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APPROXIMATE DESCRIPTION OF MULTIPLE SCATTERING IN NUCLEUS-NUCLEUS COLLISIONS AT HIGH ENERGIES

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High-energy collisions of complex nuclei are expected to provide a new tool for probing nuclear structure[1]. The theoretical studies of such interactions have generally been based upon the Glauber model of multiple scattering[2]. However, the exact evaluation of the Glauber scattering amplitude is very difficult; complete calculations have been carried out for light systems, like ^{4}He-^{4}He[1,3]. As the number of nucleons in the colliding nuclei increases, the evaluation of the full Glauber series of multiple scattering becomes intractable. That is why various approximations to the nucleus-nucleus scattering amplitude have been proposed[1,4-10]. By testing these approximate formulas it will be possible to gain a better understanding of mechanisms occurring in collisions between complex nuclei.

The discussion of ref.[9] has revealed interconnections between the approximations of the optical limit[1,6] and of the "rigid"[9] or "quasisoft"[9,4] projectile. All these approximations may be derived in a similar way, by suitable truncation in the space of intermediate excited states of the colliding nuclei. In this note we shall show that a recently proposed approximation of "swarm projectile"[10] belongs to the same category, being virtually equivalent to the approximation of "rigid target"[9]. We should also like to present a symmetric version of the rigid nucleus approximation which treats the colliding nuclei on an equal footing in compliance with the symmetry originally present in the Glauber model.

In the forthcoming discussion we shall consider only the nucleus-nucleus profile functions of elastic scattering; by evaluating their Fourier-Bessel transforms the scattering amplitudes can be obtained. The symmetric expression for the profile will be constructed from the three following approximations.

i) No-shadowing[6] or the optical limit[1] approximation where the intermediate excitations (their effect being referred to as the quasielastic shadowing[6]) in both colliding nuclei are neglected

$$\Gamma^{NS}(\underset{\sim}{b}) = 1 - \prod_{j=1}^{A}\prod_{k=1}^{B}\left[1 - \langle 0_A 0_B | \gamma_{jk}(\underset{\sim}{b} + \underset{\sim}{s}_{jA} - \underset{\sim}{s}_{kB}) | 0_A 0_B \rangle\right] . \quad (1)$$

The elementary profiles γ_{jk} depend on the transverse coordinates $\underset{\sim}{s}_{jA}$, $\underset{\sim}{s}_{kB}$ (i.e., in the plane perpendicular to the bisectrix of the c.m. scattering angle) of the constituent nucleons in the projectile nucleus A and in the target nucleus B; $|0_A\rangle, |0_B\rangle$ are the nuclear ground states.

ii) Rigid projectile[9] approximation where the virtual excitations of the projectile nucleus A are excluded:

$$\Gamma^{RP}(\underset{\sim}{b}) = 1 - \langle 0_B | \prod_{k=1}^{B}\left\{\prod_{j=1}^{A}\left[1 - \langle 0_A | \gamma_{jk}(\underset{\sim}{b} + \underset{\sim}{s}_{jA} - \underset{\sim}{s}_{kB}) | 0_A \rangle\right]\right\} | 0_B \rangle . \quad (2)$$

This expression must not be confused with the rigid-projectile approximation of ref.[4] (more correctly named as the quasisoft projectile approximation in [9]) which does admit some intermediate excitations in the projectile.

iii) Rigid target[9] approximation where the virtual excitations of the target nucleus B are excluded:

$$\Gamma^{RT}(\underset{\sim}{b}) = 1 - \langle 0_A | \prod_{j=1}^{A}\left\{\prod_{k=1}^{B}\left[1 - \langle 0_B | \gamma_{jk}(\underset{\sim}{b} + \underset{\sim}{s}_{jA} - \underset{\sim}{s}_{kB}) | 0_B \rangle\right]\right\} | 0_A \rangle . \quad (3)$$

