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PION FORM FACTOR BEYOND THE LEADING ORDER OF PERTURBATIVE QCD

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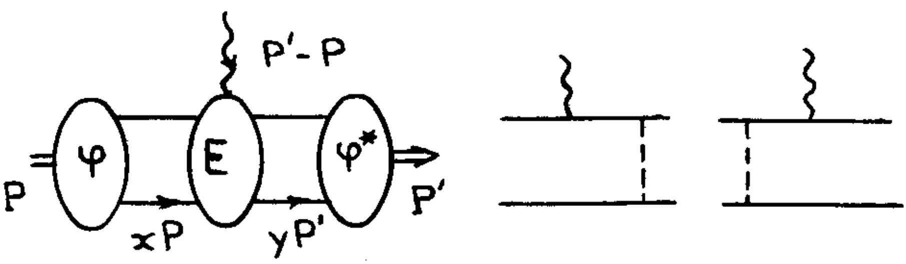
1. INTRODUCTION

Recently it has been realized that perturbative QCD is a reliable tool to study the elastic lepton-hadron and hadron-hadron processes at asymptotically large momentum transfers[1-4]. The most detailed analysis was performed for the simplest problem - the asymptotic behaviour of the pion form factor, pion treated as a bound state in a $q\bar{q}$ -system[5-8]. In refs.[5,6,8] it was shown, in particular, that for sufficiently large momentum transfer $q=P'-P$, the amplitude $T(P,P')$ related to the pion form factor may be written in the factorized form:

$$T(P,P')=\int_0^1 dx\int_0^1 dy\,\phi^*(y,\mu,\mu_R)\frac{1}{Q^2}E\left(\frac{Q^2}{\mu^2},\frac{Q^2}{\mu_R^2},x,y,\alpha_s(\mu_R)\right)\times \\ \times\phi(x,\mu,\mu_R)+O(1/Q^4), \tag{1.1}$$

where $Q^2=-q^2$, μ_R is the renormalization parameter of the ordinary R-operation, ϕ is the wave function describing the splitting of the pion into a $q\bar{q}$-state and E/Q^2 is the amplitude of the short-distance (SD) subprocess $qq\gamma^*\to q'\bar{q}'$. This picture (see Fig.1a) works to all orders and for all logarithms of perturbation theory (PT). A similar representation, as is well known, holds also for inclusive cross sections of some hard processes[9,10]. The splitting parameter $1/\mu$ in eq.



a) b) c)

Fig.1

(1.1) separates, as usual, short and long distances (or, using another language, μ is the renormalization parameter for the vertices $\bar{\psi}\gamma_5\gamma_\nu D^n\psi$ corresponding to composite operators).

In principle, the product $\phi^* \otimes E \otimes \phi$ does not depend on a particular choice of μ and μ_R. In practice, however, one can calculate only a few first terms of the perturbative expansion for $E(\ldots\alpha_s)$ and the resulting expression will depend on μ and μ_R. Furthermore, the calculations may be performed in various renormalization schemes and one can also use various recipes (schemes) for separating the contributions into Q^2- and p^2-dependent factors, and the results will depend, of course, on the schemes chosen.

In the lowest order the SD-amplitude E (see fig.1b,c) is given by

$$E(Q^2/\mu^2, Q^2/\mu_R^2, x, y, \alpha_s(\mu_R)) = \frac{2\pi\alpha_s(\mu_R)\, C_F}{N_c\, xy} \{1 + O(\alpha_s)\}, \qquad (1.2)$$

where $C_F = 4/3$ and $N_c = 3$ are the usual colour factors. Thus,

$$F_\pi^{(0)}(Q) = \frac{2\pi C_F \alpha_s(\mu_R)}{N_c Q^2} \left| \int_0^1 \frac{dx}{x} \phi(x, \mu, \mu_R) \right|^2 . \qquad (1.3)$$

