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ON QUANTIZATION OF CHIRAL THEORIES

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Introduction

A recent development in quantization of nonlinear theories with the chiral dynamical symmetry [1] is of current interest [2-6] for further construction of quantum field theory of strong interactions. The physical results obtained in the chiral quantum theory in one-loop approximation, by means of the analytic (superpropagator) methods of regularization [7], are in good agreement with the experimental data on $\pi\pi$ scattering [4], pion electromagnetic form factor [5] and on the neutral-kaon mass difference.

According to paper [8] where it has been shown that a dynamical symmetry of the chiral type is characteristic of the Einstein gravity theory, the chiral quantum theory can also be regarded as a simple model of the quantum theory of gravity.

One of the first steps towards constructing the chiral quantum theory is to formulate the chiral-invariant perturbation theory which does not depend on a choice of the coordinate system in the space of Goldstone fields [2,3].

There are two distinct approaches to this problem. In the first approach (see papers by Faddeev and Slavnov [2]) perturbation theory is formulated in terms of completely invariant currents. In the second one (work of Honerkamp et al.[3]). the starting point for construction of S-matrix in an arbitrary coordinate system is rearrangement of matrix elements due to "transfer" of derivatives from vertices onto propagators and subsequent reduction of the propagators to δ- functions.

Such a rearrangement of matrix elements which changes the structure of the Feynman diagrams is called "reduction"[9] or contraction of lines [3].

The reduced perturbation theory (i.e., the theory with all possible reductions), in the tree-diagram approximation, has been constructed by D.V.Volkov [9]. In this paper [9] it has been shown that taking into account of all possible reductions of pole diagrams to the effective contact interaction is equivalent, on the mass shell, to the explicitly covariant procedure of transition from an arbitrary to the normal coordinate system.

Therefore, in the tree-diagram approximation the reduced S-matrix in arbitrary coordinates coincides with the S-matrix in the normal coordinates, and reduction is a mechanism ensuring the equivalence theorem to be fulfilled.

The central question raised in [3,9] is: "What is a result of reductions in an arbitrary coordinate system?".

The equivalence theorem admits of a somewhat different approach to the reduction problem, viz.: "In what way can one formulate the perturbation theory without reductions ?".

The present paper is devoted to solving just this problem in quantum field theory with dynamical symmetry of the chiral type.

A foundation for formulating such a perturbation theory consists in a choice of coordinate system on the basis of the most simple properties of an interaction Lagrangian itself with respect to reductions.

In Section 2 a concise description is given for the method of phenomenological Lagrangians in terms of the Cartan forms which we shall extensively use in what follows.

In Section 3 the concept of reduction is introduced and the condition is formulated which allows one to select a coordinate system with the simple properties of Lagrangian with respect to reductions.

In Section 4 within the framework of functional integration method, the generating functional is obtained for the perturbation theory without reductions. The central point is taking into account of geometry of the curved space of Goldstone particles when dividing integration variables into "classical" and "quantized" fields.

In conclusion the Honerkamp covariant perturbation theory [3] is discussed.

The main results of this work are described briefly in [10].

2. Classical Theory

The construction of nonlinear realizations and on the basis of them of the invariants defining the structure of phenomenological Lagrangian for an arbitrary group of dynamical symmetry G can be carried out by a standard procedure [9,11]. In describing this procedure we shall follow the classical work by E.Cartan [12].

Let G be $(r+n)$-parameter semisimple symmetry group which leads to degenerating vacuum and producing the Goldstone particles, H be its maximal subgroup leaving vacuum invariant.

All infinitesimal transformations of the group G are linearly expressed through $n+\tau$ infinitesimal linear-independent transformations

$$G(da^i, d\eta^\alpha) = i\left[da^\kappa X_\kappa + d\eta^\alpha Y_\alpha\right] \tag{1}$$

$$\kappa = 1, \ldots, n \quad ; \quad \alpha = 1, \ldots, \tau .$$

Here a^κ, η^α are the group parameters, Y_α are the generators of transformations of the subgroup H , X_κ – the generators of transformations of the coset G/H which complements H to the whole group G , with the following algebra of commutation relations

$$\begin{aligned} [Y_\alpha, Y_\beta] &= iA^\gamma_{\alpha\beta} Y_\gamma \\ [X_\kappa, Y_\alpha] &= iB^i_{\kappa\alpha} X_i \\ [X_i, X_\kappa] &= iC^\alpha_{i\kappa} Y_\alpha . \end{aligned} \tag{2}$$