All these approximations are quite appealing because of a relative ease to evaluate them. When $B > A$ it turns out to be profitable [9] to apply the approximation of rigid nucleus rather to the target than to the projectile. This follows from the fact that in heavier nuclei the correlations between nucleons (including those arising from the translational invariance[11]) are weaker resulting in reducing the size of quasielastic shadowing. It appears that eq.(3) is numerically more complicated than eq.(2) since it is easier to proceed with evaluation of a large power (B) of the polynomial composed of a few terms (A+1) than vice versa (when $B \rightleftarrows A$). However, we wish to point out that the profile of the rigid target Γ^{RT} can be cast in a form which is convenient for numerical calculation. For sufficiently large B, one can write:

$$\Gamma^{RT}(\underset{\sim}{b}) \cong 1 - \left\langle 0_A \right| \exp\left[-B \sum_{j=1}^{A} \left\langle 0_B \right| \gamma_{j1}(\underset{\sim}{b} + \underset{\sim}{s}_{jA} - \underset{\sim}{s}_{1B}) \left| 0_B \right\rangle \right] \left| 0_A \right\rangle \tag{4}$$

$$\cong 1 - \left\langle 0_A \right| \prod_{k=1}^{B} \left[1 - \sum_{j=1}^{A} \left\langle 0_B \right| \gamma_{j1}(\underset{\sim}{b} + \underset{\sim}{s}_{jA} - \underset{\sim}{s}_{1B}) \left| 0_B \right\rangle \right] \left| 0_A \right\rangle . \tag{4'}$$

Eq.(4) can easily be recognized as the leading, and the only readily calculable term in the so-called "swarm projectile" model of Fäldt and Hulthage[10]. Thus, their approximation turns out to be closely related to the approximations based upon the concept of quasielastic shadowing[9]. This conclusion can be confirmed by a numerical comparison of eqs.(3) and (4'). To this end a simple nuclear model has been used:

$$\begin{aligned} \left| \left\langle 0_A \middle| \underset{\sim}{r}_{1A}, \dots, \underset{\sim}{r}_{AA} \right\rangle \right|^2 &= \prod_{j=1}^{A} \rho_A(\underset{\sim}{r}_{jA}) , \\ \left| \left\langle 0_B \middle| \underset{\sim}{r}_{1B}, \dots, \underset{\sim}{r}_{BB} \right\rangle \right|^2 &= \prod_{k=1}^{B} \rho_B(\underset{\sim}{r}_{kB}) , \end{aligned} \tag{5}$$

with the single-particle densities ρ_A, ρ_B chosen as Gaussians:

$$\rho(r) = \pi^{-3/2} R^{-3} \exp[-r^2/R^2] \qquad (6)$$

As it is well known [12] the use of Gaussians allows one to simply impose the constraint of translational invariance on the nuclear densities (5).

The elementary profile has been assumed as independent of spin and isospin:

$$\gamma(b) = \frac{\sigma(1-i\alpha)}{4\pi a} \exp[-b^2/2a] \qquad (7)$$

which corresponds to a Gaussian q -dependence of the N-N elastic scattering amplitude. The parameters σ (total N-N cross-section) α (Re/Im ratio of the forward scattering amplitude) and a (slope) are, in general, energy dependent.

The results for the ^2H-^{12}C elastic scattering presented in the Table prove the equivalence of the rigid target and swarm projectile approximations. At the same time the comparison with the complete multiple scattering calculation shows that for large momentum transfers both the approximations considerably underestimate the effect of quasielastic shadowing.

To find a better approximation to the Glauber model we shall symmetrize [13] the rigid nucleus formulae (2) and (3). This can be done by making use of a probabilistic interpretation of the Glauber profile [1]. At high energies where the nuclear amplitudes are prevalently imaginary, the nucleus-nucleus profile describes, at a given impact parameter, the total probability of whichever interaction between constituent nucleons. Considering the probability