To this order, E has only an implicit dependence on μ_R (through α_s) and does not depend on Q and μ. A logarithmic dependence on Q, μ, μ_R appears only in the next order. These logs $(\ln Q^2/\mu^2, \ln Q^2/\mu_R^2)$ tend to compensate the μ- and μ_R-dependence of $F_\pi^{(0)}$. It is clear that for some choice of μ, μ_R the lowest-order term $F_\pi^{(0)}(Q; \mu, \mu_R)$ may strongly differ from the "true" value of $F_\pi(Q)$ (which is the sum over all orders), and in this case the higher-order correction to $F_\pi^{(0)}$ will be large. A natural question is how to choose μ and μ_R in such a way that these corrections are as small as possible. If one takes, e.g., $\mu = \mu_R = Q$ then E is free from $\ln Q^2/\mu^2$- and $\ln Q^2/\mu_R^2$- factors which may be responsible for large higher corrections if $Q \gg \mu, \mu_R$ or $Q \ll \mu, \mu_R$. The meaning of the choice $\mu_R = Q$ is clear: one must equal μ_R to a scale characterizing the virtualness of the particles taking part in the SD-subprocess, and the latter is proportional to Q^2: $\langle k^2 \rangle \sim -a^2 Q^2$. Taking $\mu_R^2 = -\langle k^2 \rangle$ we include vertex- and propagator-correction into the effective coupling constant $\alpha_s(\langle |k^2| \rangle)$. In a similar way, the notation $\phi(x, \mu^2)$ implies that the pion wave function is probed at distances of the order of $1/\mu$, so the $\ln(Q^2/\mu^2)$ -terms indicate that μ must also be proportional to Q.

However, if the ratio $a^2 = \langle k^2 \rangle / q^2$ is very small (or large) compared to 1, then the perturbative expansion for $F_\pi(Q)$ will contain $\ln a^2$ -terms, and the choice $\mu = \mu_R = aQ$ is more preferable. In particular, if quarks inside the pion have roughly equal fractions of the pion momentum (i.e., if $\phi(x) \sim \delta(x-1/2)$), then the momentum of the exchanged gluon (fig.1b,c) is $q/2$ and the expansion of $F_\pi(Q)$ over $\alpha_s(Q/2)$ is the most natural one. Of course, 1/2 does not strongly differ from 1. However, for accessible momentum transfers ($Q \lesssim 2$ GeV) the difference between $a=1$ and $a=1/2$ is very essential.

The wave functions $\phi(x,\mu^2)$ describe the long-distance interactions, and this means they cannot be reliably calculated in PT. Perturbative QCD predicts only their evolution with growing μ^2 [3,5]. In ref.[11] it was shown, in particular, that

$$\phi(x,\mu^2) \to 6 f_\pi x(1-x) \qquad (1.4)$$

as $\mu^2 \to \infty$. Here $f_\pi = 133$ MeV is the pion decay constant. It appears due to the normalization condition[1,2]

$$\int_0^1 \phi(x,\mu^2)\, dx = f_\pi \,. \qquad (1.5)$$

However, for $\mu^2 \lesssim 1$ GeV the wave function $\phi(x,\mu^2)$ may strongly differ from its limiting form (1.4). For non-interacting particles having equal masses one may expect that $\phi(x) \sim \delta(x-1/2)$. When the interaction is switched on, the wave function (w.f.) broadens (Fig.2). The width Γ_0 of the "soft" wave function $\phi(x,M^2)$ (where $M \simeq 1/R_{\text{confinement}} \sim 200$–$500$ MeV) may be estimated as

$$\Gamma_0 \simeq (E_{int}/m_q)^2, \qquad (1.6)$$

where E_{int} characterizes the strength of the interaction and m_q is the typical mass of constituents. Hence, for hadrons built up of heavy quarks (e.g., for J/ψ and Υ-mesons) $\phi(x,M^2)$ is rather narrow since $E_{int} \sim M \lesssim 0.5$ GeV and $m_q > 1$ GeV. On the other hand, the pion w.f. must be very broad since $m_q \ll M$. One can also estimate the width of $\phi_\pi(x,M^2)$ using the data on the pion structure function: $f_{\bar{q}/\pi}(x,Q^2) \sim (1-x)$ for $Q^2 \sim$ 30-40 GeV^2 [12]. Starting with the well-known relation valid in the $x \sim 1$ region

