

ОБЪЕДИНЕННЫЙ  
ИНСТИТУТ  
ЯДЕРНЫХ  
ИССЛЕДОВАНИЙ  
ДУБНА



26/8-74

S-94

E2 - 8018

Yu.S.Surovtsev, F.G.Tkebuchava.

3363/2-74

COMPENSATION EFFECT  
IN PION PHOTOPRODUCTION  
AND RELATED PROCESSES  
AND POSSIBILITY OF INVESTIGATING  
HADRON ELECTROMAGNETIC FORM FACTORS.

**1974**

ЛАБОРАТОРИЯ  
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

**Yu.S.Surovtsev, F.G.Tkebuchava.\***

**COMPENSATION EFFECT  
IN PION PHOTOPRODUCTION  
AND RELATED PROCESSES  
AND POSSIBILITY OF INVESTIGATING  
HADRON ELECTROMAGNETIC FORM FACTORS.**

**Submitted to *JΦ***

---

\* Tbilisi State University.

In single photon approximation pion photoproduction and related processes:

$$\gamma + N \rightarrow \pi + N, \quad (I)$$

$$e + N \rightarrow e + \pi + N, \quad (II)$$

$$\pi + N \rightarrow e^+ + e^- + N, \quad (III)$$

are described by the unique hadron electromagnetic current  $J_\mu(s, t, \lambda^2)$  and they correspond to different values of virtual photon mass  $\lambda^2 = k_0^2 - \vec{k}^2$ . The first process corresponds to  $\lambda^2 = 0$ , electroproduction (II) corresponds to the space-like values,  $\lambda^2 < 0$ , and electron pair production (III) to the time-like values,  $\lambda^2 > 0$ .

The theory of pion photoproduction on nucleon, within the framework of dispersion relations, has been elaborated in papers <sup>/1-4/</sup> where the main features of the process were revealed. In subsequent papers various approximations for specifying  $\pi N$  final-state interaction effects were used (see, for example, Review <sup>/5/</sup>). From these papers it becomes obvious, however, that at the present time only the Born terms for the process amplitude (I) are well known while the calculation of the remainder part of the amplitude requires some model considerations. The same model uncertainties propagate for processes (II) and (III). Besides, the  $\lambda^2$ -dependence makes the situation more complicated. At the same time owing to the amplitude dependence on  $\lambda^2$  processes (II) and (III) represent the most interest since this dependence is associated with the fundamental hadron characteristics - electromagnetic form factors.

The analysis of experiment on  $\pi^+$ -electroproduction in the first resonance region <sup>/6/</sup> shows that the extracted values of pion form factor essentially depend on the applied models. The picture can be regarded as a better one in the experiments at higher energies <sup>/7/</sup>, where due to a great contribution of the Born terms at small  $t$  the dependence on a model becomes weaker <sup>/8/</sup>.

Information on pion and nucleon electromagnetic form factors in the time-like region can in principle be obtained from pair production process (III). Moreover, the time-like values of  $\lambda^2$  have no lower limit in this process ( $0 \leq \lambda^2 \leq [\sqrt{s} - M]^2$ ) in contrast to the processes  $e^+ + e^- \rightarrow \pi^+ + \pi^-$  and  $e^+ + e^- \rightarrow N + \bar{N}$  where the  $\lambda^2$ -values are restricted to the limits  $4m_\pi^2 \leq \lambda^2 \leq s$  and  $4M^2 \leq \lambda^2 \leq s$  respectively. Process (III) has been observed at Dubna <sup>/9/</sup> where the estimations of electromagnetic form factor  $F_\pi$  and the isovector Dirac nucleon form factor  $F_1$  were obtained. However, in this case the problem of the result dependence on the model remains still open. Theoretical aspects of process (III) and the possibility of investigating hadron electromagnetic form factors were considered in papers <sup>/10-13/</sup>. In <sup>/10/</sup> it is shown that the amplitude of the pair production process in the  $N^*(1236)$ -resonance region is characterized by a quasi-threshold behaviour at  $\lambda^2 \rightarrow (\sqrt{s} - M)^2$ . In this case the amplitude is determined by the Born terms. But at higher energies even quasi-threshold region rescattering effects become essential and the description of process (III)-model-dependent <sup>/17/</sup>.

In <sup>/13/</sup> it is offered to apply the electric Born model to process (III) at high energies and small  $t$ , where the cross sections of charged pion photoproduction on nucleons <sup>/14/</sup> and process  $\pi^- + p \rightarrow \rho^0 + n$  <sup>/15/</sup> are mainly determined by the electric Born terms. However, this model also gives no complete quantitative agreement with the experiments. As it follows from above it is rather difficult to extract unambiguous information about electromagnetic form factors of hadrons from the data of processes (II) and (III) because of uncertainties in the theoretical model that is used.

We suggest that the problem should be considered

differently: to find out such a region of variables  $s, t, \lambda^2$  where the cross sections of processes (I - III) are described only by the Born terms.