Consider the group parameters (a, η) as coordinates of a point in $(\tau+n)$ – dimensional space called the group space. To each point of this space (a, η) there is made correspond a transformation of the group $G_{(a,\eta)}$ and vice versa; to the identity transformation the point zero corresponds and to the transformation (1) the infinitesimal vector $(0, 0; da, d\eta)$ of space.

Definition of the equality of vectors in the group space makes it possible to introduce the transformation corresponding to an infinitesimal vector with origin in an arbitrary space point (a,η) . For instance, we suppose that a vector $(o,o;da',d\eta')$ equals a vector $(a,\eta\ ;\ a+da,\eta+d\eta)$ provided $G_{(a,\eta)}$ reduces to $G_{(oo)}=I$ and $G(a+da,\eta+d\eta)$ to $G(da',d\eta')$ through the same transfromation by the rule

$$G(da',d\eta')=G^{-1}_{(a,\eta)}G_{(a+da,\eta+d\eta)}\equiv G^{-1}_{(a,\eta)}\,dG_{(a,\eta)}$$
$$G^{-1}_{(a,\eta)}\,dG_{(a,\eta)}=i\left[\omega^i(a,\eta,da,d\eta)X_i+\theta^\alpha(a,\eta,da,d\eta)Y_\alpha\right] \tag{3}$$

Equation (3) proceeding from the finite group transformations defines the Cartan forms ω^i,θ^α which are of primary importance in the method of phenomenological Lagrangians. This method consists in that the parameters a^i are identified with the Goldstone fields and the group transformations

$$G(a',\eta')=G(g)\,G(a,\eta)\ , \tag{4}$$

where $G(g)$ is an arbitrary group element, define the non-linear realization of group on the coordinates of space of the Goldstone particles a^i

$$a^{i'}=a^{i'}(a,g). \tag{5}$$

From definition (3) it follows that the Cartan forms ω,θ are invariant relative to the group transformations (4),(5).

The forms ω^i determine with respect to some basis components of an infinitesimal displacement da^i from a point <u>a</u> to a point $a+da$, the forms θ^α define a change of the basis and are used to determine the covariant differentiation of a various "geometrical" quantities which are identified with fields of particles interacting with the Goldstone particles.

The Lagrangian invariant under the group transformations (5) is expressed via the Cartan forms in the following way:

$$\mathcal{L} = \tfrac{1}{2}\,\omega^i(a,da)\,\omega^i(a,da) + \mathcal{L}_0\left(\psi, d\psi + \theta^\alpha(a,da)T_\alpha\psi\right) . \tag{6}$$

Here

$$\begin{aligned} \omega^i(a,da) &= \omega^i(a,0,da,0) \\ \theta^\alpha(a,da) &= \theta^\alpha(a,0,da,0) \\ d\psi = \frac{\partial}{\partial x_\mu}\psi \quad &; \quad da = \frac{\partial}{\partial x_\mu}a \end{aligned} \tag{7}$$

$\mathcal{L}_0(\psi,d\psi)$ is the Lagrangian of free fields ψ which are classified over the linear representations T_α of subgroup H.

Let us find the Cartan form for the **finite** group transformation in the exponential **parametrization:**

$$G_a^N = e^{iX_\kappa a^\kappa} \quad ; \quad G_a \equiv G(a,0) , \tag{8}$$

which corresponds to the normal coordinates in space of the Goldstone field (*along geodesics*) [12,13] . For the equation (3) rewritten in the exponential form (8)

$$e^{-iX_\kappa a^\kappa}\, d\, e^{iX_\kappa a^\kappa} = i\left[\omega^i(a,da)X_i + \theta^\alpha(a,da)Y_\alpha\right] \tag{9}$$

one of the methods of solving this equation is to reduce eq. (9) to the so-called fundamental Cartan equations.

