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THE HADRON PROPERTIES WITHIN THE MODEL OF QUASI-INDEPENDENT QUARKS

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The modern state of quark models of the new and old hadrons makes actual the consideration of their properties within the relativistic formalism. For this purpose one may use relativistic equations for bound states of two and three particles (as the Bethe-Salpeter equation or quasipotential approach). One should note, however, that the consideration within these equations even of the three-body problem (unlike the two-body case) would require a more complicated mathematical method.

It is known that the transition from the two-body problem to the three- and more body problem has no further difficulties if the quark motion inside hadron to the first approximation is regarded as the quasi-independent motion /1/. In this case the two-, three- and n -body problem may be investigated similarly, as the wave function in such an approximation is a product of the wave functions of the individual quarks. The quark wave functions satisfy the one-body Dirac equation with the potential V(r):

$$[\vec{\gamma}\vec{P} - \gamma_0 E + m_q + V(r)]\Psi(\vec{r}) = 0, \qquad (1)$$

where E, m_q are energy and quark mass, respectively. As to the potential V(r) we shall assume that it transforms as the Lorentz scalar and may have a confining part.

The aim of this work is to use the model of quasi-independent quarks for the investigation of mass spectrum and electromagnetic characteristics both of mesons and baryons. We choose the potential as follows:

$$V(r) = -\frac{\alpha_S}{r} + br, \qquad (2)$$

where we suppose that α_S is subjected to the dependence of the strong coupling constant on E resulting from the quantum chromodynamics /2/:

$$\alpha_S(E) = \frac{\alpha_S(E_0)}{1 + \frac{33-2N}{12\pi}\alpha_S(E_0)[\ln(E/E_0)^2]}, \qquad (3)$$

where N is the number of types of quarks used, E_0 is an arbitrary chosen normalization energy. In our case N=5.

We shall also use an assumption that forces acting on the quark are independent of the quark type in the hadron (that concept was introduced earlier in ref. [3] *). It was shown there that this assumption provides a satisfactory agreement of the calculated meson-mass spectrum with the experimental one of the ρ, ψ, Υ families. To consider the families of particles that are bound states of quarks of different types we introduce, as in ref. [3], the concept of "averaged quarks".

It should be noted that our model has only two independent parameters: namely, the potential coefficients α_S and b that will be used to describe both the ρ, Ψ and Υ meson families and the baryon octet.

We shall also discuss the case when the potential V(r) in (1) is transformed with respect to the Lorentz group as the fourth vector component.

1. By expanding over spherical spinors ($\ell' = 2j - \ell$)

$$\Psi_{Ej\ell m}(\vec{r}) = \begin{pmatrix} g(r)\,\Omega_{j\ell m}(\vec{n}) \\ if(r)\,\Omega_{j\ell' m}(\vec{n}) \end{pmatrix}$$

the Dirac equation is reduced to the following system of equations for radial functions g(r) and f(r):

$$\begin{aligned} &\frac{d}{dr}(rg(r)) + \frac{k}{r}(rg(r)) - (E - V(r) + m_q)(rf(r)) = 0, \\ &\frac{d}{dr}(rf(r)) - \frac{k}{r}(rf(r)) + (E + V(r) - m_q)(rg(r)) = 0, \end{aligned} \qquad (4)$$

where

$$k = \begin{cases} -(\ell + 1) & j = \ell + \frac{1}{2} \\ \ell & j = \ell - \frac{1}{2} \end{cases}$$

First, consider vector mesons. The parameters $\alpha_S(E_0)$, b and quark masses m_c and m_b may be fixed by requiring that the mass values of the ground and first radial excited states calculated by equations (4) coincide with the experimental masses of Ψ, Ψ', Υ, Υ' particles. The numerical solution gives us the following values: $\alpha_S(3.1) = 0.35$, $b = 0.078\ (\text{GeV})^2$, $m_c = 1.47$ GeV, $m_b = 4.78$ GeV.

