объединенный
институт
ядерных
исследований
дубна

E2-80-818

S.M.Bilenky, B.Pontecorvo

NEUTRINO OSCILLATIONS IN NEW MIXING SCHEMES WITH EITHER DIRAC OR MAJORANA MASSES

Submitted to "Physics Letters"

1980

1) Recently [1] we have discussed neutrino oscillations [2] in a theory in which Dirac and Majorana mass terms coexist [3], and, moreover, in the weak interaction Lagrangian and in the mass term there are present the same neutrino field components. The last circumstance means that in this scheme there are no "exotic" oscillations, that is, those oscillations which during the last year sometimes are being called "second class" oscillations ($\nu_{eL} \rightleftarrows \bar{\nu}_{eL}$, $\nu_{eL} \rightleftarrows \bar{\nu}_{\mu L}$ ...) [3]. The starting point in ref. [1] is the extension of the old idea [4] that e^- and μ^- have opposite values of the lepton number (only one!) to the case of more than two charged lepton types, the number of which must necessarily be even. Such schemes are of physical interest if, in addition to the well known charged leptons, e, μ, τ, at least one more charged lepton (say ξ) does exist. In the theory considered in ref. [1] oscillations between all types of neutrinos $\nu_\ell \rightleftarrows \nu_{\ell'}$, $\bar{\nu}_\ell \rightleftarrows \bar{\nu}_{\ell'}$ ($\ell, \ell' = e, \mu, \tau, \xi$; $\ell \neq \ell'$) are possible*. There we assumed that the oscillations of the type $\nu_e \rightleftarrows \nu_\mu$, $\nu_\tau \rightleftarrows \nu_\xi$ are suppressed, because in this scheme ν_e and ν_μ, ν_τ and ν_ξ are particles and antiparticles respectively**.

In this paper we discuss different schemes of neutrino mixing in which the oscillations $\nu_e \rightleftarrows \nu_\mu$, $\nu_\tau \rightleftarrows \nu_\xi$ (oscillations within one "neutrino flavour", which now is defined by two distinct charged leptons) are not merely suppressed but

---

* We keep the usual notation ν_e, ν_μ, ν_τ ... , $\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$..., for the "phenomenological" neutrinos, undergoing the weak interaction.

**Of course, the grouping of e with μ and τ with ξ is an arbitrary choice. A different choice could be for example e,τ and μ,ξ. For the sake of concreteness we shall use the first choice. Moreover we limit ourselves to the case of four lepton types.

entirely forbidden. The charge lepton current in our schemes is*:

$$j_{\alpha} = 2[(\bar{\nu}'_{1L} \gamma_{\alpha} e_L) + (\bar{\mu}^c_R \gamma_{\alpha} \nu'_{1R}) + + (\bar{\nu}'_{2L} \gamma_{\alpha} \tau_L) + (\bar{\xi}^c_R \gamma_{\alpha} \nu'_{2R})] , \qquad (1)$$

where the indices 1 and 2 correspond to the "flavours" (e, μ) and (τ, ξ), μ^c, ξ^c are the charge conjugated spinors (e, μ, τ and ξ indicating here the fields of e^-, μ^-, τ^- and ξ^-). In ref. /1/ we considered the case in which Dirac and Majorana mass terms coexist. Below we shall consider theories with only <u>either</u> Dirac <u>or</u> Majorana mass terms.

2) The most general Dirac mass term which may be written in terms of the fields ν'_1 and ν'_2 is

$$\mathcal{L}_{\mathfrak{D}} = -\bar{\nu}'_R M \nu'_L + h.c., \qquad (2)$$

where

$$\nu' = \begin{pmatrix} \nu'_1 \\ \nu'_2 \end{pmatrix} \qquad (3)$$

and M is a complex 2x2 matrix. The matrix M can be diagonalized through the transformation

$$M = U_R m U_L^+, \qquad (4)$$

where U_R and U_L are unitary 2x2 matrices (in general different) and $m = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$; $m_{1,2} > 0$. From expressions (2) and (4) one gets

$$\mathcal{L}_{\mathfrak{D}} = - \sum_{i=1,2} m_i \bar{\nu}_i \nu_i \, . \qquad (5)$$

The mixing is defined by the relations

$$\nu'_L = U_L \nu_L, \quad \nu'_R = U_R \nu_R, \qquad (6)$$

---

*Let us notice that it is possible to construct /5/ an SU(2)xU(1) gauge theory in which the charged lepton current is given by eq. (1). Moreover E.Fradkin and O.Kalashnikov /5/ showed that the Zeldovich-Konopinsky-Mahmoud scheme may be well accomodated within the framework of GUT.

where

$$\nu = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{7}$$

ν_1 and ν_2 are neutrino Dirac fields with masses m_1 and m_2. It is clear from (6) that only oscillations of the type $\nu_e \rightleftarrows \nu_\tau$ and $\nu_\mu \rightleftarrows \nu_\xi$ are possible in the case considered.