To **this** end, we introduce into (9) a **parameter** t by means of the substitution

$$a^\kappa \rightarrow a^\kappa t$$

and we get

$$e^{-iX_\kappa t a^\kappa} d\, e^{iX_\kappa t a^\kappa} = i\left[\omega^i(ta, tda) X_i + \theta^\alpha(ta, tda) Y_\alpha\right]. \quad (10)$$

Differentiating both sides of eq. (10) with respect to t we obtain the fundamental Cartan equations

$$\frac{\partial \omega^i}{\partial t} = da^i + a^\kappa \theta^\beta B^i_{\kappa\beta} \quad (11)$$

$$\frac{\partial \theta^\alpha}{\partial t} = a^j \omega^\ell C^\alpha_{j\ell}$$

with the zeroth boundary conditions

$$\omega^i(0,0) = \theta^\alpha(0,0) = 0, \quad (12).$$

where $B^i_{\kappa\beta}$, $C^\alpha_{j\ell}$ are the structure constants of group (2).

In general case the solution to eqs. (11) can be written as the series

$$\omega^i(ta, tda) = \sum_{n=0}^{\infty} \left(\mathcal{M}_a^n\right)^i_\ell da^\ell \frac{(-1)^n t^{2n+1}}{(2n+1)!}$$

$$(13)$$

$$\theta^\beta(ta, tda) = a^j C^\beta_{j\kappa} \sum_{n=0}^{\infty} \left(\mathcal{M}_a^n\right)^\kappa_\ell da^\ell \frac{(-1)^n t^{2n+2}}{(2n+2)!},$$

where

$$\left(\mathcal{M}_a^0\right)^i_\kappa = \delta_{i\kappa}\,;\ \left(\mathcal{M}_a\right)^i_\ell = -B^i_{\kappa\beta} a^\kappa C^\beta_{j\ell} a^j\,;\ \left(\mathcal{M}_a^2\right)^i_\ell = \left(\mathcal{M}_a\right)^i_\kappa \left(\mathcal{M}\right)^\kappa_\ell\,; \ldots$$

For the SU(2)xSU(2) theory (for dimensionless variables $a^i = \pi^i / F_\pi$)

$$\left(\mathcal{M}_a\right)^i_\ell = -\varepsilon_{i\kappa\beta}\varepsilon_{\beta j\ell}\, a^\kappa a^j = a^2\left(\delta_{i\ell} - \frac{a_\ell a_i}{a^2}\right)$$

$$\left(\mathcal{M}_a^n\right)^i_\ell = a^{2n}\left(\delta_{i\ell} - \frac{a_\ell a_i}{a^2}\right) \quad ; \quad a = \sqrt{a^i a^i}$$

the series (13) are summed and we have

$$\omega^i = F_\pi \left[da^i + \left(\delta_{il} - \frac{a_i a_l}{a^2}\right)\left(\frac{\sin a}{a} - 1\right) da^l \right]$$
$$\theta^\beta = - a^j da^k \varepsilon_{\beta jk} \frac{\cos a - 1}{a^2} . \tag{14}$$

Here F_π = 92 MeV, $\varepsilon_{\beta jk}$ is the antisymmetric tensor, $\varepsilon_{123} = 1$

3. Reductions

A specific feature of the chiral Lagrangian is the presence of derivatives in it. The fact that the derivatives are present in an interaction Lagrangian of the type

$$\mathcal{L}^I = da^i da^j g^I_{ij}(a) + da^i \mathcal{B}_i(a, \Psi) \tag{15}$$

may, generally speaking, lead to rearrangements of matrix elements in perturbation theory due to integrating by parts ("transfer" of derivatives) and reducing some propagators to δ- functions. Such a rearrangement which changes the structure of the Feynman diagrams, in terminology of ref. [9], is called reduction (or contraction of lines[3]).

The main purpose of our paper is to formulate the perturbation theory without reductions.

As a starting point of such formulation we suggest the choice of a Lagrangian (i.e. a coordinate system) proceeding from the most simple properties of this Lagrangian itself with respect to reductions.

