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NON-ABELIAN FIELDS AS GLUON BAGS

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1. Introduction

To solve the problem of confinement in QCD it is necessary to find a simple physical analogy, adequate to the confinement mechanism. Recently there have appeared some analogies claiming such an adequacy.

The most popular one is the analogy of non-Abelian fields with ferromagnetic: the QCD coupling constant, depending on distances, plays the role of temperature, and the problem is just to find the phase transition point and to describe dynamically this transition from short to long distances [1].

In my works [2,3] a different analogy of the non-Abelian theory with theories of superfluidity [4,5,6] is developed. It is assumed that due to the infrared divergence the interaction at long distances plays so essential role that one may speak only about the quantum states and spectrum of the whole interacting system and it is impossible to consider the states of free gluons and quarks. Just that idea of collective dynamics is leading in theories of superfluidity.

The physical methods of "removing infrared catastrophe" consist briefly in the following. The two types of the collective excitations are distinguished in the superfluid liquid:

1st - weak quasiparticle excitations (local dynamics); the mathematical apparatus of describing the quasiparticles practically coincides with that of the usual quantum field theory [7] with its quantization principles (finiteness of the observables, stability of the physical states, Hermitean character of Hamiltonian, etc.).

2nd - excitations of the whole system (global dynamics); in the superfluid liquid this is the coordinates of center of inertia or angle of turning for a rotating liquid as a unique object. Just the "global" movement as a whole is the essence of the superfluid phenomenon. As a rule, the global dynamics has a physical meaning in the presence of a singular condensate (i.e., c-number field, defined on the class of functions nonvanishing at spatial infinity in $R(3)$ or nondifferentiable at some points of $R(3)$).

Theory, which simultaneously describes the Bose-condensate with the global dynamics (superfluid component) and the quasi-particles (normal component) is called the two-component theory.

Thus, we have the following logical scheme:

Physical principle		Physical methods
1)Relativistic invariance	$\xleftarrow{\quad ? \quad}$	1) Global dynamics [4]
2)Gauge invariance		2) Condensate [5,6]
3) SU_c -invariance	$\xrightarrow[\text{catastrophe}]{\text{Infrared}}$	3) Local dynamics[5,6]

At first sight such a scheme is contradictory because the existence of the condensate and the global dynamics (i.e., the simultaneous movement of all the field system as a whole) contradicts the relativistic invariance principle.

I shall show that the relativistic invariance may be reproduced within the scheme just following the ideas of the theory of superfluidity. The problem raised here is as follows: to reconstruct the relativistic-invariant two-component theory of non-Abelian fields without infrared catastrophe. Mathematically such a reconstruction is formulated as a realization of a unitary (physical) representation of the topological nontrivial gauge group. The reconstruction is carried out in three steps :

1)The proof of existence of the global dynamics .
2)The proof of existence of the local dynamics .
3)The restoration of the relativistic invariance.

2. Global Dynamics and Characteristic Classes

The existence of the global characteristics of non-Abelian fields has been pointed out for classical solutions of Yang-Mills equations with the finite Euclidean action /8/

$$S = \frac{1}{2g^2}\int d^4_x \, tr\left(F_{\mu\nu} F^{\mu\nu}\right) \quad ; F_{\mu\nu} = \frac{F^a_{\mu\nu}\tau^a}{2i} g \tag{1}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc} A^b_\mu A^c_\nu .$$

It can be shown /8/, that vacuum Yang-Mills solutions, minimizing the functional (1), satisfy the duality equation

$$F_{\mu\nu} = {}^*F_{\mu\nu} \quad ; \quad {}^*F_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta} F^{\alpha\beta} \tag{2}$$

and are characterized by an integer Pontryagin index x)

$$\nu[A] = -\frac{1}{16\pi^2}\int d^4_x \, tr\left(F_{\mu\nu} \, {}^*F^{\mu\nu}\right) . \tag{3}$$

The functionals of a kind of (3) are called characteristic classes in topology /9/ . They have the remarkable property

$$\delta \nu[A] / \delta A = 0 \tag{4}$$

and reflect the global feature of non-Abelian fields as a whole.

The vacuum solutions (instantons) are supposed /10,11/ to give a basic contribution to the Faddeev-Popov functional integral, like classical trajectories of tunnelling in quantum mechanics of periodic potentials.

The Hamilton formalism in gauge $A_0 = 0$ /12/ gives the most clear statement of the problem of quantization of such a theory with nontrivial topology.