The matrices U_L and U_R in general are complex but for our case of only four leptons we may consider them as a real*. This means that in the oscillations at issue there cannot be CP violation [8] and that the matrices $U_{L,R}$ can be written as

$$U_{L,R} = \begin{pmatrix} \cos\theta_{L,R} & \sin\theta_{L,R} \\ -\sin\theta_{L,R} & \cos\theta_{L,R} \end{pmatrix} \tag{8}$$

Thus the following relations must hold

$$\begin{aligned} P_{\nu_\tau;\nu_e}(R) &= P_{\bar{\nu}_\tau;\bar{\nu}_e}(R), \\ P_{\nu_\xi;\nu_\mu}(R) &= P_{\bar{\nu}_\xi;\bar{\nu}_\mu}(R), \end{aligned} \tag{9}$$

where $P_{\nu_\tau;\nu_e}(R)$ is the probability of finding ν_τ at a distance R from a source of ν_e, etc. We get

$$\begin{aligned} P_{\nu_\tau;\nu_e}(R) &= \frac{1}{2}\sin^2 2\theta_L\left(1-\cos 2\pi\frac{R}{L}\right), \\ P_{\nu_\xi;\nu_\mu}(R) &= \frac{1}{2}\sin^2 2\theta_R\left(1-\cos 2\pi\frac{R}{L}\right), \\ P_{\nu_e;\nu_e}(R) &= 1-P_{\nu_\tau;\nu_e}(R);\quad P_{\nu_\mu;\nu_\mu}(R) = 1-P_{\nu_\xi;\nu_\mu}(R), \end{aligned} \tag{10}$$

where

$$L = 4\pi\frac{p}{|m_1^2-m_2^2|} \tag{11}$$

is the oscillation length and p is the neutrino momentum.

---

*Notice that the matrices U_L and U_R cannot be made real through the usual procedure a la Kobayashi-Maskava [6]. In order to see that these matrices are orthogonal, one can use the transformation proposed in ref. [7] (see equation (20) therein).

One can see that the oscillations $\nu_e \rightleftarrows \nu_\tau$, $\nu_\mu \rightleftarrows \nu_\xi$ in the present scheme with only the Dirac mass term are characterized by the following features: 1) there is one oscillation length for both types of oscillations, 2) in the general case the mixing angles θ_L and θ_R are different*. It should be noticed that the neutrino mixing theory considered here is the first example in which a Dirac mass term contains the same neutrino field components which are present in the weak current.

3) The most general Majorana mass term which may be written in terms of the fields ν'_1 and ν'_2 is

$$\mathcal{L}_M = -\bar{\nu}'^c_R M_L \nu'_L - \bar{\nu}'^c_L M_R \nu'_R + \text{h.c.}, \tag{12}$$

where M_L and M_R are symmetrical complex matrices. We have [9]

$$M_L = (V_L^+)^T m^L V_L^+,$$
$$M_R = (V_R^+)^T m^R V_R^+, \tag{13}$$

where $V_{L,R}$ are unitary 2x2 matrices and $m^{L,R}$ are diagonal 2x2 matrices with positive elements. From equations (12) and (13) we get

$$\mathcal{L}_M = -\bar{\chi} m^L \chi - \bar{\phi} m^R \phi \tag{14}$$

and

$$\nu'_L = V_L \chi_L,$$
$$\nu'_R = V_R \phi_R, \tag{15}$$

where

$$\chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \quad \text{and} \quad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \tag{16}$$

are Majorana fields of masses m_1^L, m_2^L, m_1^R, m_2^R. Clearly the neutrino mixing (15) leads to oscillations of the types $\nu_e \rightleftarrows \nu_\tau$, $\nu_\mu \rightleftarrows \nu_\xi$ just as in the preceding Dirac case. The probability of oscillations are given by expressions similar to eq. (10), but here we are faced with two different oscillation lengths

$$L_L = 4\pi \frac{p}{|(m_1^L)^2 - (m_2^L)^2|}, \quad L_R = 4\pi \frac{p}{|(m_1^R)^2 - (m_2^R)^2|} \tag{17}$$

---

*It is easy to see that $\theta_L = \theta_R$ in the case of a symmetrical matrix M.

for the two possible oscillation types. Notice that here too relations (9) do take place too.

4) Thus we have considered neutrino mixing schemes with an even number of charged leptons, based on the Zeldovich-Konopinsky-Mahnoud [4] ideology. These schemes are attractive, because they may be constructed so that in the mass term and in the weak current the same neutrino field components are present.