Consider, for instance, a matrix element of the type

$$\langle 0 | T^* \left(\int d^4x : \mathcal{L}^I(x) : \right)^2 | 0 \rangle$$

It is obvious that reductions will be absent if after integrating by parts the integrand will not change, i.e., if the interaction Lagrangian obeys the condition

$$da^i\left[g^I_{ij}\,da^j + \beta_i(a,\psi)\right] = -a^i d\left[g^I_{ij}\,da^j + \beta_i(a,\psi)\right] . \qquad (16)$$

It is easy to prove that the coordinate system satisfying condition (16) does exist and it is unique. It is just the normal coordinate system (13).

Indeed, in these coordinates the derivatives of the Goldstone fields enter into the interaction Lagrangian in the form of a combination with the group structure constants

$$da^\ell C^\beta_{j\ell}\, a^j\, \Phi_\beta(a,\psi,da) \qquad (17)$$

(Φ_β stands for all the remaining factors).

The expression (17) satisfies the condition (16) due to antisymmetry of the group structure constants $C^\beta_{j\ell}$ in lower indices.

To pass over to some other coordinate system is made via the transformations

$$a = a' f(a') \quad ; \quad f(0) = 1 . \qquad (18)$$

The interaction Lagrangian in the normal coordinates (13) after the transformations (18) again obeys the condition (16). (In this sense, the Lagrangian in normal coordinates, before and after the transformations (18), resembles the Lagrangian without derivatives in nonchiral theories of the $\lambda\varphi^4$ type, where there are no reductions as derivatives are absent).

However, the new interaction Lagrangian contains, in addition to the transformed expression (17), also the "kinetic" part which arises due to the transformations (18) in the "free" Lagrangian

$$\frac{1}{2} da_i da_i = \frac{1}{2} da_i' da_i' + \frac{1}{2}\left\{ d\left[a_i' f(a')\right] d\left[a_i' f(a')\right] - da_i' da_i' \right\} . \qquad (19)$$

It is just the latter term in (19) which violates the condition (16) and, as in the $\lambda \varphi^4$ theory, is responsible for reductions.

4. Quantum Theory

As the generating functional for S-matrix it is convenient to use expressions in the form of the continual integral with sources [x]

$$S(\pi^{(in)}) = \frac{1}{N}\int \mu(a)\prod_x da \, \exp\left\{ i\int d^4x \left[\mathcal{L}(\omega(a,da), \theta(a,da), \psi) - da^i d\pi_i^{(in)}\right]\right\} . \qquad (20)$$

Here N is the normalization, $\pi^{(in)}$ the asymptotic field (source), $\mu(a)\prod_x da$ the invariant measure over the group, i.e.,

$$\mu(a)\prod da = \mu(a')\prod da' \qquad (21)$$

if

$$G_{a'} \rightarrow G_b G_a , \qquad (22)$$

where G_b is a group transformation.

In integral (20) one can take any integration variables. In our case, following Sect.3, we take the normal coordinates (13):

$$\mathcal{L} \rightarrow \mathcal{L}^N \quad ; \quad \mu(a) \rightarrow \mu^N(a) \quad ; \quad G \rightarrow G^N .$$

Consider the quasi-classical expansion of functional (20). For this expansion the change is made for integration variables separating the "classical" fields φ obeying the equation

$$\frac{\delta \mathcal{L}(\varphi)}{\delta \varphi^i} = - d^2 \pi_i^{(in)}$$

and "quantized" fields Γ over which integration is carried out. The usual change of variables

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[x] The fields ψ in (6) will be considered to be classical.

$$\vec{q} \rightarrow \vec{\varphi} + \vec{\Gamma} \tag{23}$$

breaks the condition (16). We should make a change of variables such that:

1. the condition of absence of reductions with respect to the fields Γ be fulfilled;

2. the Lagrangian in (20) at $\Gamma = 0$ be the Lagrangian of the "classical" fields φ in the normal coordinates. In this case the generating functional for S-matrix in the tree-diagram approximation

$$S(\pi^{in})_{tree} = exp\left\{ i\int d\tilde{x}\left[\mathcal{L}(\omega^N(\varphi, d\varphi), \theta^N(\varphi d\varphi) - d\varphi^i d\pi_i^{in} \right]\right\} \tag{24}$$

according to results of [9], gives the matrix elements without reductions.

