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ON OBLATE-PROLATE TRANSITION  
IN THE GROUND STATE ROTATIONAL  
BAND OF LIGHT MERCURY ISOTOPES

**1974**

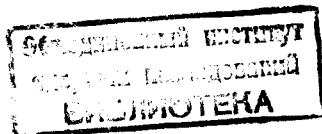
ЛАБОРАТОРИЯ  
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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**ON OBLATE-PROLATE TRANSITION  
IN THE GROUND STATE ROTATIONAL  
BAND OF LIGHT MERCURY ISOTOPES**

*Submitted to Physics Letters*



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Переход от сплюснутой к вытянутой форме в основной ротационной полосе в легких изотопах ртути

Рассчитывалась зависимость от деформации уровней ротационной полосы с моментом до  $8^+$  в изотопах ртути. Показано, что состояния с  $0^+$  и  $2^+$  имеют сплюснутую деформацию, а  $4^+$ ,  $6^+$  и  $8^+$  - вытянутую. Переход от сплюснутой к вытянутой форме в нечётных изотопах ртути происходит между  $A=187$  и  $A=185$ , в соответствии с экспериментом.

Препринт Объединенного института ядерных исследований.  
Дубна, 1974

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On Oblate-Prolate Transition in the Ground State Rotational Band of Light Mercury Isotopes

Transitions from the oblate shape to the prolate one in the rotational bands of even-mass mercury isotopes and in the ground states of odd-mass isotopes are studied. These transitions are shown to occur in agreement with the available experimental data.

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Dubna, 1974

The optical pumping experiments <sup>/1/</sup> have drawn attention to light mercury isotopes. The observed sharp increase in the mean-square radius of  $^{185}\text{Hg}$  with respect to  $^{187}\text{Hg}$  has been interpreted as evidence for the transition from a nearly spherical to a deformed shape. The recently measured characteristics of the yrast levels in  $^{184}\text{Hg}$  <sup>/2/</sup> and  $^{186}\text{Hg}$  <sup>/3/</sup> support this interpretation. Theoretical investigations of the even-mass Hg isotopes <sup>/4-6/</sup> show that the deformation energy has an oblate and prolate minima of nearly the same depth. Therefore the addition of an odd particle or the excitation of a rotational level may cause transition from one minimum to the other.

We calculated the deformation energy  $E_1$  of the rotational levels of the ground state band of the even-mass Hg isotopes, as well as  $E$  of the ground state of the odd-mass Hg isotopes. We consider the ground state energy to be the sum of the liquid drop energy  $U_{\text{LDM}}$  and Strutinsky's shell correction energy  $\delta U$  <sup>/7/</sup>. The rotational energy of the excited levels with angular momentum  $I$  is assumed to be equal to  $I(I+1)/2 \mathcal{J}$ . The moment of inertia  $\mathcal{J}$  is calculated by means of the cranking model including pairing. As a single particle basis, we used the states in the Woods-Saxon potential <sup>/8/</sup> with the parameters of ref. <sup>/9/</sup>. The strength of the pairing interaction is determined by the average gap <sup>/7/</sup>, for which we choose  $12A^{1/2}\text{MeV}$  and  $13A^{1/2}\text{MeV}$  for neutrons and protons respectively.

We have found from our calculations that all Hg isotopes have oblate and prolate minima at the respective

deformations  $\epsilon \approx -0.12$  and  $0.22$ . The odd-mass isotopes  $^{181-185}\text{Hg}$  have a large prolate equilibrium deformation, whereas the  $^{187-191}\text{Hg}$  isotopes have a small oblate equilibrium deformation (see fig. 1 and table 1). This

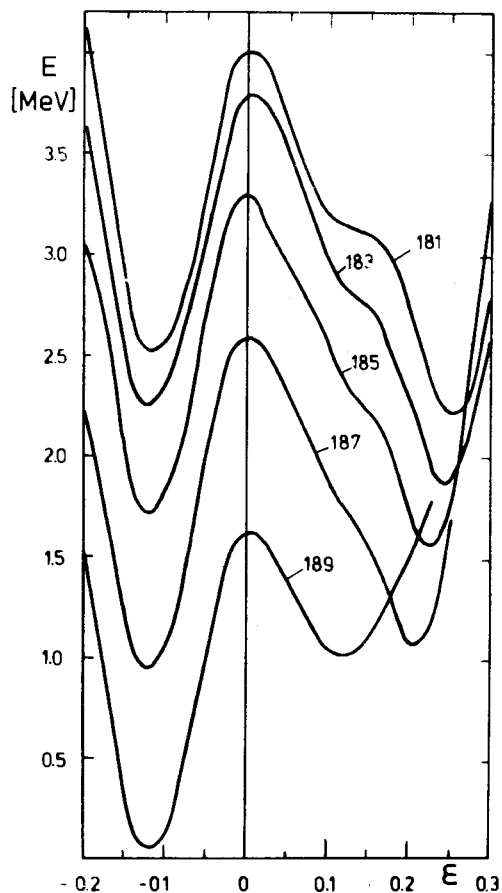


Fig. 1. Deformation energy of the odd-mass Hg isotopes. Hexadecapole deformation  $\gamma_8$  is equal to  $-0.02$ .