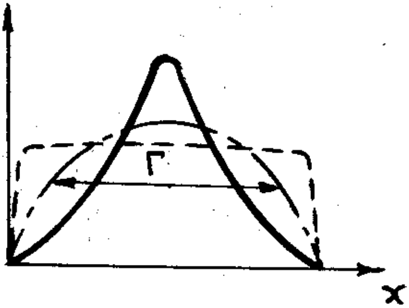


Fig.2

$$f(x,Q^2) \sim f(x,M^2)(1-x)^{\delta(Q^2,M^2,\Lambda^2)}, \tag{1.7a}$$

$$\delta(Q^2,M^2,\Lambda^2) = \frac{16}{27}\left(\ln\ln\frac{Q^2}{\Lambda^2} - \ln\ln\frac{M^2}{\Lambda^2}\right) \tag{1.7b}$$

(see, e.g., ref.[13]), we obtain that $\delta \simeq 0.6-0.8$ for $\Lambda \simeq 0.1-0.2$ GeV. Using the formula $f(x,M^2) \sim \phi^2(x,M^2)$ we obtain $\phi(x,M^2) \sim (x(1-x))^{0.2\div 0.1}$, i.e., a very broad w.f.

Note, that the amplitude (1.2) is singular for $x,y = 0$. Hence, for sufficiently broad w.f., the main contribution into the integral (1.1) is given by the region $x,y \ll 1$, where xQ^2 and xyQ^2 (i.e., the momenta squared of quark and gluon lines related to the SD-subprocess) are much smaller than Q^2 or, in other words, the distances at which the pion w.f. is really probed ($r^2 \sim 1/xyQ^2 \div 1/xQ^2$) are much larger than the most naive estimate $r \sim 1/Q$. In such a situation the choice μ^2, $\mu^2_R \sim xyQ^2 + xQ^2$ should be favoured over the naive choice $\mu^2 = \mu^2_R = Q^2$. To find a particular choice that provides the best convergence of the α_s-expansion, we must know $E(x,y)$ at least in the next-to-leading order.

In this paper we present (in Sec.2) our results for the $O(\alpha_s^2)$ -contribution into the SD-amplitude $E(x,y)$. In Sec.3 we discuss the magnitude of the resulting corrections to the pion form factor $F_\pi(Q)$. We analyze, in particular, their μ, μ_R- and w.f.-dependence. We observe there that for a broad w.f. the corrections are very large if we take $\mu = \mu_R = Q$, but just as expected, their magnitude is much smaller if we take μ^2, μ^2_R of the order of $xyQ^2 + xQ^2$. The perturbative QCD-analysis (neglecting power corrections) may be relied upon only if all "large" variables (e.g., $\langle xyQ^2 \rangle$, the average virtualness of the exchanged gluon), are larger than 1 GeV^2. So, to analyze where the perturbative QCD breaks down, we study in Sec.3 also the Q^2-dependence of $\langle xyQ^2 \rangle$. In Sec.4 we discuss the structure of higher-order corrections. In conclusion, we give a short summary of our results and formulate some further problems.

2. THE SHORT-DISTANCE AMPLITUDE IN THE 1-LOOP APPROXIMATION

The contributions of the multi-loop diagrams contain usually ultra-violet (or renormgroup) and mass logarithms, i.e., terms $(\alpha_s \ln Q^2/\mu_R^2)^N$ and $(\alpha_s \ln Q^2/p^2)^N$, $(\alpha_s \ln^2 Q^2/p^2)$ respectively. The RG logs appear after the subtraction of the UV-divergences, whereas the mass logs are present in some UV-convergent integrals. We assume that all masses are zero, and the IR cut-off (its magnitude is characterized by p^2) is provided, e.g., by the non-zero off-shellnesses of the external particles. To establish the factorization theorem (1.1) one must prove first the cancellation of the double-logarithmic contributions $(\alpha_s \ln^2 Q^2/p^2)^N$. The remaining single-logarithmic terms $(\alpha_s \ln Q^2/p^2)^N$ are splitted then into the "short-distance" and "long-distance" parts: $\ln Q^2/p^2 = \ln Q^2/\mu^2 + \ln\mu^2/p^2$. Then one must prove that these logs form two factors, i.e., that the amplitude $T(Q^2,p^2)$,