In this gauge fields A_i are defined up to the gauge stationary transformation, and the functional (3) is of the form of difference of two functionals.

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x) For finite action (1) in R(4) non-Abelian fields are gauge on the boundary S(4): $A_\mu = v^{-1}(x)\partial_\mu v(x)$. Integer ν is a degree of mapping of S(4) to SU(2) given by $v(x)$.

$$V_{t_1 t_2} = \int_{t_1}^{t_2} dt \int d^3x \left(-\frac{1}{16\pi^2} tr F_{\mu\nu} {}^*F^{\mu\nu}\right) = \int_{t_1}^{t_2} dt \dot{N}_t = N_{t_1}[A_i] - N_{t_2}[A_i] \quad (5)$$

$$N_t[A_i] = \frac{1}{8\pi^2} \int d^3x \, \varepsilon_{ijk} \left(\frac{1}{2} \partial_i A_j^a A_k^a + \frac{g}{6} \varepsilon^{abc} A_i^a A_j^b A_k^c\right)$$

which are transformed under the gauge groups as

$$N_t[v(\vec{x})^{-1}(A_i + \partial_i) v(\vec{x})] = N_t[A_i] + n(v)$$

$$n(v) = -\frac{1}{12\pi^2} \int d^3x \, \varepsilon_{ijk} \, tr[(v^{-1}\partial_i v)(v^{-1}\partial_j v)(v^{-1}\partial_k v)]. \quad (6)$$

As we have shown above, the definition of the smooth function class plays the basic role for the topological properties. Because of the finiteness of observables (energy, momentum, etc.) we should choose smooth A_i and therefore $v(\vec{x})$, in particular

$$\lim_{|\vec{x}| \to \infty} v(\vec{x}) = I. \quad (7)$$

For matrix $v(\vec{x})$ (7) number $n(v)$ (6) is integer and means the degree of smooth mapping of R(3) to SU(2) given by matrix $v(\vec{x})$. ($n(v)$ indicates how frequently the space R(3) has turned about SU(2)). Thus the gauge group is topologically disconnected and, besides continuous, has discrete transformations. The Total gauge group is a product of a "small" continuous group G_0 (n=0) by infinite cyclic group of all integers Z (The factor group G/G_0 is called the homotopy group $\pi_3(SU(2)) = Z$).

The functional N (6) is a classical realization of the group Z representation.

While solving the Schrödinger equation

$$\hat{H} \Psi_\varepsilon[A] = \varepsilon \Psi_\varepsilon[A] \quad ; \quad \hat{H} = \frac{1}{2} \int d^3x [\hat{E}^2 + B^2]$$

$$E_i^a = i \, \delta / \delta A_i^a \quad ; \quad B_i^a = \frac{1}{2} \varepsilon_{ijk} F_{jk}^a \quad (8)$$

besides the Gauss condition of invariance under group G_0

$$\nabla_i(A)\hat{E}_i \Psi_\varepsilon[A] = 0 \quad ; \quad \nabla_i(A)^{ac} = \delta^{ac}\partial_i + g\,\varepsilon^{abc}A_i^b \qquad (9)$$

it is necessary to impose the condition of covariance of "wave function" Ψ_ε under the transformations of group Z

$$T\Psi_\varepsilon = e^{i\theta}\Psi_\varepsilon \qquad ; \quad T \in Z\,, \qquad (10)$$

where θ is the quasimomentum $0 \leqslant \theta \leqslant \pi$. Equation (10) is an analogy of the "periodic condition" of the wave function in the above-mentioned quantum mechanics of the periodic potentials, in which for the representation of the operator T one usually uses the classical variable N (6), changing by an integer

$$T = \exp\left(\frac{d}{dN}\right). \qquad (11)$$

But the representation of T in form (11) is contradictory for the Yang-Mills theory: operator T does not commute with the Hamiltonian

$$[T,H] \neq 0 \quad , \quad [[T,H],H] \neq 0 \text{ , etc.}$$

The operators T, H have no common eigenstates besides the "vacuum state" with zero-energy ($\varepsilon = 0$).

$$\Psi_0 [T,H] \Psi_0 = 0 \qquad (\text{as } H\Psi_0 = 0)$$

Such a state is easily constructed, if we substitute the solution of eqs. (10), (9) in the form of a "plane wave"

$$\Psi_0 = \exp\{i(2\pi k + \theta)N\} \qquad (k \text{ - integer})$$

to the Schrödinger eq.(8). As a result we obtain the equation for the momentum

$$(2\pi k + \theta) = \pm i\left(\frac{8\pi^2}{g^2}\right). \qquad (12)$$

As the momentum is imaginary, the solution Ψ_0 is a nonphysical (nonunitary) representation of group Z . Wave function Ψ_0 obeys simultaneously the Euclidean duality equation in the operator form $\hat{E}\Psi_0 = \pm B\Psi_0$ and, probably, reflects the nonphysical meaning of the classical instantons.

