrangian (3a) and (3b) for the matrix element corresponding to diagram (3a) one can get the following expression

$$A_1 = \frac{e^2\,(\varepsilon_1 \varepsilon_2)}{(2\pi)^6\, 2\sqrt{q_1^0 q_2^0 p_1^0 p_2^0}\, F_\pi^2} \iint dx_1 dx_2\, e^{i(q_1+q_2)x_1 - i(p_1+p_2)x_2} \times \tag{8}$$

$$\times\left\{\left[(p_1p_2)+\tfrac{4}{3}m^2\right]\Delta^2(x_1-x_2)+\tfrac{i}{2}(p_1+p_2)^\mu \partial_\mu^{(2)}\Delta^2(x_1-x_2)-\left[\partial_\mu^{(2)}\Delta(x_1-x_2)\right]^2\right\}.$$

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[x] Note that considering the infinite set of such loops in all the perturbation expansion orders of $(1/F_\pi)$ with the use of the superpropagator method also results in zero contribution from these diagrams [11].

Here $\Delta(x_1-x_2)$ is the propagator of a free scalar field of mass m. Index (2) of the derivative means that it operates on x_2. Transfering then the derivatives onto $e^{-i(p_1+p_2)x_2}$ and omitting the terms having the δ- function $\delta^{(4)}(x_1-x_2)$ in the integrand ("contracted" diagrams) we obtain

$$\bar{A}_1 = -i\frac{2}{\pi^2}e^2\gamma\, t_{\lambda_1\lambda_2}^{\mu\nu} g^{\mu\nu}\left[q_1q_2-\tfrac{2}{3}m^2\right]\int\frac{d^4k}{[m^2-(k+q_1)^2-i\varepsilon][m^2-(k-q_2)^2-i\varepsilon]}. \tag{9}$$

In a similar way one can find for the three-vertex diagram (Fig.3b) the expression

$$\bar{A}_2 = -i\frac{8}{\pi^2}e^2\gamma\, t_{\lambda_1\lambda_2}^{\mu\nu}\left[q_1q_2-\tfrac{2}{3}m^2\right]\int\frac{d^4k\, k^\mu k^\nu}{[m^2-k^2-i\varepsilon][m^2-(k+q_1)^2-i\varepsilon][m^2-(k-q_2)^2-i\varepsilon]}. \tag{10}$$

It is easy to see that only the logarithmic divergences remain in integrals (9) and (10). Now we combine both the integrals and consider the integral $A_{\mu\nu}^{(m)}(q_1q_2)$

$$A_{\mu\nu}^{(m)}(q_1q_2) = \int\frac{d^4k}{[m^2-(k+q_1)^2-i\varepsilon][m^2-(k-q_2)^2-i\varepsilon]}\left[g^{\mu\nu}+\frac{4k^\mu k^\nu}{m^2-k^2-i\varepsilon}\right]. \tag{11}$$

It appears that this integral does not contain the logartihmic divergences either. To be convinced of this, let us calculate $A_{\mu\nu}^{(m)}$ in the case $q_1 = q_2 = 0$. It is just that part of $A_{\mu\nu}^{(m)}$ where the logarithmic divergences can be expected to appear. It is of the form

$$A_{\mu\nu}^{(m)}(0) = im^2 g^{\mu\nu}\lim_{\ell\to 0}\int_\ell^\infty\frac{d\lambda}{\lambda}e^{-\varepsilon\lambda - im^2\lambda}\left[\iint dt_1dt_2\,\delta\!\left(1-\sum_1^2 t_i\right)-2\iiint dt_1dt_2dt_3\,\delta\!\left(1-\sum_1^3 t_i\right)\right]=0. \tag{12}$$

Eq. (12) is hold, if we go to limit $\ell = 0$ at the end of the our calculations. Note, that $A_{\mu\nu}^{(m)}(0)$ is required to be zero due to gauge invariance also.