$$T(Q^2,p^2) = \alpha_s t_0 + \alpha_s^2 (t_1 + t_0 \quad \gamma_1 \ln Q^2/p^2) + \dots \qquad (2.1a)$$

can be represented as $E(Q^2/\mu^2) \otimes \Gamma(\mu^2/p^2)$, where

$$\Gamma = 1 + \alpha_s(\gamma_1 \ln\mu^2/p^2 + a_1) + \qquad (2.1b)$$

$$E = \alpha_s t_0 + \alpha_s^2((t_1 - a_1) + t_0 \otimes \gamma_1 \ln Q^2/\mu^2) + \dots \qquad (2.1c)$$

A detailed discussion of the factorization machinery may be found in refs.[5,6,8-10].

In eq. (2.1b) it is taken into account that the Γ-factor, usually related to matrix elements of some local operators, contains in general a constant non-logarithmic term $\alpha_s a_1$. Hence, to find the $O(\alpha_s^2)$-contribution into E, one must calculate the 1-loop diagrams both for $T(q,p)$ (i.e., for a process $q\bar{q}\gamma^* \to q\bar{q}$) and for matrix elements of the relevant operators (see eq. (2.1c)). Note, that if the IR cut-off is provided by the nonzero off-shellnesses of the external quarks, then both t_1 and a_1 are not gauge invariant and only the difference $e_1 = t_1 - a_1$ does not depend on the gauge choice (see ref.[14]). In QCD, the most convenient IR cut-off is based on the dimensional regularization

$$\frac{d^4k}{(2\pi)^4} \to \frac{d^{4+2\epsilon}k}{(2\pi)^{4+2\epsilon}} (4\pi e^{-\gamma_E})^{+\epsilon} (\mu^2)^{-\epsilon} \qquad (2.2)$$

combined with the subsequent removal of the $1/\epsilon$ -poles (these correspond formally to $\ln(\mu^2/p^2)|_{p^2=0}$). Our choice (2.2) corresponds to the $\overline{MS}$-scheme/14/ The $4\pi e^{-\gamma_E}$ - factor compensates the artifact contributions $\ln(4\pi)$ and γ_E present in the standard minimal-subtraction (MS) scheme. External lines are now taken on-shell: $p^2 = m^2 = 0$. Hence, the original non-regularized amplitude is(formally) gauge-invariant. The dimensional regularization preserves the gauge invariance, hence t_1 is also gauge-invariant now. Furthermore, if one uses the IR cut-off based on eq. (2.2) then, according to a straightforward calculation, $a_1 = 0$ and it is sufficient to calculate only the 1-loop diagrams for the SD-subprocess $qq\gamma^* \to q'\bar{q}'$ (cf. refs./15, 16/).

A few words about kinematics. The initial state quarks have momenta $xP, (1-x)P$ and the final state ones - $yP', (1-y)P'$. Moreover, $P^2 = P'^2 = 0$. The pion form factor is usually defined by

$$\langle P' | J_\nu(0) | P \rangle = (P_\nu + P'_\nu) F_\pi(Q). \tag{2.3}$$

It is convenient to get rid of the ν-index multiplying eq. (2.3) by P'_ν , i.e., to define F_π by

$$F_\pi(Q) = \frac{2P'_\nu}{Q^2} \langle P' | J^\nu(0) | P \rangle. \tag{2.4}$$

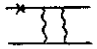
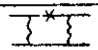
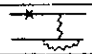
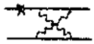
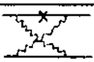
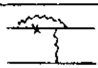
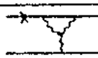
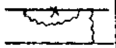
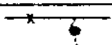
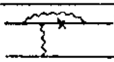
If we adhere to this definition, then the total lowest-order contribution into $F_\pi(Q)$ (see eq. (1.3)) is given by the diagram 1b only. The contribution of fig.1c is zero, because the photon vertex $\hat{P}'$-factor "kills" the $\gamma_5 \hat{P}'$ -factor due to the matrix element of the final state $\bar{\psi}\gamma_5\gamma_\lambda D^n \psi$ -operator.