These facts and experience of quantum mechanics give doubts about the existence of the exact physical solutions of the set of eqs. (8)-(11). Then, the question arises: How does one construct the unitary (physical) representations of group Z ?

To answer this question, one considers the exactly solvable model $QED_{(1+1)}$, which is topologically equivalent to Yang-Mills theory

$$S = \frac{1}{2}\int d^2x\, F_{01}^2 \quad ; \quad \nu[A] = \frac{1}{2\pi}\int d^2x\, F_{01} \quad ; \quad F_{01} = \partial_0 A_1 - \partial_1 A_0$$

(We have here the map of $R(1)$ into $U(1)$: $\pi_1(U(1)) = Z$).
For gauge $A_0 = 0$ the analogies of eqs. (8)-(11) are

$$\hat{H}\Psi = \varepsilon\Psi \quad ; \quad H = \frac{1}{2}\int dx_1\, \hat{E}_1^2 \quad ; \quad \hat{E}_1 = i\,\delta/\delta A_1$$

$$\partial_1 \hat{E}_1 \Psi = 0 \quad ;$$

$$T\Psi = e^{i\theta}\Psi \quad ; \quad T = e^{\frac{d}{dN}} \quad ; \quad N_t = \frac{1}{2\pi}\int dx_1 A_1 \quad ; \quad \nu_{t_1 t_2} = N_{t_1} - N_{t_2} \, .$$

In that case the "plane wave" $\Psi = \exp\{i(2\pi k + \theta)N\}$ is the exact physical solution with the finite energy density $E/V_1 \sim (2\pi k + \theta)^2$. According to the usual way of quantization there are no local dynamic variables in this model, as there are no transverse degrees of freedom. However, we see that the nontrivial dynamics exists which is described by variable N, canonical conjugate to the operator of a constant electric field $\hat{E}\Psi \sim (2\pi k + \theta)\Psi$, nonvanishing at infinity $R(1)$.

And such a dynamics corresponds to the excitation of the system as a whole, i.e., it is global. (The stationary state of $QED_{(1+1)}$ is equivalent to the ground state of the superfluid, moving along the closed ring /6/).

Thus, to construct the unitary (physical) representation of the homotopy group it is sufficient to consider the characteristic class $\nu[A]$ (3) as a global dynamical variable, and to go out of the smooth-function class).

Let us prove that Yang-Mills theory allows the existence of the global dynamic variable $\dot{\underset{\sim}{\nu}}$, bound with the characteristic class $\nu[A]$ by the relation

$$\nu = \int dt\,(\dot{\underset{\sim}{\nu}} + \dot{N}[A]) \, . \tag{13}$$

The basic quantities are the functionals of action (1) and characteristic class (3) in Minkowsky space. We shall not restrict ourselves to the smooth-function class. To separate the dynamical variables in gauge theory one needs to solve an equation of constraint. In the Abelian gauge theory this is the classical equation for the scalar potentials A_0 /14/

$$\delta S[A] / \delta A_0 = 0 \tag{14}$$

which is on a distinct status as the canonical momentum, conjugate to A_0 equals zero (A_0 is not an operator, but c-number in contrast to the spatial component A_i). We shall consider eq. (14) as an equation constraint in non-Abelian theory also, with two comments: eq. (14) is defined beyond the points of singularities, and the action itself allows arbitrariness: in view of eq. (4) the functional (3) may be added to the action and the constraint (14) does not change.