Let us write the differential cross sections of virtual photoproduction as a sum of the two terms:

$$\frac{d\sigma}{dt} = \frac{d\sigma^{\text{Born}}}{dt} + \Phi(s, t, \lambda^2), \quad (1)$$

where the first term is the Born one and  $\Phi(s, t, \lambda^2)$  takes into account the final-state interaction and its interference with the Born part of the amplitude.

To establish the conditions under which

$$\Phi(s, t, \lambda^2) = 0, \quad (2)$$

we use the existence theorem for implicit functions. By this theorem if equation (2) allows the solution  $s = s_0$ ,  $t = t_0$ ,  $\lambda^2 = \lambda_0^2$  and function  $\Phi(s, t, \lambda^2)$  and its partial derivatives of the first order are continuous in the vicinity of the point  $M_0(s_0, t_0, \lambda_0^2)$  and the derivative  $\Phi_t$  at this point is different from zero:

$$\Phi_t(s_0, t_0, \lambda_0^2) \neq 0, \quad (3)$$

then there exists the only function

$$t = f(s, \lambda^2), \quad (4)$$

which satisfies equation (2) in some vicinity of the point  $M_0$  and it takes the value  $t = t_0$  at  $s = s_0$ ,  $\lambda^2 = \lambda_0^2$ . This function and its partial derivatives are continuous in the vicinity of the point  $M_0$ .

According to this theorem if we observe at least one point in the space of  $s, t, \lambda^2$ , where the effects of re-scattering and their interference with the Born terms compensate each other then, since the cross section is continuous in the physical region, there is a surface of compensation in this space on which the cross section is the Born one. The intersections of this surface with every plane  $\lambda^2 = \text{const}$  define some curves in the plane  $s, t$  each

of them being characterized by its own value of  $\lambda^2$  and cross section (1) being the Born one along them; thus we obtain a one-parameter set of the compensation curves with  $\lambda^2$  as a parameter.

Let us consider the curve  $\lambda^2 = 0$  in more detail, i.e., pion photoproduction process (I). We shall use  $x = \cos \theta$  instead of variable  $t$ ;  $\theta$  - scattering angle in the c.m.s. If the compensation takes place then in the plane  $x, s$  we ought to have the compensation curve  $x = f(s)$  continuous in the physical region. Generally speaking, there may be several curves of this kind. This follows from the fact that the function  $\Phi(s, x)$  can always be represented as a polynomial in powers of  $x$ :

$$\Phi(s, x) = \sum_i a_i(s) x^i \quad (5)$$

and the equation  $\sum_i a_i(s) x^i = 0$  can have several real roots  $x_j = f_j(s)$ ,  $j = 1, 2, \dots$ . However, due to the theorem mentioned above the curves do not intersect since the only curve can pass through the given point.

In the case of pion photoproduction the available experimental data are sufficient to construct the compensation curves<sup>16</sup>. For this purpose it is necessary to find out the intersection points of the experimental differential cross section with the calculated Born cross section and plot these points in the plane  $(x, s)$  or  $(\theta, s)$ . In figures 1 and 2 the compensation curves for charged  $\pi^-$ -photoproduction are shown. Calculating the Born cross section we take into account the interaction of photon only with a charge. Therefore along these curves the experimental cross sections are completely determined by the electric Born terms. One can see that at high energy the compensation curves pass through small angles in accordance with the electric Born model<sup>14</sup>. Figures 1 and 2 show that the compensation curves extend a range of applicability of the electric Born model onto the whole energy range giving a more complete and transparent picture. Of course, there are also the compensation curves corresponding to the total Born amplitude.

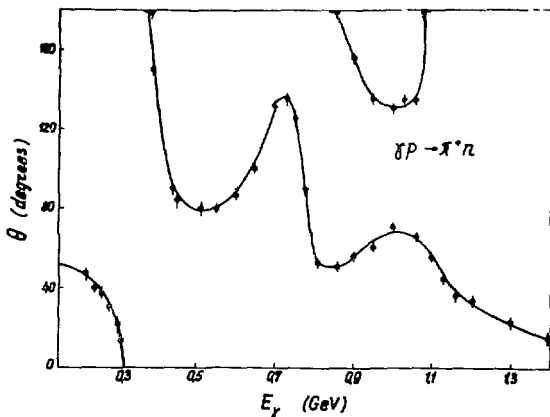


Fig. 1. The compensation curves for  $\gamma p \rightarrow \pi^+ n$ . Along these curves differential cross section is described by the electric Born model. At high energies the compensation curve passes through the scattering angles corresponding to the value of the invariant momentum transfer

$t = -0.04 \left(\frac{\text{GeV}}{c}\right)^2$ . The experimental data are taken from the Review <sup>[16]</sup>.