A natural way for separating the classical fields, without violating the condition (16), is to use the geometric properties of the curved Riemann space of the Goldstone particles, namely to understand the sum of vectors (23) as the addition of vectors in the curved isospace of the Goldstone particles (addition of vectors in the quotient space G/H), i.e.,

$$G_a^{\prime N} \rightarrow G_\varphi^{\prime N} G_\Gamma^{\prime N} \tag{25}$$

where $G_a^{\prime N}$ is defined by (8).

Transformation (25) has simple geometrical interpretation. It gives the normal coordinate system with the origin at the point φ , coordinates of the point φ themselves being also the normal coordinates.

The Cartan forms are obtained in the new coordinates Γ substituting (25) into (9)

$$[G_\varphi^N G_r^N]^{-1} d[G_\varphi^N G_r^N] = \bar\omega^i(\Gamma,d\Gamma|\varphi,d\varphi)X_i + \bar\theta^\alpha(\Gamma,d\Gamma|\varphi,d\varphi)Y_\alpha \quad (26)$$

Using the substitution with parameter t

$$\Gamma^i \to t\Gamma^i$$

and differentiating both sides of (26) with respect to t we find the fundamental Cartan equations, the same as in the classical case, eq. (11),

$$\frac{\partial\bar\omega^i}{\partial t} = d\Gamma^i + \Gamma^k\bar\theta^\beta B^i_{k\beta} \quad (27)$$
$$\frac{\partial\bar\theta^\alpha}{\partial t} = \Gamma^j\bar\omega^k C^\alpha_{jk}$$

but with the nonzero boundary conditions in the normal coordinates

$$\bar\omega^i(0,0|\varphi,d\varphi) = \omega^i(\varphi,d\varphi)\ ; \quad \bar\theta^\alpha(0,0|\varphi,d\varphi) = \theta^\alpha(\varphi,d\varphi). \quad (28)$$

Solution to these equations has the form

$$\bar\omega^i\Big|_{t=1} = \sum_{n=0}^\infty(-1)^n(\mathcal{M}_\Gamma^n)^i_\ell\left[\frac{\omega^\ell}{(2n)!} + \frac{(\mathcal{D}\Gamma)^\ell}{(2n+1)!}\right] \quad (29)$$
$$\bar\theta^\beta\Big|_{t=1} = \Gamma^j C^\beta_{jk}\sum_{n=0}^\infty(-1)^n(\mathcal{M}_\Gamma^n)^k_\ell\left[\frac{\omega^\ell}{(2n+1)!} + \frac{(\mathcal{D}\Gamma)^\ell}{(2n+2)!}\right],$$

where

$$(\mathcal{D}\Gamma)^i = d\Gamma^i + \Gamma^k B^i_{k\beta}\theta^\beta(\varphi,d\varphi)\ ; \quad (\mathcal{M}_\Gamma)^i_\ell = -B^i_{k\beta}\Gamma^k C^\beta_{j\ell}\Gamma^j .$$

For the SU(2) x SU(2) chiral theory in the dimensionless variables (see (14)) we get

$$\bar\omega^i = \omega^i + F_\pi(\mathcal{D}\Gamma)^i + \left(\delta_{i\ell} - \frac{\Gamma_i\Gamma_\ell}{\Gamma^2}\right)\left[(\mathcal{D}\Gamma)^\ell F_\pi\left(\frac{\sin\Gamma}{\Gamma} - 1\right) + \omega^\ell(\cos\Gamma - 1)\right]$$
$$\bar\theta^\beta = \Gamma^j\varepsilon_{\beta jk}\left[\frac{\sin\Gamma}{\Gamma}\frac{\omega^k}{F_\pi} + (\mathcal{D}\Gamma)^k\frac{1-\cos\Gamma}{\Gamma^2}\right] \quad (30)$$
$$(\mathcal{D}\Gamma)^i = d\Gamma^i + \varepsilon_{ik\beta}\Gamma^k\theta^\beta(\varphi,d\varphi)\ ; \qquad \Gamma = \sqrt{\Gamma^i\Gamma^i},$$

where $\omega^i(\varphi, d\varphi)$, $\theta^\beta(\varphi, d\varphi)$ are defined by (14).