A solution of the classical equation for A_0

$$\nabla_i^2 A_0 = \nabla_i \partial_0 A_i \quad ; \quad \nabla_i^{ac} = \delta^{ac} \partial_i + g \varepsilon^{abc} A_i^b \tag{14'}$$

is given as a sum of a solution of the homogeneous equation

$$\nabla_i^2 \Phi = 0 \tag{15}$$

and a solution of the nonhomogeneous one

$$A_0 = \dot{\underset{\sim}{\nu}}(t) \left(C_B^{-1} \right) \Phi + \frac{1}{\nabla_i^2} \nabla_j \partial_0 A_j , \tag{16}$$

where $\dot{\underset{\sim}{\nu}}$ is the global dynamical variable, common for the whole space. The factor C_B is defined by substituting eq. (16) into eq. (3) from the condition (13) . Operator in (16) should be defined in the class of functions where substitution of (16) into (1), (3) gives the action

$$S' = \int dt \, L \tag{17}$$

$$L = \frac{1}{2} \int d^3x \left(E_T^2 - B^2 \right) + \frac{1}{2} \dot{\underset{\sim}{\nu}}^2 C_\Phi C_B^{-2} - \dot{\underset{\sim}{\nu}} C_E C_B^{-1} ,$$

$$E_{T i}^{a} = \left(\delta_{ij} - \nabla_i \frac{1}{\nabla_k^2} \nabla_j\right)^{ab} \partial_0 A_j^{b} \quad ; B_i^{a} = \frac{1}{2} \varepsilon_{ijk} F_{jk}^{a} , \tag{18}$$

$$C_B = \frac{g^2}{8\pi^2} \int d^3x\, B_i^a (\nabla_i \Phi)^a ; \; C_E = \int d^3x (E_T)_i^a (\nabla_i \Phi)^a ; \; C_\phi = \int d^3x (\nabla_i \Phi)^2 . \tag{19}$$

Fields $B, E, \nabla\Phi$ satisfy the transversality condition

$$\nabla_i B_i = \nabla_i E_i = \nabla_i (\nabla_i \Phi) = 0 \tag{20}$$

beyond the points of singularities. Owing to eq. (20) the functionals (19) are the characteristic classes. They are nonzero only for singular fields $B, E_T, \nabla\Phi$. Thus, the condition for existence of the global variable is the singular fields. Let us represent gauge fields A_i as a sum of a singular field $\underset{\sim}{b}_i$ (nonvanishing at infinity or nondifferentiable at some points of space $R(3)$) and regular field $\underset{\sim}{a}_i$ (smooth and vanishing at singularities) which describes the local dynamical variables (quasiparticles)

$$A_i = \underset{\sim}{b}_i + \underset{\sim}{a}_i . \tag{21}$$

In this case coefficients C_φ, C_B, C_E in eq. (17) depend on the "condensate" $\underset{\sim}{b}$ only. Local and global variables in eq.(17) are separated completely. The wave function of the corresponding quantum system is factorized: $\Psi = \Psi_{glob}(\underset{\sim}{\nu})\, \Psi_{loc}(\underset{\sim}{a})$. The plane wave $\Psi_{glob}(\underset{\sim}{\nu}) = e^{iP\underset{\sim}{\nu}}$ is a stationary state of the system, because the Lagrangian (17) depends on the "velocity" $\dot{\underset{\sim}{\nu}}$ only. The spectrum of momenta is defined by the condition

$$\Psi_{glob}(\underset{\sim}{\nu}+1) = e^{i\theta}\, \Psi_{glob}(\underset{\sim}{\nu})$$

$$P = 2\pi k + \theta . \tag{22}$$

For the stationary state the number $\underset{\sim}{\nu}$ is fuzzy, and classical field with fixed ν has no physical meaning. Recall that the basic task of quantization of a strong-interacting system is to find the energy spectrum of weak excitations, or, in other words, to find stationary states. To solve this task, it is necessary first to pass to the stationary state of the global variable ν (as is made in the microscopic theory of superfluidity).

3. Local Dynamics

We have defined the global dynamics as a common variable for the whole space and have found that the condition of its existence is the singular condensate.

Let us prove analogously the existence of the local dynamic variable, defining its properties following quantum field theory /7/ and Bogolubov work /6/ ; and then construct explicitly the condensate and local variables x) .

Define the local variables a_i as the weak stable excitations with finite energy, momentum and other observables. This means that the fields a_i are defined in the class of smooth functions, vanishing at singularities of the condensate.

Let us prove that a singular condensate allows the existence of weak local excitations with finite energy.