In a previous paper [1] we considered the case of coexistence of Dirac and Majorana mass terms. Here we have been considering either Dirac or Majorana mass terms. In both cases only oscillations of the types $\nu_e \rightleftarrows \nu_\tau$, $\nu_\mu \rightleftarrows \nu_\xi$ are possible (see footnotes on page 1). We have seen that in principle the Dirac and Majorana mass cases are distinguishable through the character of their oscillations: in the first case there is one oscillation length, in the second there are two oscillation lengths. This is due to the fact that for the Dirac case there are two mass eigenvalues and for Majorana one there are four mass eigenvalues. As far as we know, this is the first physical example* where from difference in the character of oscillations one can draw conclusions about either the Dirac or the Majorana nature of the mass eigenstates**.

In connection with the solar neutrino problem [10] we notice that in the schemes considered here with four charged leptons, the detectable flux of solar neutrinos may be reduced [11] at most by a factor 2 with respect to the expected flux in the absence of oscillations. In the general case of N (Neven) charged leptons the maximum reduction of the detectable neutrino flux is N/2 [12].

In conclusion we are very grateful to L.Wolfenstein for illuminating discussions concerning four component massive neutrinos and to E.Fradkin for useful discussions.

---

*Of course we do not consider "exotic" oscillations $\nu_{\ell L} \rightleftarrows \bar{\nu}_{\ell' L}$ [3], since in that case the Majorana character of the mass eigenstates is obvious.

** Let us notice that some statements made in ref. [1,7] about the impossibility of distinguishing Dirac and Majorana masses through oscillation experiments are too categorical (in the sense that they cannot be applied to the cases considered here).

REFERENCES

1. Bilenky S.M., Pontecorvo B. Phys.Lett., 1980, 95B, p.233; Proc. of the "ν-80" Conf. in Erice, June, 1980.
2. Pontecorvo B. JETP, 1957, 33, p.549; 1958, 34, p.247; 1967, 53, p.1717; Gribov V., Pontecorvo B. Phys.Lett., 1969, 28B, p.493; A review of the subject with the literature therein up to 1978-79 is given in Bilenky S., Pontecorvo B. Phys. Rep., 1978, 41, p.226 and Maiani L. Talk given at the meeting "Astrophysics and Elementary Particles - Common Problems". Preprint TH 2846-CERN, 1980.
3. For a previous theory with Majorana and Dirac mass terms see Bilenky S.M., Pontecorvo B. Lett.Nuovo Cim.,1976, 17, p.569. See also for example. Kobzarev I.Yu. et al. Preprint ITEP-90, 1980; Barger V. et al. Phys.Rev.Lett., 1980, 45, p.692.
4. Zeldovich Yu.S. DAN SSSR, 1952, 86, p.505; Konopinsky E.J., Mahmoud H. Phys.Rev., 1953, 92, p.1045. See also: Kawakami I. Progr.Theor.Phys., 1958, 19, p.459; Lipmanov E.M. JETP, 1959, 37, p.1054; Sokolov A.A. Phys.Lett., 1963, 3, p.21.
5. Jafarov I.G. Yad.Fiz., 1979, 30, p.215; Fradkin E., Kalashnikov O. Phys.Lett., 1976, 64B, p.177; Prepr.Lebedev Phys. Inst., No.4, 1980.
6. Kobayashi M., Maskawa K. Progr.Theor.Phys., 1973, 49, p.652.
7. Bilenky S.M., Hošek J., Petcov S.T. Phys.Lett., 1980, 94B, p.495.
8. Cabibbo N. Phys.Lett., 1978, 72B, p.333; Doi M. et al. Preprint OS-GE80-23, 1980.
9. See, e.g., Gaillard M.K., Maiani. LAPP-TH-09, 1979.
10. Davis R. et al. Proc. of the Int.Conf. "ν-78", p.53, 1978; Bahacall J.N. "40 years of Steller Energy", Bethe Symp., Stony Brook, N.Y., 1979.
11. Pontecorvo B. JETP, 1967, 53, p.1717; Gribov V., Pontecorvo B. Phys.Lett., 1969, 28B, p.493.
12. For different neutrino mixing schemes of N types of neutrinos see Pontecorvo B. Pisma JETP, 1971, 13, p.281; Bilenky S.M., Pontecorvo B. Proc. XVIII Int. Conf. on High Energy Physics, Tbilisi, 1976. JINR, D1,2-10400, Dubna, 1977, vol.II, p.B122; JINR, E2-10032, Dubna, 1976.

Received by Publishing Department
on December 12 1980.