The integral (11) is exactly calculated by a method given in [12] (see Appendix in [12]). As a result we have

$$A^{(m)}_{\mu\nu}(q_1q_2) = i\pi^2 \frac{(g^{\mu\nu}q_1q_2 - q_1^{\nu}q_2^{\mu})}{q_1q_2}\left\{\frac{2m^2}{q_1q_2}\left[\mathrm{arctg}\left(\frac{2m^2}{q_1q_2}-1\right)^{-1/2}\right]^2 - 1\right\}. \tag{13}$$

Thus the contribution to the amplitude $T^{\mu\nu}_{(\pi^+\pi^-)}$ from the pion loops drawn in Fig.3 may be thought to be completely calculated.

Up to now we have considered the process of photon annihilation with production of charged pions. The contribution from pion loops to the amplitude for the neutral-pion production will be two times larger than for charged pions and of the same sign. Writing the obtained result for the amplitude in a form similar to (5) we arrive at formula (6) for the function $\beta^{(\pi)}(q_1q_2)$.

We would like to note here one important property of the obtained function. With increasing energy $S = 2q_1q_2$ from zero to the threshold of two-pion production this function rapidly varies and even changes its sign. At $q_1q_2 = 0$ it equals to

$$\beta^{(\pi)}(0) = -1/9 . \tag{14}$$

At the threshold value of argument $q_1q_2 = 2m^2$ it increases nine times in magnitude and changes its sign:

$$\beta^{(\pi)}(2m^2) = \frac{2}{3}\left[\frac{\pi^2}{4} - 1\right] \approx 1 . \tag{15}$$

With this we complete the discussion of the pion-loop contribution and proceed to considering the nucleon diagrams.

4. <u>The nucleon-loop contribution</u> (e^2/F_π^2).

The dominant contribution to the $\gamma\gamma \to \pi^+\pi^-$ amplitude comes from the three-vertex diagram with the nucleon loop drawn in Fig.4a. Using the Lagrangians $\mathcal{L}_{\psi A}$ and $\mathcal{L}^{(2)}_{\varphi\psi}$ (see eqs. (3c) and (3d)) we obtain for the corresponding matrix element the expression

$$B = i8\gamma\left(\frac{e g_A M}{\pi}\right)^2 t^{\mu\nu}_{\gamma\gamma}\left\{A^{(M)}_{\mu\nu} - [g^{\mu\nu}q_1q_2 - q_1^{\nu}q_2^{\mu}]A^{(M)}\right\}. \tag{16}$$

All the integrals in (16) again converge and the whole expression (16) has the gauge-invariant form. Indeed, $A^{(M)}_{\mu\nu}(q_1q_2)$ is the same function as we have found for two-pion loops (formula (11)) with the only difference that the nucleon masses M enter here instead of the pion masses m . The function $A^{(M)}$ corresponds to the triangle loop with scalar particles of mass M

$$A^{(M)}(q_1q_2) = \int \frac{d^4k}{[M^2-k^2-i\varepsilon][M^2-(k+q_1)^2-i\varepsilon][M^2-(k-q_2)^2-i\varepsilon]} . \tag{17}$$

Since the higher-order terms of expansion of these functions in powers of (q_1q_2) have the form of $(q_1q_2/M^2)^n$, we may take with good accuracy only the first terms of such an expansion. Then we get

$$B \approx \frac{8}{3} g_A^2 e^2 \gamma\, t^{\mu\nu}_{\gamma\gamma}(g^{\mu\nu}q_1q_2 - q_1^{\nu}q_2^{\mu}) . \tag{18}$$

Consider next the four-vertex diagram drawn in Fig.4b. Unlike the previous diagram, in this case we meet the logarithmic divergences in the corresponding integrals and the finite part of the matrix element has no longer the gauge-invariant form. To obtain the finite gauge-invariant result the diagram (4b) must be considered together with the whole set of diagrams shown in Fig.6.