To be convinced of that, it is sufficient to express the action (17) in terms of the conserving global momentum (22):

$$S = \int dt\, L(p, a)$$

$$L(p,a) = \frac{1}{2}\left[\int d^3x\, E(a+b)^2 - \left(\int d^3x (\nabla\Phi) E\right)^2 \Big/ \int d^3x (\nabla\Phi)^2\right] -$$

$$- \frac{1}{2}\left[\int d^3x\, B(a+b)^2 - \rho^2 \left(\int d^3x (\nabla\Phi) B\right)^2 \Big/ \int d^3x (\nabla\Phi)^2\right] \tag{23}$$

$$\rho^2 = \left(\frac{g^2}{8\pi^2}\right)^2 (2\pi k + \theta)^2 .$$

It is easy to show that for $\rho^2 = 1$ and the potential condensate

$$E(b) \sim B(b) \sim \nabla(b)\, \Phi \tag{24}$$

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x) Such a proof of the existence apparently has first been applied by D.Hilbert in the Invariant Theory /15/ .

the expansion of the action in fields $\underset{\sim}{a}_i$ starts from the second order in $\underset{\sim}{a}_i$

$$S(\underset{\sim}{a}+\underset{\sim}{b}) = S(\underset{\sim}{b}) + S'(\underset{\sim}{b})\cdot\underset{\sim}{a} + \frac{1}{2}S''(\underset{\sim}{b})\cdot\underset{\sim}{a}^2 + \dots = \frac{1}{2}S''(b)a^2 + \dots \tag{25}$$

(as the condensate (24) satisfies classical equations beyond singularities and at the points of singularities δ-functions in the second term $S'(\underset{\sim}{b})\cdot\underset{\sim}{a}$ are multiplied by zeros of fields $\underset{\sim}{a}_i$).

By our definition of fields $\underset{\sim}{a}_i$, the theory with action (25) will be stable and will have finite observables. The choice of eqs. (24) is consistent with the requirement of stationarity and transversality condition outside singularities (20). (In accordance with the Landau Theory of superfluidity /5/, potentiality of condensate is a necessary condition for the "superfluid component").

The condition $\rho^2 = 1$, generally speaking, is optional because the action (see p.7) allows arbitrariness up to the characteristical class.

The scheme of separating the dynamical variables described above permits the introduction of interaction with spinor fields /2,3,14/ . Let us represent the final result in the form of the Lagrangian density

$$\underline{\mathcal{L}}(\underset{\sim}{a},\underset{\sim}{b},\psi) = -\frac{1}{4}\left(F^a_{\mu\nu}(\underset{\sim}{a}+\underset{\sim}{b})^2 - F^a_{\mu\nu}(\underset{\sim}{b})^2\right) + i\bar{\psi}\gamma_\mu(\partial^\mu + \underset{\sim}{a}^\mu + \underset{\sim}{b}^\mu)\psi$$

$$\underset{\sim}{b}_\mu = (\Phi, b_i) \equiv g\underset{\sim}{b}_\mu^a \frac{\tau^a}{2i}\,, \tag{26}$$

where fields $\underset{\sim}{b}$ satisfy eq. (24).

Since as a final step we should restore the group of motion of Minkowsky space, we consider here the particular case, eq. (24), of the stationary solutions of duality equations in Minkowsky space

$$-\left(\nabla_j(\underset{\sim}{b})\,\Phi\right)^a = i\,B_j^a(\underset{\sim}{b})\,. \tag{27}$$

A general solution to (27) may be written like the most general solutions, obtained up to now in Euclidean space (see, for example, /16/)

$$b_\mu^a = \frac{1}{g} \Sigma_{\mu\nu}^{(+)a} \partial^\nu \ln \rho \, , \tag{28}$$

where

$$\Sigma_{0j}^{(+)a} = i\delta_{ja} \; ; \; \Sigma_{ij}^{(+)a} = \varepsilon^{aij} \; ; \; \Sigma_{\mu\nu}^{(+)a} = -\Sigma_{\nu\mu}^{(+)a} \tag{29}$$

and the function ρ satisfies the equation

$$\partial_\mu^2 \rho = 0 \, . \tag{30}$$

The condensate is stationary, $\rho \sim e^{ik_0 t} f(\vec{x})$, then due to the spherical symmetry (for the condensate at rest there is no preferable direction) it is natural to classify by the spherical functions

$$\rho(x,t) = e^{ik_0 t} P_\ell^m(\cos\theta)\, j_\ell(r)\, e^{im\varphi}$$
$$\vec{x} = (r\cos\varphi\sin\theta \, , \; r\sin\varphi\sin\theta \, , \; r\cos\theta) \, , \tag{31}$$

where P_ℓ^m , $j_\ell(r)$ are Legendre polynomials and Bessel functions.