We work in Feynman gauge. To remove the mass singularities we use the recipe (2.2). For the UV-divergent integrals we also use the dimensional regularization

$$\frac{d^4k}{(2\pi)^4} \to \left(\frac{d^{4-2\epsilon}k}{(2\pi)^{4-2\epsilon}}\right)(\mu_R^2)^\epsilon (4\pi e^{-\gamma_E})^{-\epsilon} \tag{2.5}$$

and 't Hooft's renormalization/17/. It is worth noting here that in our calculations all integrals either have mass singularities or are UV-divergent, but not both. Our results for all relevant diagrams are given in Table 1. The diagrams not included therein give zero contributions for just the same

<u>Table 1</u>

1		$-C_F y \ln y [L(x) + \ln y/2 + 1]/\bar{y}$
2		$-C_F x \ln x [L(1) + \ln y/\bar{y} + \ln x/2 + 1]/\bar{x}$
3		$-(C_F - C_A/2)[L^2(xy) - \pi^2/6 + L^{(R)}(xy) + 2L(xy) + 4]/2$
4		$(C_F - C_A/2)[L^2(x\bar{y}) - \pi^2/6 + 2L(x\bar{y}) + 2\ln y/\bar{y} \cdot (L(x) + \ln y/2 + 1) + 2\ln\bar{y}\,(L(x) + \ln\bar{y}/2 + 1)]/2$
5		$(C_F - C_A/2)[L^2(\bar{x}y) - \pi^2/6 + 2L(y)(1 + \ln x/\bar{x}) - 2\ln\bar{y}(L(x) + \ln\bar{y}/2 + 1) - 2(\bar{x} - y)(S(x,y) + \ln x \ln y/(\bar{x}\bar{y})) + \ln^2 x/\bar{x} + 2x\ln x/\bar{x} + 2\ln\bar{x} + 2\ln x]/2$
6		$-(C_F - C_A/2)[L^2(\bar{x}\bar{y}) - \pi^2/6 - 2L(\bar{x}\bar{y}) + 2L(x)(1 + y\ln y/\bar{y}) + A(x,y) + y\ln^2 y/\bar{y} + xy\ln y/(\bar{x}\bar{y}) - y\ln(xy)/(\bar{x}\bar{y}) + x\ln x/\bar{x} + \ln(\bar{x}\bar{y}) + 2]/2$
7		$(C_F - C_A/2)[2L(x)(1 + y\ln y/\bar{y}) - L^{(R)}(x) + y\ln^2 y/\bar{y} - y\ln y/\bar{y} - 2]/2$
8		$C_A[4L(xy) - 3L^{(R)}(xy) + 2]/4$
9		$C_A[2L(x)(1 + \ln y/\bar{y}) - 3L^{(R)}(x) + \ln^2 y/\bar{y} + \ln y + 4]/4$
10		$C_A[2L(1)(1 + \ln x/\bar{x}) + \ln^2 x/\bar{x} + 2x\ln x\ln y/(\bar{x}\bar{y}) + 2\ln x - 2]/4$
11		$-C_F[L^{(R)}(1) - x\ln x/\bar{x} - 1]/2$
12		$C_F[L^{(R)}(x) - 1]/2$
13		$-5C_A[L^{(R)}(xy) - 31/15]/6 + N_f[L^{(R)}(xy) - 5/3]/3$
14		$-(C_F - C_A/2)[A(y,x) + (1 - 2x)\ln y/(\bar{x}\bar{y})]/2$
15		$C_F \ln y/(2\bar{y})$