Without detailing the calculations, we give here the matrix elements corresponding to every of these diagrams allowing for all possible transpositions of the external lines. The matrix elements corresponding to the diagrams drawn in Figs. 4b, 6a, 6b, 6c and 6d are denoted by letters C,D,F,G and H. Then we get

$$C = -\frac{4}{3} g_A^2 e^2 \gamma\, t_{12}^{\mu\nu} \{ g^{\mu\nu} q_1 q_2 - q_1^{\nu} q_2^{\mu} + 2(p_1^{\mu} p_2^{\nu} + p_1^{\nu} p_2^{\mu}) - g^{\mu\nu}(m^2 - \mathcal{I}(m,M)) \}; \tag{19}$$

where

$$\mathcal{I}(m,M) = i\,\frac{6}{\pi^2} M^2 \int \frac{d^4k}{[M^2-k^2-i\varepsilon][M^2-(k-p_1)^2-i\varepsilon]} . \tag{20}$$

$$\mathcal{D} = -\frac{4}{3} g_A^2 e^2 \gamma\, t_{12}^{\mu\nu} g^{\mu\nu}(m^2 - \mathcal{I}(m,M)),$$

$$F + G + H = \frac{8}{3} g_A^2 e^2 \gamma\, t_{12}^{\mu\nu} (p_1^{\mu} p_2^{\nu} + p_1^{\nu} p_2^{\mu}). \tag{21}$$

Collecting together all the matrix elements corresponding to the considered diagrams with nucleon loops and representing them in a form similar to formulae (4) and (5) the value given by (7) is found for the quantity $\beta^{(N)}$.

Consider now the photon annihilation with production of the two neutral pions. In this case the diagrams similar to those

drawn in Fig.4 are present but with π^0 external lines instead of π^+ and π^- ones. Their contributions to the matrix elements coincide with those found for the charged pions. However, the diagrams shown in Fig.6 are replaced only by one four-vertex diagram drawn in Fig.7. Its corresponding matrix element I is

$$I = -\frac{4}{3} g_A^2 e^2 \gamma\, t_{12}^{\mu\nu} [g^{\mu\nu} q_1 q_2 - q_1^{\nu} q_2^{\mu} - 2(p_1^{\mu} p_2^{\nu} + p_1^{\nu} p_2^{\mu}) + g^{\mu\nu}(m^2 - \mathcal{I}(m,M))]. \tag{22}$$

As a result, the contribution to the $\gamma\gamma \to \pi^0\pi^0$ amplitude from two four-vertex diagrams with nucleon loops turns out to cancel completely with the contribution to this amplitude from the three-vertex diagram 4a. In this way we are convinced of that the diagrams with nucleon loops do not affect the amplitude of the process $\gamma\gamma \to \pi^0\pi^0$ in the e^2/f_π^2 order.

5. The consideration of baryons and kaons

Until now we have considered only nucleon loops when calculating the quantity $\beta^{(N)}$. An interaction with other baryons may be analyzed in a similar manner. However, as will be shown below, taking account of interaction with other terms of the baryon octet little affects the quantity $\beta^{(N)}$.

The corresponding diagrams are shown in Fig.8. Note that the number of the four-vertex diagrams is twice as many as those of the three-vertex ones due to participation of Λ particle in the first diagrams. The contributions from these diagrams will be calculated in a way similar to that described in our previous paper [3b].

The new "strong" vertices are connected with the πN vertex through the relations (see [3b,13,14]):

$$|g_{\pi\Sigma\Sigma}| = \frac{2}{3} g \ ; \ |g_{\pi\Sigma\Lambda}| = \frac{4}{3\sqrt{3}} g \ ; |g_{\pi\Xi\Xi}| = \frac{1}{3} g \ ; \ g_{\pi NN} = g \ . \qquad (23)$$

Now it is easy to see that taking account of the contributions of all the terms of the baryon octet reduces to multiplying of $\beta^{(N)}$ by the following quantity

$$1 + \frac{4}{9}\left(\frac{M_N}{M_\Sigma}\right)^2 + \frac{1}{9}\left(\frac{M_N}{M_\Xi}\right)^2 - \frac{16}{27}\left(\frac{M_N}{M_{(\Sigma\Lambda)}}\right)^2 \approx 0{,}95 \ . \qquad (24)$$