Just as the Cartan forms (13), the forms (29) satisfy the condition (16) in virtue of the construction of these forms with the help of the structure constants antisymmetric in lower indices (see eq. (17)).

Thus, allowing for the invariance of the measure $\mu(a)\prod da$ under the transformations (25) (see (21), (22) the generating functional for S-matrix without reductions in the variables (25) takes the form

$$S(\pi^{(in)}) = \int \mu^N(\Gamma) \prod_x d\Gamma \exp\left\{ i \int d^4x \left[\mathcal{L}\left(\bar{\omega}(\Gamma, d\Gamma | \varphi, d\varphi), \bar{\theta}(\Gamma, d\Gamma | \varphi, d\varphi), \psi\right) - d\varphi^i d\pi_i^{(in)} \right] \right\}. \qquad (31)$$

So the generating functional for S-matrix without reductions is the generating functional in the normal coordinates for the "quantized" fields Γ with the origin at the point, the normal coordinates of which are the "classical fields" φ.

In principle, a coordinate system may be taken arbitrary for the fields φ , i.e. an arbitrary parametrization may be used for the transformation G_φ when dividing the variables a into φ and Γ in (25). This will result in forms $\bar{\omega}$, $\bar{\theta}$ satisfying the same fundamental equations but with the boundary conditions (28) in an arbitrary coordinate system.

The quasiclassical expansion of the generating functional (31) with the covariant dependence on the fields φ is just a generalization to arbitrary chiral dynamics groups and to interactions with arbitrary particles of the so-called covariant perturbation theory by Honerkamp [3] which corresponds to the choice of the normal coordinates for the fields Γ at the point φ in arbitrary coordinates.

The method we have presented for formulating such a perturbation theory is essentially simpler than the apparatus of classical differential geometry used in [3] .

However, the latter is indispensable for the case when a chiral Lagrangian contains noninvariant the pion-mass type terms (see paper by M.K. Volkov and the author [4]).

Conclusion

In this paper we have found the generating functional for S-matrix (31) in the perturbation series expansion of which there are no reductions resulting in change of the structure of the Feynman diagrams (contractions of lines).

To formulate such a theory it suffices to take the normal coordinate system and to allow for the geometrical properties of the curved space of the Goldstone particles, when dividing integration variables into the "classical" and "quantized" fields. By the equivalence theorem, the reduced perturbation theory in an arbitrary coordinate system coincides with the perturbation theory without reductions (31) [3,9] . In this sense the perturbation theory without reductions (31) is the invariant perturbation theory and it is rather useful in applying of the regularization methods based on the selection of a definite class of diagrams, either with a fixed number of vertices [4-7] or with a fixed number of loops [14] . One can say that the perturbation theory without reduction in the coordinates (25),(29) is as simpler and more convenient than a perturbation theory in other coordinates, as the $\lambda\varphi^4$ theory is simpler and more convenient than any other equivalent theory derived from the $\lambda\varphi^4$ theory by the transformation $\varphi = \varphi' f(\varphi')$, $f(0) = 1$.

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also expresses his deep gratitude to D.V.Volkov for invitation to the Kharkov Physical Technical Institute where this work has been completed.

APPENDIX

In the Appendix we write down the minimal Lagrangian for π-N interaction allowing for the Gursey transformation [15] $\psi = \exp(\gamma_5 \frac{\tau\varphi}{2})N$.
The Lagrangian for classical fields φ (in dimensionless variables) is

$$\mathcal{L} = \tfrac{1}{2} F_\pi^2 \omega^i(\varphi, d\varphi)\omega^i(\varphi d\varphi) + \bar{\psi}[\hat{\partial} + \gamma_5 \tau_i \omega^i(\varphi, \hat{d}\varphi) + i\tau_k \theta^k(\varphi \hat{d}\varphi)]\psi - M\bar{\psi}\psi =$$
$$= \tfrac{1}{2} F_\pi^2 \omega^i(\varphi, d\varphi)\,\omega^i(\varphi, d\varphi) + \bar{N}\hat{\partial}N - M\bar{N}\exp\{\gamma_5 \tau_i \varphi_i\}N, \tag{A.1}$$

where $\omega^i\ \theta^k$ are defined by (14) ; $\hat{\partial} = \gamma_\mu \frac{\partial}{\partial x_\mu}$, $\gamma_5^2 = -1$.