transition from a small (oblate) to a large (prolate) deformation may explain the experimentally observed increase in the mean-square radius of  $^{185}\text{Hg}$  with respect to  $^{187}\text{Hg}$ . Unlike odd-mass isotopes, all even-mass isotopes

heavier than  $^{180}\text{Hg}$  are oblate in the ground state, though for lightest ones the energy difference between the two minima  $\Delta E = E_{\text{obl.}} - E_{\text{prol.}}$  is very small (see fig. 2).

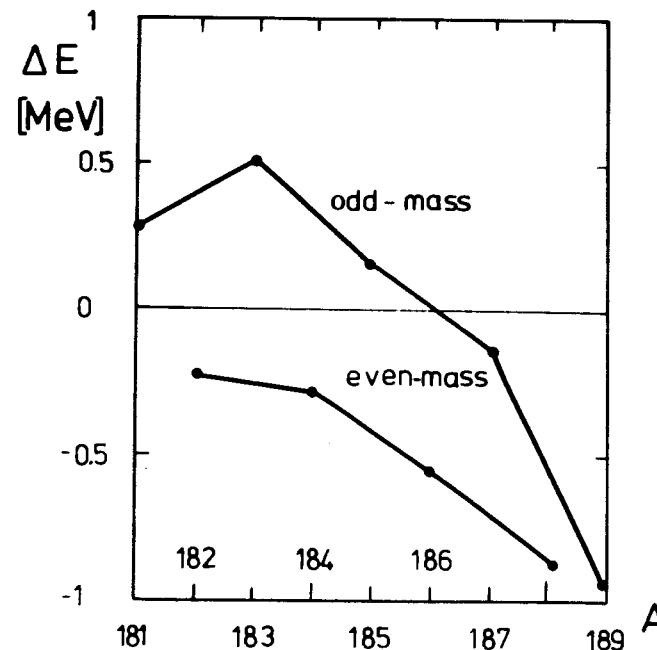


Fig. 2. The oblate-prolate energy difference  $\Delta E$  as a function of the mass number  $A$  of Hg isotopes. The hexadecapole deformation  $\gamma_8$  is the equilibrium one for even  $A$  and equal  $-0.02$  for odd  $A$ . The change of  $\Delta E$  due to minimization with respect to the hexadecapole deformation is less than  $0.06$  MeV for  $^{183-187}\text{Hg}$ .

Now, we summarize the main features of our calculations, which lead to different ground state shapes of some adjacent odd- and even-mass isotopes. The density of the single particle proton levels is very low in the almost spherical oblate minimum, because nearby, at  $Z = 82$ , a spherical shell is closed. At the same deformation the density of the neutron levels is very high. Vice versa, the neutron level density is low in the well deformed prolate minimum, because nearby, at  $N = 100$ , a deformed

shell is closed. The proton level density is high in the prolate minimum. The neutron pairing gap corresponding to the level density is large ( $\approx 1.1$  MeV) in the oblate and small ( $\leq 0.6$  MeV) in the prolate minimum. Therefore the loss of pairing energy due to the blocking of the Fermi level by the odd neutron is larger in the oblate minimum than in the prolate one, and the shift of the former with respect to the even system is larger than that of the latter.

The energy difference between the two minima is very small in the light even-mass Hg isotopes (see fig. 2). As the prolate minimum corresponds to deformation twice as large as the oblate one the rotational energy is significantly lower at the prolate shape than at the oblate one. In fig. 3 the rotational levels of  $^{184}\text{Hg}$  are shown as functions of the deformation parameter  $\epsilon$ . The states  $0^+$  and  $2^+$  have a small oblate deformation, whereas the states  $4^+$ ,  $6^+$ , ... have a large prolate deformation. As shown in table 2, the calculated transition energies in  $^{184-186}\text{Hg}$  reproduce the experimental ones with good accuracy. In refs. <sup>/2,10/</sup>, on the basis of the rotor model the deformation  $\bar{\beta}_{\text{exp}}$  is calculated from the experimental lifetimes of the levels. It is shown in table 3 that these values are comparable