The mass of u quark appears to be equal to 0.2 GeV at the same values of potential parameters provided that the calculated value of the ground state mass of ρ meson coincides with the experimental values of ρ (0.773). The parameters thus fixed define the mass spectrum of the higher radial excitation of the vector mesons. The results of calculations are shown in Table 1.

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*Independently this hypothesis was considered some later in [4].

Table 1

n		1	2	3	4
ρ	exp.	0.773	1.250(1.600)		
	theor.	0.773*	1.574	2.002	2.337
ψ	exp.	3.096	3.684	4.028	
	theor.	3.096*	3.685*	4.061	4.364
Y	exp.	9.460	10.010	10.320	
	theor.	9.472*	9.990*	10.280	10.521

To obtain the spectrum of mesons of type $q_i \bar{q}_j$ (i, j are types of quarks), we use the concept of "averaged quarks", i.e., we shall treat these mesons as composed of two quarks with equal masses:

$$m_{\tilde{q}} = \frac{m_{q_i} + m_{q_j}}{2} \quad . \tag{5}$$

The mass spectrum thus calculated with the same potential parameters as mentioned above is shown in Table 2.

Table 2

n		1	2	3	4
D	exp.	2.006			
	theor.	1.913	2.563 .	2.965	3.288
(bu)	theor.	5.027	5.570	5.914	6.191
(bc)	theor.	6.257	6.784	7.113	7.378

The ρ meson spectrum is of great interest as it is constructed using only one parameter m_u, since the model potential is considered to be fundamental. Thus, the first radial excited mass of ρ meson is calculated rather than fitted as usual. Our procedure of calculation possessing the universality of consideration of all vector-meson families gives the first excited mass of ρ' (1,6) resonance that may be regarded as a candidate for the first excited state of ρ mesons [5].

From the results for D meson, listed in Table 2, it is clear that the approximation of "averaged quark" results is about 5% deviation from experimental data. It should be noted that the recently discovered meson with mass 5.3 GeV /6/ (a preliminary result) interpreted as a bound state of heavy b and light u quarks coincides with the prediction of our model within the same 5% deviation.

2. As it was pointed above the model chosen allows us to investigate not only mesons but also baryons in a similar way. According to work /1/ we shall consider some properties of baryons. It is supposed that the baryons are bound states of three "averaged quarks". As usual, in the model of quasi-independent quarks one assumes that there exists a scalar potential constant of renormalization of the heavy mass of the free quark. Suppose that the constant renormalizing the meson quark mass is different from the corresponding baryon constant and hence

$$m_q^B = m_q^m + C_q \quad ,$$

where C_q is the only new parameter that arises in considering of baryons. The potential parameters α_S , b are chosen the same as in investigating of mesons.

If we compare the calculated proton mass with observed value we get the renormalized mass of u quark equal to $m_u = 116$ MeV. The quark wave functions found in solving of the set of equations are used for calculation of the r.m.s. proton radius:

$$\bar{r}^2 = \langle r^2 \rangle = \frac{\int d\vec{r}\, r^2 \,|\Psi(\vec{r})|^2}{\int d\vec{r}\, |\Psi(\vec{r})|^2} \tag{6}$$

and the ratio of the axial F_A and vector F_V coupling constants:

$$\frac{F_A(0)}{F_V(0)} = (1-2\delta)\Psi_B^{*\uparrow} \sum_{\ell=1}^{3} 2I_z^{(\ell)} \, r_3^{(\ell)} \; \Psi_B^{\uparrow} , \tag{7}$$

where $\delta = \frac{\langle L_z \rangle}{2I_z}$; $L_z^{(\ell)}$, $I_z^{(\ell)}$ are projections of the total and orbital angular momentum of ℓ quark on z axis, $\Psi_B^{\uparrow}$ is the baryon wave function with the projection of total angular momentum equal to $+\frac{1}{2}$. The values are: $\bar{r}_p = 0.76 \frac{1}{m_\pi}$ $(\bar{r}_p^{exp} = 0.629 \frac{1}{m_\pi})$ and $\frac{F_A}{F_V} = 1.23$ $((\frac{F_A}{F_V})^{exp} = 1.18 \pm 0.02)$. Using the formula for the baryon magnetic momentum:

$$\mu_B = \frac{(1-\delta)m_p}{E_B}(\Psi_B^{*} \uparrow \sum_{\ell=1}^{3} Q_\ell \, 2I_z^{(\ell)} \Psi_B \uparrow), \qquad (8)$$