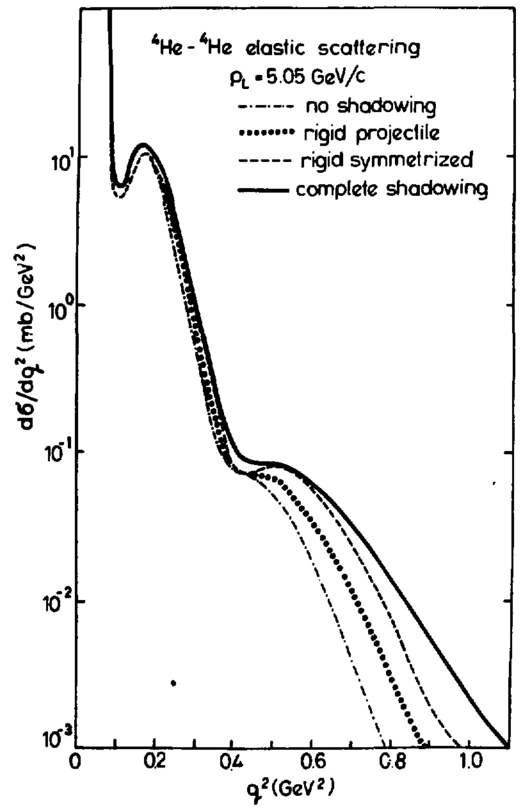
Table

q^2 $[GeV^2]$	$d\sigma/dq^2$ $[mb/GeV^2]$		
	rigid target	swarm projectile	complete shadowing
0.0	$0.174 \cdot 10^5$	$0.175 \cdot 10^5$	$0.181 \cdot 10^5$
0.1	$0.149 \cdot 10^2$	$0.150 \cdot 10^2$	$0.171 \cdot 10^2$
0.2	$0.446 \cdot 10^0$	$0.487 \cdot 10^0$	$0.566 \cdot 10^0$
0.3	$0.292 \cdot 10^{-1}$	$0.302 \cdot 10^{-1}$	$0.425 \cdot 10^{-1}$
0.4	$0.870 \cdot 10^{-2}$	$0.974 \cdot 10^{-2}$	$0.161 \cdot 10^{-1}$
0.5	$0.121 \cdot 10^{-2}$	$0.156 \cdot 10^{-2}$	$0.308 \cdot 10^{-2}$

The invariant differential cross-section for the ^2H-^{12}C elastic scattering in the function of squared momentum transfer. The rigid target (eq.3) and swarm projectile (eq.4') approximations are compared with the complete Glauber calculation using the nuclear densities (6) and elementary profiles (7) with the parameters: $R_2 = 2.28$ fm, $R_{12} = 1.93$ fm, $\sigma = 39.3$ mb, $\alpha = -0.4$, $a = 3.4$ GeV^{-2}.

of virtual excitations in colliding nuclei, one can write down the profile as follows:

$$\begin{aligned} \Gamma^{SYM} = \Gamma^{NS} &+ \left(\Gamma^{RP} - \Gamma^{NS}\right)\left[1 - \left(\Gamma^{RT} - \Gamma^{NS}\right)\right] \\ &+ \left(\Gamma^{RT} - \Gamma^{NS}\right)\left[1 - \left(\Gamma^{RP} - \Gamma^{NS}\right)\right] \\ &+ \left(\Gamma^{RP} - \Gamma^{NS}\right)\left(\Gamma^{RT} - \Gamma^{NS}\right). \end{aligned} \tag{8}$$



The invariant differential cross-section for the ^{4}He-^{4}He elastic scattering in the function of squared momentum transfer. The rigid nucleus approximation and its symmetric version (9) are compared with the complete Glauber calculation. For completeness, the no-shadowing approximation is also presented. The nuclear Gaussian radii are $R_A = R_B = 1.37$ fm. The N-N parameters are $\sigma = 39.3$ mb, $\alpha = -0,4$, $a = 3.4$ GeV^{-2}, corresponding to a 650 MeV/ nucleon collision.

The first term in the right-hand side represents the probability that none of the two nuclei is virtually excited, the second term corresponds to a situation where the target becomes excited but the projectile remains permanently in its ground state, the third term describes the opposite case, and the last term gives the probability of a simultaneous excitation of the colliding nuclei. Summing up these four contributions one obtains:

$$\Gamma^{SYM} = \Gamma^{RP} + \Gamma^{RT} - \Gamma^{NS} - \left(\Gamma^{RP} - \Gamma^{NS}\right)\left(\Gamma^{RT} - \Gamma^{NS}\right). \qquad (9)$$

The symmetric formula (9) is the main result of this paper. The example of its application to the $^4He-^4He$ scattering, given in the figure, shows that the agreement with the complete multiple-scattering calculations is quite good except for very large momentum transfers where some shadowing is still missing. It should be stressed that the profiles occurring in eq.(9) are those of the rigid projectile (RP) and of the rigid target (RT); they must not be replaced with the quasisoft profiles [9,4] since this would lead to an overcounting of excitations.

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