As a result, $\beta^{(B)}$ becomes

$$\beta^{(B)} \approx 0{,}99 \ . \qquad (25)$$

For the $\gamma\gamma \to \pi^0\pi^0$ the interaction with the whole baryon octet is as nonessential as that with nucleons only. The pion-kaon interaction can be neglected because allowing for this interaction results in a small correction of the type $O\left(\left(\frac{m_\pi}{m_K}\right)^2\right)$ to the pion-loop contribution $\beta^{(\pi)}(q_1 q_2)$.

6. Discussion of the results

At the present time an extensive literature exists which is devoted to various approaches of the theoretical description of the process $\gamma\gamma \to \pi\pi$ (see e.g. [15-17]). Leaving aside many aspects of this process described in detail in review articles [15-16], we now consider briefly the pion polarizability following from the formulae we have already obtained.

The pion polarizability α_π is defined as the coefficient of effective interaction of a pion with an external electromagnetic field [15-16]:

$$V_{int.} = -\frac{\alpha_\pi}{2}\left(E^2 - H^2\right). \qquad (26)$$

It is interesting to note that it follows from the structure of formula (5) for the amplitude of $\gamma\gamma \to \pi\pi$ that the electric and magnetic pion polarizabilities are equal to each other in magnitude and are opposite in sign (see Appendix II).

The coefficient α_π is expressed via the quantities γ and β entering into amplitude (5) in the following way [15]:

$$\alpha_{\pi^\pm} = \alpha_{\pi^\pm}(q_1q_2)\Big|_{q_1q_2=0} = \gamma\frac{e^2}{m}\left(\beta^{(\pi)}(0)+\beta^{(B)}\right) = 0{,}16\ \alpha/m^3 \qquad (27)$$

$$\alpha_{\pi^0} = \alpha_{\pi^0}(q_1q_2)\Big|_{q_1q_2=0} = 2\gamma\frac{e^2}{m}\beta^{(\pi)}(0) = -0{,}04\ \alpha/m^3\ , \qquad (28)$$

where $\alpha = e^2/4\pi = 1/137$ x) .

Now we compare our results with the values for the pion polarizability obtained by Terent'ev [15] with the use of current algebra and PCAC conditions:

$$\alpha_{\pi^\pm} = 0{,}16\ \alpha/m^3\ , \quad \alpha_{\pi^0} = 0\ . \qquad (29)$$

For the charged pion both of the results differ little especially if one neglects the hyperon influence which is not taken into account in [15] . For the neutral pions we have obtained essentially different result xx) .

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x) To make the parameter γ dimensionless it is natural to use the pion mass squared in the energy range under consideration. Then it is found that $m^2\gamma = 2\alpha$ with the very good accuracy. The quantity $m^2\gamma$ is just that small parameter which we always obtain in the two first orders of pertrubation expansion (see also ref. [3]).

xx) Almost the same value for $\alpha_{\pi^\pm}$ has been found in the quark models for quarks of charge 1/3 as well (see ref. [18]).

It is interesting to investigate the influence of the pion-loop contributions on the threshold behaviour of the functions $\alpha_{\pi^{\pm}_{0}}(q_1q_2)$. In processing experimental data one should bear in mind that the function $\beta^{(\pi)}(q_1q_2)$ varies rapidly in the threshold energy region so that at $q_1q_2 = 2m^2$ the magnitudes of $\alpha_{\pi^\pm}(q_1q_2)$ and $\alpha_{\pi^0}(q_1q_2)$ become equal to each other and exceed more than twice the magnitude of $\alpha_{\pi^\pm}(0)$.

$$\alpha_{\pi^\pm}(2m^2) = 0{,}36\ \alpha/m^3\ ;\quad \alpha_{\pi^0}(2m^2) = 0{,}36\ \alpha/m^3 \qquad (30)$$

As to the total cross sections of the process $\gamma\gamma\to\pi\pi$, we notice only that the cross section for $\gamma\gamma\to\pi^+\pi^-$ little changes as compared with that calculated in [15] , for the process $\gamma\gamma\to\pi^0\pi^0$ the cross section differs from zero though being considerably smaller than that for $\gamma\gamma\to\pi^+\pi^-$ (in [15] the value: $\sigma_{\gamma\gamma\to\pi^0\pi^0}=0$ has been found).