(where Q_A is the quark charge, m_p , E_B are the masses of the proton and baryon, respectively), one finds the value $\mu_p = 2.60$ for the proton ($\mu_p^{exp} = 2.79$).

In such a way one can calculate the characteristics of other baryons. The agreement with experiment should be worse as the violation of SU(3) symmetry becomes considerable.

Table 3

	P	N	Λ	Σ^+	Σ^0	Σ^-	Ξ^0
μ_B exp.	2.79	-1.91	-0.60	2.83		-1.48	-1.8
μ_B theor.	2.60	-1.74	-0.88	2.67	0.89	-0.89	-0.78
$\bar{r}(\frac{1}{m_\pi})$	0.76	0	0.68	0.65	0	0.65	0.38

As is clear from the results of the calculation (Tables 1-3) our model that provides the same approach for investigation of both mesons and baryons is in sarisfactory agreement with the experimental data.

3. In this work, we have used so far the potential that is a combination of the Coulomb and linear confining terms and is transformed like a Lorentz scalar.

In general, the modern theory does not allow /7/ one to define the Lorentz structure of potential V(r). In this section we assume that under the Lorentz transformations V(r) is transformed as fourth vector component. Then the Dirac equation for quark wave functions is written as

$$[\vec{\gamma}\vec{P} - \gamma_0(E - V(r)) + m_q]\Psi(\vec{r}) = 0. \qquad (9)$$

The set corresponding to this equation is:

$$\begin{aligned} &\frac{d}{dr}(rg(r)) + \frac{k}{r}(rg(r)) - (E - V(r) + m)(rf(r)) = 0, \\ &\frac{d}{dr}(rf(r)) - \frac{k}{r}(rf(r)) + (E - V(r) - m)(rg(r)) = 0. \end{aligned} \qquad (10)$$

To define the influence of altering the Lorentz structure of the potential on the meson-mass spectrum we calculate the spectrum at the same quark mass as in ref.[3] where the scalar potential has been used. Considering the $\rho(0.77)$ and $\rho'(1.25)$ meson states as the ground and first excited states with the orbital momentum $\ell = 0$ and taking the u quark mass equal to 0.30 GeV we get potential parameters $\alpha_S = 0.56$, $b = 0.029(\mathrm{GeV})^2$ and the spectrum of ρ particles. In the same way from the $\Psi(3.096)$ and $\Psi'(3.685)$ resonances we get potential parameters $\alpha_S = 0.43$, $b = 0.063\ (\mathrm{GeV})^2$. The mass of c quark is chosen as 1.58 GeV. Finally if we choose $\alpha_S = 0.098$, $b = 0.157\ (\mathrm{GeV})^2$ and $m_b = 4.78$ GeV, then the calculated masses of the ground and first excited states coincide with the masses of $\Upsilon(9.46)$ and $\Upsilon(10.01)$ particles. All the results are presented in Table 4.

Table 4

n	1	2	3	/3/	4	/3/
ρ	0.773	1.250	1.480	15.46	1.720	1.780
Ψ	3.096	3.685	4.050	4.035	4.350	4.313
Υ	9.460	10.010	10.430		10.790	

From comparison of the results obtained under the assumption of the different Lorentz structure of potential it is seen that the mass spectra coincide within experimental errors. In this sense both the potentials are applicable in the particle spectroscopy. So, the Lorentz structure of potential is not yet clear and further analysis is required (See also discussion in[3,8,9]).

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