reason as the diagram shown in <u>fig.1c.</u> The total contribution is

$$E^{(1)}(x,y,\alpha_s,Q^2/\mu^2,Q^2/\mu_R^2) = \frac{2\pi C_F}{N_c xy}\alpha_s\{1 + \frac{\alpha_s}{2\pi}[C_F((2+\ln x)L(\sqrt{xy}) -$$

$$-\frac{1}{2}L^{(R)}(1)+\frac{\ln x}{2}(3-x/\bar{x})-1/3)-\frac{1}{4}(\frac{11}{3}C_A-\frac{2}{3}N_f)\times$$

$$\times(L^{(R)}(xy)-5/3)+(C_F-C_A/2)(Sp(\bar{x})-Sp(x)+\ln\bar{x}\ln(y/\bar{y})-$$

$$-5/3)+(C_F-C_A/2)\frac{1}{(y-x)^2}[\frac{y^2\bar{y}+x^2\bar{x}}{y-x}(Sp(\bar{x})-Sp(x)- \tag{2.6}$$

$$-\ln\bar{x}\ln y)+2xy\ln x+(x+y-2xy)\ln\bar{x}]+\{x\leftrightarrow y\}]\},$$

where $\bar{x}\equiv 1-x$, $\bar{y}\equiv 1-y$, $C_A\equiv N_c=3$, $L(a)\equiv\ln(aQ^2/\mu^2)$, $L^{(R)}(a)\equiv\ln(aQ^2/\mu_R^2)$ and $Sp(a)$ is the Spence function

$$Sp(a)=-\int_0^1\frac{dz}{z}\ln(1-az). \tag{2.7a}$$

The functions $A(x,y)$ and $S(x,a)$, present in Table 1, are given by the formulas

$$A(x,y)=[2y^2\bar{y}S(x,\bar{y})-\bar{y}(3y-x)\ln(\bar{x}\bar{y})+y^2(3y-x-2)\ln(xy)/\bar{y}]/(y-x)^2 \tag{2.7b}$$

$$S(x,a)=[Sp(x)-Sp(\bar{x})+Sp(a)-Sp(\bar{a})+\ln\bar{x}\ln\bar{a}-\ln x\ln a]/(\bar{x}-a). \tag{2.7c}$$

Note, that although there are terms containing $(y-x)^{-3}$ in eq. (2.6), $E^{(1)}(x,y)$ is not singular for $x=y$.

3. THE STRUCTURE OF 1-LOOP CORRECTION

To get a notion about the structure and magnitude of the 1-loop corrections, let us represent $F_\pi(Q)$ as

$$F_\pi(Q)=F_\pi^{(0)}\{1+\frac{\alpha_s(\mu_R)}{\pi}B(Q,\mu,\mu_R)+O(\alpha_s^2)\}, \tag{3.1}$$

where $F_\pi^{(0)}$ is the lowest-order contribution eq. (1.3) and

$$B=[-A\ln\frac{Q^2}{\mu^2}-\frac{2}{3}\ln\frac{\mu^2}{\mu_R^2}]-\frac{1}{4}(11-\frac{2}{3}N_f)\ln\frac{Q^2}{\mu_R^2}+C. \qquad (3.2)$$

The term in the square brackets in eq. (3.2) corresponds to the contributions which tend to change $\phi(x,\mu^2)\to\phi(x,a_1^2Q^2)$ whereas the $\ln Q^2/\mu_R^2$ -term tends to substitute $g(\mu_R)$ by the effective coupling constant $\bar{g}(a_2Q)$. We emphasize that there are no a priori grounds for taking a_1 and a_2 equal to 1.

The coefficients A and C depend on a particular choice of the wave functions $\phi(x)$, $\phi(y)$. To analyse this dependence we use the simplest parameterization

$$\phi_r(x)=f_\pi\frac{\Gamma(2+2r)}{\Gamma^2(1+r)}x^r(1-x)^r. \qquad (3.3)$$

The $f_\pi\Gamma/\Gamma^2$-factor is necessary to satisfy the normalization condition (1.5). The magnitude of A and C for some r is given in Table 2.