with the mean value  $\bar{\epsilon} = \left( \frac{\epsilon_1^2 + \epsilon_2^2}{2} \right)^{1/2}$  calculated from

our values of  $\epsilon_1$  for equilibrium deformation (see table 2). Our calculations confirm the assumption made in ref. <sup>/3/</sup> about an oblate-prolate transition in the rotational band. The energy of the transition  $4^+ - 2^+$ , which is very sensitive to the energy difference between the two minima, is reproduced within 0.15 MeV. We did not investigate the problem of whether there is a barrier between the two minima in the  $\gamma$ -degree of freedom, i.e., at triaxial shapes. The experimental fact that the  $B(E2)$  value of the  $4^+ \rightarrow 2^+$  transition has the same order of magnitude as those of the other transitions within the band <sup>/3/</sup> supports the assumption that no barrier exists. In the calculations of ref. <sup>/4/</sup> only a very flat and broad barrier was found. Therefore the wavefunctions of the  $2^+$  and  $4^+$  states should be spread out in the  $\gamma$ -direction. However the upper part of the

**Table 1**  
The equilibrium deformation  $\epsilon$ , the oblate-prolate energy difference  $\Delta E$  and the difference  $\delta\beta^2$  between the square of the equilibrium deformation of  $^{181-185}\text{Hg}$  and  $^{187}\text{Hg}$  for odd-mass Hg isotopes. Hexadecapole deformation  $^{8/}$  is equal to  $-0.02$ .

Isotope	$\epsilon$	$\Delta E$ (MeV)	$\delta\beta^2$	$\delta\beta_{\text{exp}}^{2/1/}$
$^{181}\text{Hg}$	0.251	0.27	0.066	0.065
$^{183}\text{Hg}$	0.244	0.35	0.062	0.061
$^{185}\text{Hg}$	0.225	0.15	0.055	0.059
$^{187}\text{Hg}$	-0.122	-0.13	-	-
$^{189}\text{Hg}$	-0.119	-0.96	-	-

**Table 2**  
The transition energies, equilibrium deformations and  $\Delta E$  for even-mass Hg isotopes. The equilibrium hexadecapole deformation  $^{8/}$  is close to  $-0.02$  for all cases but the prolate minimum in  $^{188}\text{Hg}$ , for which it is equal to  $-0.04$ .

Transition	Energy (MeV)		Final state	
	exp.	theor.	$\epsilon$ eq.	$\Delta E$ (MeV)
<b><math>^{184}\text{Hg}</math></b>				
$2^+ \rightarrow 0^+$	0.367	0.38	-0.117	-0.29
$4^+ \rightarrow 2^+$	0.288	0.25	-0.123	-0.01
$6^+ \rightarrow 4^+$	0.340	0.36	0.229	0.57
$8^+ \rightarrow 6^+$	0.418	0.48	0.233	1.35
<b><math>^{186}\text{Hg}</math></b>				
$2^+ \rightarrow 0^+$	0.405	0.37	-0.118	-0.54
$4^+ \rightarrow 2^+$	0.403	0.54	-0.124	-0.29
$6^+ \rightarrow 4^+$	0.357	0.39	0.215	0.25
$8^+ \rightarrow 6^+$	0.424	0.52	0.218	0.95
<b><math>^{188}\text{Hg}</math></b>				
$2^+ \rightarrow 0^+$	0.413	0.37	-0.117	-0.97
$4^+ \rightarrow 2^+$	0.591	0.79	-0.122	-0.69
$6^+ \rightarrow 4^+$	0.504	0.43	-0.134	-0.11
$8^+ \rightarrow 6^+$	0.460	0.43	0.212	0.67

Table 3  
The mean values of deformation for transitions in the rotational bands of  $^{184}\text{Hg}$  and  $^{186}\text{Hg}$ .

Transition	$\bar{\epsilon}$	$\bar{\beta}$	$\bar{\beta} / 2, 10 / \text{exp.}$
$^{184}\text{Hg}$			
$2^+ \rightarrow 0^+$	0.12	0.13	$0.15 \pm 0.02$
$4^+ \rightarrow 2^+$	0.18	0.20	$0.22 \pm 0.01$
$6^+ \rightarrow 4^+$	0.23	0.26	$0.28 \pm 0.05$
$^{186}\text{Hg}$			
$2^+ \rightarrow 0^+$	0.12	0.13	$0.13 \pm 0.01$
$4^+ \rightarrow 2^+$	0.18	0.20	$0.16 \pm 0.03$
$6^+ \rightarrow 4^+$	0.22	0.25	$0.27 \pm 0.05$

rotational spectrum and, to some extent, the ground state also have a comparatively large  $\Delta E$ . Hence both parts should be axially symmetric and the account of triaxial shapes cannot very much change their relative position.