In Appendix A the spectra of quasiparticles for the condensate (31) with $\ell = 0$ are calculated and it is shown that the complex stationary solution (31) does not lead to physical difficulties with the Hermitean character of Hamiltonian. The spectrum of operator $[i\nabla_j(b)]^2$ is positive definite and the eigenvalue $[i\nabla_j]^2 = 0$ does not belong to the physical spectrum. (The corresponding solution is not normalized). Thus, the condensate (31) is energetically favourable and represents bags [x] for coloured quasiparticles with the confinement parameter k_0^{-1}, which appears like the dimensional photon momentum in the conformal invariant

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x) One of the first hadron models of the bag type has been considered in ref. /18/ .

4. The Restoration of Translation and Relativistic Invariance

The new perturbation theory in coupling constant g coincides with expansion in $\underset{\sim}{a}$. In such cases one usually restores the initial translation and relativistic invariance by zero-modes of physical fields $\underset{\sim}{a}$, which are obtained by the action of generators of a restoring group on classical field $\underset{\sim}{b}$ (It is just the way used for the expansion around the instantons /11/ and in soliton theory /16/). However, in our case, zero-modes do not belong to the physical spectrum and the Lorentz-group is not covariance group of eq. (27). (We destroy the usual Lorentz-invariance, when we introduce the global dynamics). Condensate (31) transformed under the Lorentz-group does not belong to classical solutions and the corresponding perturbation theory becomes unstable.

The covariance group of eq. (27) (i.e., group of transformations, by which eq. (27) becomes covariant $F_{\mu\nu} = i\, {}^{*}F_{\mu\nu}$) is the group, in which the usual Lorentz generators $L_{\mu\nu}$ are replaced by the generators.

$$L'_{\mu\nu} = L_{\mu\nu} + \sum_{\mu\nu}^{(+)a} T^{a} \qquad (32)$$

(where T^{a} are generators of the colour group). These generators with the Lorentz transformation make a rotation in the colour space by a constant.

The corresponding stationary solution (31) (bag at rest) transforms into a bag, moving with arbitrary velocity $\vec{V}$ from arbitrary point $\vec{X}$ of space $R(3)$. The dynamics of such a motion will be described in analogy with the two-component theory of superfluidity /5/.

Equations (26), (30) allow us to formulate the "relativistic" invariant two-component theory with the Lagrangian density

$$\mathcal{L} = \underline{\mathcal{L}}\left(\underset{\sim}{a}, \underset{\sim}{b}_{\mu} = \frac{1}{g}\sum_{\mu\nu}\partial^{\nu}\ln\rho, \psi\right) + \underset{\sim}{j}_{\lambda}\,\partial^{\lambda}\rho\,, \qquad (33)$$

where $\underline{\mathcal{L}}$ is defined by eq. (26), and $\lambda(x)$ is the Lagrange factor. Lagrangian (33) describes the condensate, the equation of motion of which does not depend on the presence of quasiparticles (analogously, in statphysics the weak quasiparticle excitations practically do not influence the condensate dynamics).

For the dynamical system (33), in a usual way one may construct the energy-momentum and angle-momentum tensors. The last is Lorentz-invariant for colourless states.

The asymptotic states of hadron-bags are described by the Lagrangian without the quasiparticle interaction ($g=0$). In that case we have four-dimensional nonlinear and exactly solvable model. Consider for illustration the case of the spinor particles

$$\mathcal{L} = i\bar{\Psi}\gamma_\mu\partial^\mu\Psi + J_\mu\partial^\mu \ln\rho + \partial_\mu\lambda\partial^\mu\rho \;;\quad J_\nu = i\bar{\Psi}\gamma^\mu\Sigma^{(-)}_{\mu\nu}\Psi .$$

Define the complex-conjugated variable

$$\pi_\lambda = \frac{\partial\mathcal{L}}{\partial\dot{\lambda}} = \dot{\rho} + J_0/\rho \;;\quad \pi_\Psi = \frac{\partial\mathcal{L}}{\partial\dot{\Psi}} = i\bar{\Psi}\gamma_0 \;;\quad \pi_\rho = \frac{\partial\mathcal{L}}{\partial\dot{\rho}} = \dot{\lambda} .$$

Let us accomplish the canonical quantization of this system and define the physical state of the condensate as a coherent state of the field ρ

$$|0\rangle_{phys} = \exp\{i\int d^3x\, \hat{\pi}\,\rho_{class}\}\,|0\rangle_{Fock} ,$$

where ρ_{class} is one of the classical solutions of eq. (26) and $|0\rangle_{Fock}$ is the Fock vacuum

$${}_{Fock}\langle 0|\, \hat{\pi}_\lambda \,;\, \hat{\pi}_\rho \,;\, \hat{\pi}_\Psi \;;\, \hat{\lambda} \;;\, \hat{\rho} \,;\, \hat{\Psi}\, |0\rangle_{Fock} = 0 .$$

The energy spectrum for the condensate at rest is calculated in Appendix A.