A more detailed consideration of the behaviour of the $\gamma\gamma\to\pi\pi$ amplitude in energy regions of physical interest will be dealt with in subsequent papers of the authors.

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APPENDIX I

The $\pi\pi$ interaction Lagrangian in the order $1/F_\pi^2$ in an arbitrary parametrization has the form

$$\mathcal{L}_{\varphi\varphi} = -(2F_\pi)^{-2} : [\vec{\varphi}^2(\partial_\mu\vec{\varphi})^2 - Cm^2(\vec{\varphi}^2)^2] : , \qquad \text{(A.I1)}$$

where $C_G = 1/3$, $C_W = 1/4$ respectively for the Gursey and Weinberg parametrizations. The change $C_G \to C_W$ little affects the final results. We have preferred here the Gursey parametrization as this one is connected naturally with the geometry of curved isospace of Goldstone fields (the choice of pion coordinates along geodesics). There also exists a parametrization — independent definition of the mass term of chiral Lagrangian based on the use of the linear realization of chiral symmetry [19]. The obtained within this approach coefficient C in (A.I.1) coincides with C_W.

Note also that independently of the group considerations there exist the physical arguments in favour of bounds on possible values of the constant C.

Assuming that the $\pi\pi$ scattering phases independent of the pion mass at energies $\sim 2\pi F_\pi$ [3a] do not alter their signs up to the threshold energy values (the smoothness hypothesis), we get the following constraints on the scattering lengths:

$$a_0^0 = const(3-5C) \geq 0, \quad (const. = \frac{m}{8\pi F_\pi^2})$$

$$a_0^2 = -const \cdot 2C \leq 0 \quad (const > 0)$$

This results in the possible values for the constant C

$$0 \leq C \leq 3/5 .$$

Finally, we write down the amplitude (5) for an arbitrary constant C :

$$T_{ab}^{\mu\nu} = 2e^2(\delta_{ab} - \delta_{3a}\delta_{3b})\Big\{g^{\mu\nu} - \frac{p_1^\mu p_2^\nu}{p_1q_1} - \frac{p_1^\nu p_2^\mu}{p_1q_2} +$$
$$+ \gamma(g^{\mu\nu}q_1q_2 - q_1^\nu q_2^\mu)\Big[\beta^{(\omega)} + \frac{[(1-2C) - \frac{q_1q_2}{2m^2}]}{[\frac{1}{3} - \frac{q_1q_2}{2m^2}]}\beta^{(\pi)}(q_1q_2)\Big]\Big\} +$$
$$+ 4e^2\delta_{3a}\delta_{3b}\gamma(g^{\mu\nu}q_1q_2 - q_1^\nu q_2^\mu)\frac{[1 - C - q_1q_2/m^2]}{[2/3 - q_1q_2/m^2]}\beta^{(\pi)}(q_1q_2).$$

APPENDIX II

The amplitude $T_{ab}^{\mu\nu}$ in the order e^2/F_π^2 contains only one gauge-invariant combination

$$M = C\,\varepsilon_1^\mu\varepsilon_2^\nu(g^{\mu\nu}q_1q_2 - q_1^\nu q_2^\mu) . \qquad \text{(A.II.1)}$$

This results in that the defined in this order electric and magnetic pion polarizabilities are the same in magnitude and opposite in sign.