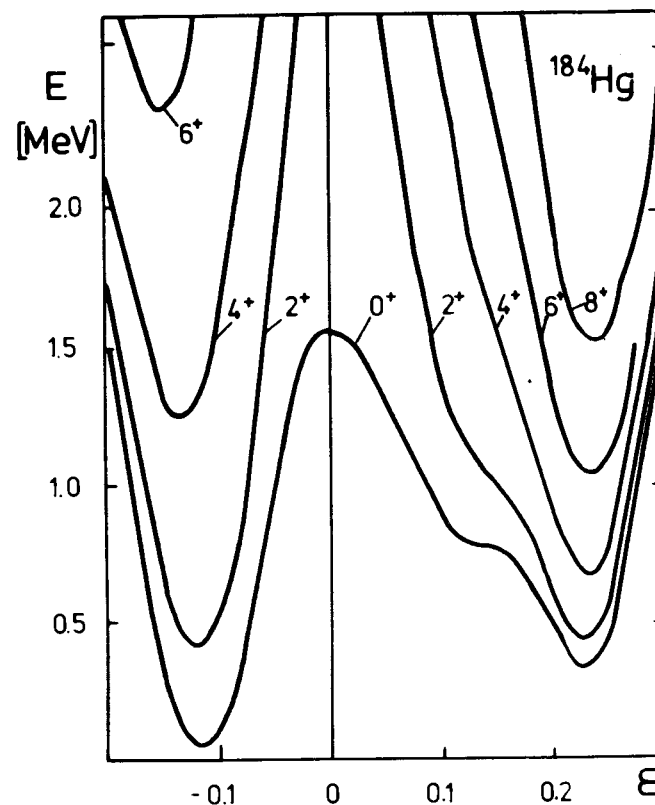


Fig. 3. Deformation energy of the rotational states in  $^{184}\text{Hg}$ . Hexadecapole deformation  $^{18/8}$  is equal to  $-0.02$ , which is very close to the equilibrium value both in the oblate and in the prolate minima.

In isotopes heavier than  $^{186}\text{Hg}$  the oblate-prolate transition occurs at larger angular momenta  $^{10,12}$ . The transition energies for  $^{188}\text{Hg}$  are shown in table 2. For still heavier isotopes, the oblate-prolate transition is shifted to very high values of  $l$  so that our collective model

becomes inapplicable in the case of the small oblate deformations.

We have found that the Pt isotopes with the same number of neutrons as the Hg isotopes considered are prolate already in the ground state. The deformation is nearly the same as in the "prolate" part of the rotational bands of the Hg isotopes. In accordance with the experiment<sup>/3/</sup>, the calculated distances between the higher rotational levels of these adjacent nuclei approximately coincide.

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## References

1. J.Bonn, G.Huber, H.J.Kluge, U.Kopf, L.Kugler, E.W.Otten. Phys.Lett., 36B, 41 (1971); 38B, 308(1972). J.Bonn, G.Huber, H.J.Kluge, E.W.Otten. Proc.Intern. Conf. on Nuclear Physics, Munich, vol. 1, p. 318 (1973).
2. N.Rud, O.Ward, H.R.Andrews, R.L.Graham, J.S.Geiger. Phys.Rev.Lett., 31, 1421 (1973).
3. D.Proetel, R.M.Diamond, P.Kienle, J.R.Leigh, K.H.Maier, F.S.Stephens. Phys.Rev.Lett., 31, 896 (1973).
4. A.Faessler, U.Gotz, B.Slavov, T.Ledergerber. Phys. Lett., 39B, 579 (1972).
5. M.Gallian, J.Letessier, H.Flocard, P.Quentin. Phys.Lett., 46B, 11 (1973).
6. S.G.Nilsson. J.R.Nix, P.Moller, I.Ragnarsson. Nucl. Phys., A222, 221 (1974).
7. M.Brack, J.Damgaard, A.S.Jensen, H.C.Pauli, V.M.Strutinsky, C.Y.Wong. Rev.Mod.Phys., 44, 320 (1972).
8. V.V.Pashkevich. Nucl.Phys., A169, 275 (1971).
9. V.G.Soloviev, U.M.Fainer. Izv.Akad. Nauk SSSR, Ser. Fiz., 36, 698 (1972).
10. D.Proetel, R.M.Diamond, F.S.Stephens. Phys.Lett., 48B, 102 (1974).
11. D.Proetel, R.M.Diamond, F.S.Stephens. Proc.Intern. Conf. on Reactions Between Complex Nuclei, Nashville, USA, vol. 1, p. 162 (1974).
12. J.H.Hamilton et al. *ibid*, p. 178.

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