Using the conservation laws, it is easy to show that the physical observables, averages of the Hamiltonian, momentum, and angular momentum

$$H = \int d^3x \left(i\bar{\Psi}\gamma_i\nabla_i(b)\Psi + \dot{\rho}\dot{\lambda} + \partial_i\rho\,\partial_i\lambda \right) \equiv \int d^3x\, T^{00}$$

$$P_i = \int d^3x \left(i\bar{\Psi}\gamma_0\partial_i\Psi + J_0\partial_i\ln\rho + \dot{\lambda}\partial_i\rho + \dot{\rho}\,\partial_i\lambda \right) = \int d^3x\, T^{0i}$$

$$M^{\mu\nu} = \int d^3x \left\{ x^\nu T^{0\mu} - x^\mu T^{0\nu} + \bar{\Psi}\gamma^0\left[\tfrac{1}{4}[\gamma^\nu,\gamma^\mu] + \Sigma^{\nu\mu}\right]\Psi \right\}$$

for "empty" bags (without quasiparticles) are equal to zero. For a moving bag with quasiparticles in the considered case the relativistic relations are fulfilled between the energy and momentum

$$\langle E\rangle_v = \frac{M}{\sqrt{1-\vec{v}^2}} \quad , \quad \langle \vec{P}\rangle = \frac{\vec{v}\,M}{\sqrt{1-\vec{v}^2}}$$

$$M = \langle E\rangle_{v=0} = {}_{Phys(v=0)}\langle \zeta^{(-)}\dots\zeta^{(-)} | H(\hat{\rho},\dots) | \zeta^{(+)}\dots\zeta^{(+)}\rangle_{Phys(v=0)}$$

$$= {}_{Fock}\langle \zeta^{(-)}\dots\zeta^{(-)} | H(\rho_{class(v=0)},\dots) | \zeta^{(+)}\dots\zeta^{(+)}\rangle_{Fock},$$

where $\zeta^{(-)}, \zeta^{(+)}$ are the operator of "death" and "birth" of quasiparticles. Thus, the spectrum obtained in Appendix A is the spectrum of hadron masses

One may consider also classical solutions of the condensate, describing N bags, moving with arbitrary velocities from arbitrary points of space

$$\rho_N = \sum_{i=1}^{N} \rho(k_0, l_i, m_i, \vec{v}_i, \vec{X}_i).$$

Due to the quasiparticle "confinement", asymptotical states for an N -bag solution ρ_N are completely factorized. Hadrons can interact only by a quasiparticle at the moment of the bag intersection. At the decay of hadron, when all quarks are transformed to leptons, the "empty" bag disappears in the physical vacuum which is the continuum of "empty bags".

Conclusion

The basic distinction of the approach, developed here, from analogous approaches with the gluon condensation is the introduction of the global dynamic variable. The global dynamics is the essence of the superfluidity phenomenon, and in non-Abelian theory the introduction of the global variable is at least one of the way of the construction of stationary quantum states as a unitary representation of the homotopy group. We have shown that the singular condensate, satisfying the duality equation in Minkowsky space allows simultaneously the existence both of global and local dynamic variables.

The classification of the solutions of duality equation naturally leads to hadron bags and to two-component relativistic theory, in which "empty" bags are not observable.

Experience of studies of Yang-Mills theory in Euclidean space has shown that, probably, the most adequate mathematical apparatus for the new perturbation theory is the twistor formalism.

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Appendix A

Let us calculate quasiparticle spectra for the condensate

$$\rho = e^{-ik_0 t}\,\frac{\sin(k_0 r)}{r} \quad ; \quad k_0 = 1$$

$$b_\mu^a = (\varphi^a, b_i^a) = \frac{1}{g}\Sigma^{(+)a}_{\mu\nu}\,\partial^\nu \ln\rho .$$