Now, if one assumes that for processes (II) and (III), when  $\lambda^2 \neq 0$ , the compensation curves are not much different from the corresponding curves for photoproduction ( $\lambda^2 = 0$ ) then along these curves the dependence of the description of the processes on a model reduces to minimum. Plausibility of this assumption is justified by successful application of the electric Born model to the description of the process  $\pi^- + p \rightarrow \rho^0 + n$  at high energies and small angles  $|t| \lesssim 2m_\pi^2$  [15]. And in any case the compensation curves help one to reveal the optimal experimental conditions for studying the form factors  $F_\pi(\lambda^2)$  and  $F_\rho(\lambda^2)$  in processes (II) and (III).

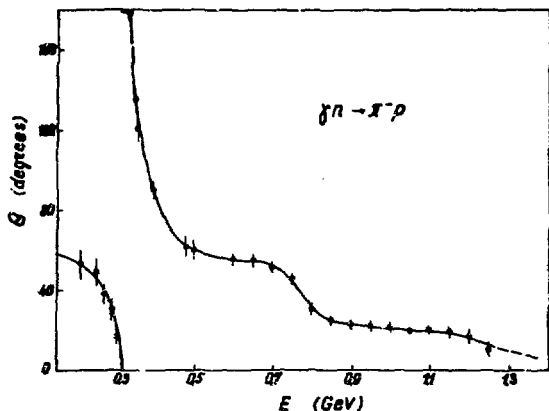


Fig. 2. The compensation curves for  $\gamma n \rightarrow \pi^- p$  (see caption of Figure 1).

We wish to thank V.A.Mescheryakov for his interest in this work and valuable comments; A.M.Baldin, S.M.Bilenky, T.D.Blokhintseva, S.B.Geracimov, E.M.Leikin and L.L.Nemenov for helpful discussions.

#### References

1. G.F.Chew, M.L.Goldberger, F.E.Low and Y.Nambu. Phys.Rev., 106, 1345 (1957).
2. A.A.Logunov, A.N.Tavkhelidze, L.D.Solovyov. Nucl. Phys., 4, 427 (1957).
3. A.M.Baldin. Zh.Eksp. i Teor. Fiz., 38, 579 (1960). A.M.Baldin, A.I.Lebedev. Nucl.Phys., 40, 44 (1963).
4. J.S.Ball. Phys.Rev., 124, 2014 (1961).
5. A.Donnachie. Hadron Interactions of Electrons and Protons. Proceedings of the 11th Session of Scottish



- Universities Summer School in Physics, p. 106, New York, Academic Press, 1971.
6. C.W.Akerlof et al. *Phys.Rev.*, 163, 1483 (1967).  
C.Mistretta et al. *Phys.Rev.*, 184, 1437 (1969).
  7. C.N.Brown et al. *Phys.Rev.*, D8, 92 (1973).  
C.Driver et al. *Phys.Lett.*, 35B, 77 (1971); *ibid.*, 35B, 81 (1971).  
P.S.Kumer et al. *Lett. Nuovo Cim.*, 1, 1026 (1971).
  8. F.A.Berends. *Phys.Rev.*, D1, 2590 (1970).
  9. S.F.Berezhnev et al. *Yad.Fiz.*, 18, 102 (1973).
  10. Yu.S.Surovtsev, F.G.Tkebuchava. *Yad.Fiz.*, 16, 1204 (1972).
  11. V.A.Matveev, R.M.Muradyan, A.N.Tavkheldidze. "Particles and Nucleus", vol. 2, part 1, p. 5. Atomizdat, Moscow, 1971.
  12. R.J.Oakes. *Nuovo Cim.*, 44A, 440 (1966).  
Yu.S.Surovtsev, F.G.Tkebuchava. JINR Preprint P2-4561, P2-4524, Dubna, 1969.  
A.V.Tarasov, L.G.Tkachev. JINR Preprint, P2-4970, Dubna, 1970.  
M.Karatshentzeff, G.Cochard, P.Kessler, B.Roehner. *College de France, P.A.M.*, 71-05, 1971.  
Yu.V.Kulish. *Yad.Fiz.*, 16, 1102 (1972).  
G.I.Smirnov, N.M.Shumeiko. *Yad.Fiz.*, 17, 1266 (1973).
  13. A.M.Baldin, V.A.Suleimanov. *Phys.Lett.*, 37B, 305 (1971); JINR Preprint P2-7096, Dubna, 1973.
  14. B.Richter. Proc. 1967 Intern. Symposium on Electron and Photon Interactions at High Energies, Stanford, California, 1967.
  15. C.F.Cho, J.J.Sakurai. *Phys.Lett.*, 30B, 119 (1969).  
N.N.Achasov, G.N.Shestakov. *Yad.Fiz.*, 11, 1090 (1970).
  16. G. von Holtey. *Springer Tracts in Modern Physics*, 59, Springer-Verlag, Berlin, Heidelberg, New York, 1971, p. 3; H.Fischer, *ibid.* p. 188.
  17. A.Bietti, S.Petrarca. *Ist. Fis. G.Marconi*, N 541, Roma, 1974.

Received by Publishing Department  
on June 13, 1974.