The Lagrangian for classical and quantized fields is

$$\mathcal{L}(\varphi, \Gamma, N) = \frac{1}{2} F_\pi^2 \bar{\omega}^i(\Gamma, d\Gamma|\varphi, d\varphi)\bar{\omega}^i(\Gamma, d\Gamma|\varphi, d\varphi) + \bar{N}\hat{\partial}N -$$
$$- M\bar{N}\exp\{\gamma_5 \tau_i \frac{\varphi_i}{2}\}\exp\{\gamma_5 \tau_i \Gamma_i\}\exp\{\gamma_5 \tau_i \varphi_i/2\}N, \tag{A.2}$$

where $\bar{\omega}^i(\Gamma, d\Gamma|\varphi, d\varphi)$ are given by (30).

If one applies the superpropagator method of regularization[7], which leads to the normal ordering of fields Γ (see paper by the author [3]) to calculate matrix elements, it is convenient to employ the following expression for the generating functional

$$S(\pi^{in}, N) = \langle o_\Gamma | T^* \exp\{ i\int d^4x [:\mathcal{L}^I(\varphi, \Gamma, N): - d\varphi^i d\pi_i^{(in)}]\} | o_\Gamma \rangle \tag{A.3}$$

where $\mathcal{L}^I(\varphi, \Gamma, N) = \mathcal{L}(\varphi, \Gamma, N) - \mathcal{L}_0(\Gamma, N)$,

$\mathcal{L}_0(\Gamma, N)$ is the "free" Lagrangian

$\langle o_\Gamma | \quad | o_\Gamma \rangle$ — the vacuum average over fields Γ

$$\langle o | T^* [\Gamma_i(x) \Gamma_k(y)] | o \rangle = \frac{1}{iF_\pi^2} \Delta^c(x-y)\delta_{ik} . \tag{A.4}$$

References:

1. S.Weinberg, Phys.Rev.Lett.,18, 507 (1971).
2. L.D.Faddeev, A.A.Slavnov, Teor.Mat.Fiz., 8,297 (1971); A.A.Slavnov, Nucl.Phys.,B31, 301 (1971); S.Gerstein, R.Jackiw, B.W.Lee, S.Weinberg, Phys.Rev.,D3, 2486 (1971).
3. J.Honerkamp, Nucl.Phys.,B36, 130 (1972); G.Ecker and J.Honerkamp, ibid.,B52, 211 (1973); J.Honerkamp et al. Bonn University preprint, July (1973). V.N.Pervushin, JINR E2-7540, Dubna (1973).
4. H.Lehman and H.Trute, Nucl.Phys. 52, 380 (1973). G.Ecker and J.Honerkamp, ibid., B62, 509 (1973). V.N.Pervushin, M.K.Volkov, JINR E2-7661, Dubna (1974).
5. M.K.Volkov, V.N.Pervushin, JINR E2-7283, Dubna (1973).
6. V.N.Pervushin, M.K.Volkov, JINR E2-7835, Dubna (1974). G.V.Efimov, JETP 44, 1207 (1963). E.S.Fradkin, Nucl.Phys., B49, 624 (1963).
8. A.B.Borisov, V.I.Ogievetsky, JINR E2-7684, Dubna (1974).
9. D.V.Volkov, Preprint ITF-69-75, Kiev, 1969, in "Particles and Nucleus", 4, 3 (1973).
10. V.N.Pervushin. JINR preprint P2-7644, Dubna, (1973).
11. S.Coleman, G.Wess, B.Zumino, Phys.Rev.,177, 2239 (1969). G.C.Callan et al. ibid.,177, 2447 (1969).
12. E.Cartan, Journal Math.pures et appl.9, v.6, 1 (1927).
13. E.Cartan, Lecons sur la geometrie des espaces de Riemann, Paris, 3785, 1946.
14. S.Coleman, E.Weinberg, Phys.Rev.,D7, 1888 (1973).
15. P.Chang, F.Gursey, Phys.Rev.,164, 1752 (1967).

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