Consider, first, the equation for a scalar coloured field

$$\left(\nabla_\mu(b)^2\right)^{ab} \underset{\sim}{a}^b = 0$$

Substituting $\underset{\sim}{a}^b$ in the form

$$\underset{\sim}{a}^b = n^b \sum_\ell \frac{U_\ell(r)}{r}\left(e^{+iE_\ell t}\, z_\ell^{(+)} + e^{-iE_\ell t}\, z_\ell^{(-)}\right); \; n^b = \frac{x^b}{r}$$

we obtain the equation for U_ℓ

$$\left(-\partial_r^2 + \frac{1}{2}\left[\left(1/\sin^2\tfrac{r}{2}\right) + \left(1/\cos^2\tfrac{r}{2}\right)\right]\right) U_\ell(r) = E_\ell^2 U_\ell(r).$$

The normalized solutions and spectrum have the form:

$$U_\ell = (\sin r)\,\partial_r\left[\frac{1}{\sin r}\,\sin(\ell+2)r\right]\frac{1}{\pi}\sqrt{\frac{1}{(\ell+3)(\ell+1)}}$$

$$E_\ell = (\ell+2) \; ; \quad 4\pi\int_0^\pi dr\, U_{\ell_1} U_{\ell_2} = \delta_{\ell_1\ell_2} \; ; \quad \ell = 0, 1, 2, \ldots$$

The operator $[i\nabla_j(b)]^2$ is defined in the class of normalized functions for which $[i\nabla_j(b)]^2 > 0$.

The spinor equation

$$\gamma^\mu \nabla_\mu(b)\psi = 0 \quad ; \quad \psi = \begin{pmatrix} (\psi_L)^i_B \\ (\psi_R)^i_B \end{pmatrix}$$

may be solved by the Grossmann substitution /19/

$$\psi_\ell^{(+)} = \sum_{\ell=0}^{\infty}\left(\xi_\ell^{(+)}\psi_\ell^{(+)}e^{iE_\ell t} + \xi_\ell^{(-)}\psi^{(-)}e^{-iE_\ell t}\right)$$

$$\psi_\ell^{(+)} = \left(\frac{\sin z}{z}\right)^{1/2}\begin{pmatrix} u_1^{(\ell)} \\ \frac{\sin z}{z}u_2^{(\ell)} \end{pmatrix} b_2 N_\ell^{1/2}$$

$$\psi_\ell^{(-)} = \left(\frac{\sin z}{z}\right)^{1/2}\begin{pmatrix} u_2^{(\ell)} \\ \frac{\sin z}{z}u_1^{(\ell)} \end{pmatrix} b_2 N_\ell^{1/2}$$

$$u_1^{(\ell)} = \left(1 - i\frac{b_i\partial_i}{\ell+2}\right)\frac{\sin(\ell+1)z}{\sin z} \quad ;$$

$$u_2^{(\ell)} = \left(1 + i\frac{b_i\partial_i}{\ell+1}\right)\frac{\sin(\ell+2)z}{\sin z} \quad ;$$

$$E_\ell = \left(\ell + 3/2\right) \quad ; \quad \ell = 0, 1, 2, \ldots$$

N_ℓ is defined from the condition $\int d^3x\, \bar{\psi}_\ell^{(+)}\gamma_0\psi_\ell^{(-)} = 1$

$$N_\ell^{-1} = 4\pi^2\left[2 + \frac{1}{(\ell+2)(\ell+1)}\right].$$

Or, in a more compact form:

$$\psi \sim \sum_{\ell=0} \rho(x)^{1/2}\begin{pmatrix} \partial^{(+)}\left(\left[\xi_\ell^{(-)}\rho((\ell+2)x) + \xi_\ell^{(+)}\rho^*((\ell+1)x)\right]/\rho(x)\right) \\ \rho^*(x)\partial^{(-)}\left(\left[\xi_\ell^{(-)}\rho((\ell+1)x) + \xi^{(+)}\rho^*((\ell+2)x)\right]/\rho^*(x)\right) \end{pmatrix} b_2 N_\ell^{1/2}$$

$$\partial^{(\pm)} = \alpha_\mu^{(\pm)}\partial^\mu \quad ; \quad \alpha_\mu^{(\pm)} = (1, \mp b_i)$$

For gluons we have the solution

$$a_\mu^c = \sum_{\ell=0} \bar{u}_1 \left\{ \alpha_\mu^{(+)} \rho(x)\, \partial^{(-)} \tau^c \frac{1}{\rho^2(x)}\, \partial^{(+)} \left[\xi_\ell^{(-)} \rho((\ell+3)x) + \xi_\ell^{(+)} \rho^*((\ell+1)x) \right] \right\} u_2$$

$\bar{u}_1, u_2$ are constant spinors, $E_\ell = (\ell+2)$.

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