Indeed, if the amplitude can be written in the form

$$M = \alpha\omega_1\omega_2(\vec{\varepsilon}_1\vec{\varepsilon}_2) + \beta(\vec{S}_1\vec{S}_2), \qquad \text{(A.II.2)}$$

where ω_i – the photon energy, $\varepsilon_i = \{0, \vec{\varepsilon}_i\}$, $\varepsilon_1^{\mu}\varepsilon_2^{\mu} = -\vec{\varepsilon}_1\vec{\varepsilon}_2$, $\vec{S}_i = [\vec{\varepsilon}_i \times \vec{q}_i]$, then the quantity α will determine the electric pion polarizability, the quantity β – the magnetic one [16].

Now let us reduce both (A.II.1) and (A.II.2) to one and the same form and express α and β through C. Formula (A.II.1) can easily be written as follows

$$M = C\{-\omega_1\omega_2(1-\cos\theta)(\vec{\varepsilon}_1\vec{\varepsilon}_2) - (\vec{\varepsilon}_1\vec{q}_2)(\vec{\varepsilon}_2\vec{q}_1)\}. \qquad \text{(A.II.3)}$$

On the other hand, using the well-known relation

$$[\vec{\varepsilon}_1\times\vec{q}_1][\vec{\varepsilon}_2\times\vec{q}_2] = (\vec{\varepsilon}_1\vec{\varepsilon}_2)(\vec{q}_1\vec{q}_2) - (\vec{q}_1\vec{\varepsilon}_2)(\vec{q}_2\vec{\varepsilon}_1),$$

formula (A.II.2) can be written as

$$M = (\alpha + \beta\cos\theta)\omega_1\omega_2(\vec{\varepsilon}_1\vec{\varepsilon}_2) - \beta(\vec{\varepsilon}_1\vec{q}_2)(\vec{\varepsilon}_2\vec{q}_1). \qquad \text{(A.II.4)}$$

Comparing (A.II.3) and (A.II.4), we get

$$C = \beta = -\alpha.$$

References

1. H.Lehmann. a) Phys.Lett, 41B, 529 (1972); b) Preprint DESY 72/54 (1972); c) Preprint DESY 73/26 (1973).
2. G.Ecker, J.Honerkamp. a) Nucl.Phys. B52, 211 (1973); b) B62, 509 (1973).
3. V.N.Pervushin, M.K.Volkov a) Preprint JINR, E2-7661, Dubna, 1974; b) Yadernaya Fisika, 19, 652 (1974).
4. М.К.Волков "Использование суперпропагаторов для описания неполиномиальных квантовых теорий поля". Лекции на школе по физике элементарных частиц, Тбилиси, 1973.
   M.K.Volkov. Preprint JINR, D2-7161, 84, Dubna 1973.
5. L.S.Brawn and R.L.Coble. Phys.Rev., D4, 723 (1971).
6. M.K.Volkov. Ann.Phys. 49, 202 (1968).
7. P.Chang and F.Cursey. Phys.Rev., 164, 1752 (1967); F.Cursey ITP-Buda-Pest Report No.249. November (1968).
8. Н.Н.Боголюбов, Д.В.Ширков. "Введение в теорию квант... полей". Гостехиздат 1957 г.
9. R.Dashen and M.Weinstein. Phys.Rev. 183, 1261 (1969).
10. W.A.Bardeen and Wu-Ki Tung. Phys.Rev. 173, 1423 (1968).
11. V.N.Pervushin. Preprint JINR E2-7540, Dubna (1973).
12. М.К.Волков. ТМФ 6, 2I (I97I).
13. Ю.В.Новожилов. "Введение в теорию элементарных частиц", "Наука", Москва (I972).
14. J.J.Sakurai "Currents and Mesons" Chicaga and London the University of Chicaga Press.
15. М.В.Терентьев. ЯФ I6, I62 (I972); УФН II2, 37 (I974).

I6. В.А.Петрунькин. Труды ФИАН 4I, "Наука" I968.
I7. П.С.Исаев, В.И.Хлестков. ЯФ I6, I0I2 (I972); Письма в ЖЭТФ I6, I90 (I972).
I8. T.E.O.Ericson and J.Hüfner. Nuclear Phys. B47, 205 (1972)
I9. S.P.Rosen. Phys.Rev. D1, 3392 (1970).

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