Table 2

r	A	C	$B(\mu^2=\sqrt{xy}\,Q^2)$
1	0	+7.25	+3.04
0.5	+1.2	+13.4	+2.32
0.2	+5.0	+54.4	+0.39
0.1	+11.5	+204	−2.63
0.05	+24.8	+803	−8.53
0.01	+131	+19953	−55.2

If we take $a_1=a_2=1$ (i.e., $\mu=\mu_R=Q$), then the magnitude of the $O(\alpha_s^2)$ -correction is proportional to that of C. From Table 2 it is seen that even for $r=1$, i.e., for a rather narrow w.f., the correction is rather large: B=C= 7.25, i.e., 70% - correction if $\alpha_s/\pi\sim 0.1$. As it was argued in the Introduction, the best choice for a narrow wave function should be $\mu^2=\mu_R^2=Q^2/4\div Q^2/2$. Really, if we take μ^2, μ_R^2 in this interval, then B= 4.1 ÷ 5.7, i.e., the $O(\alpha_s^2)$ -correction is smaller, just as expected. Strictly speaking, the choice $\mu^2=\mu_R^2=Q^2/4\div Q^2/2$ optimizes the convergence of perturbative expansion only if we use a "physical" or "momentum-space-subtraction" scheme, i.e., if $\bar{g}(\mu)$ corresponds to a vertex with momenta satisfying $k_i^2=-\mu^2$. The meaning of the parameter μ_R in the $\overline{MS}$-scheme that we use is less transparent. It is known, however, that if one expands the effective coupling constant $\bar{g}_i(\mu)$ (related to the i-th scheme) over $\ln^{-1}(\mu^2/\Lambda^2)$

$$\frac{\bar{g}_i^2(\mu^2)}{(4\pi)^2}=\frac{1}{b_0\ln(\mu^2/\Lambda_i^2)}\{1-\frac{b_1}{b_0^2}\frac{\ln\ln\mu^2/\Lambda_i^2}{\ln\mu^2/\Lambda_i^2}+\ldots\}, \qquad (3.4)$$

then the series expansions obtained for a renormgroup-invariant quantity like $F_\pi(Q)$ in, say, i-th and j-th scheme differ only by rescaling $\Lambda_i=\kappa_{ij}\Lambda_j$, where κ_{ij} is a universal number relating these schemes. In particular, Λ_{PH} weakly depends on the vertex chosen to define $\bar{g}_{PH}(\mu)$: $\Lambda_{PH}/\Lambda_{\overline{MS}}\simeq 2$ (see ref. /19/). This means that the choice $\mu^2=\mu_R^2=Q^2/4\div Q^2/2$ in a physical scheme corresponds to the choice $\mu^2=\mu_R^2=Q^2/16\div Q^2/8$ in the MS-scheme. For this choice $B=1.0\div 2.6$ for $r=1$, i.e., the $O(\alpha_s^2)$-correction is now sufficiently smaller (10÷26% if $\alpha/\pi\simeq 0.1$).

As r decreases (i.e., as the w.f. broadens), the coefficient C (and, consequently, the magnitude of the $O(\alpha_s^2)$-correction for $\mu=\mu_R=Q$) rapidly (like $1/r^2$) increases. At the same time (though not so very fast, only like $1/r$) increases the coefficient A. For very small r, the main contribution into A and C is given by terms which are more singular at $x=y=0$ than $E^\circ(x,y)$. To estimate these terms we represent $E^1(x,y)$ in the following way

$$E^1(x,y)=E^\circ(x,y)\{1+\frac{\alpha_s(\mu_R)}{4\pi}[C_F(\ln^2\frac{xyQ^2}{\mu^2}-\ln^2\frac{Q^2}{\mu^2})+$$
$$+4C_F\ln(xy(\frac{Q^2}{\mu^2})^2)-2C_F\ln\frac{Q^2}{\mu_R^2}+3C_F\ln(xy)- \qquad (3.5)$$
$$-(11-\frac{2}{3}N_f)\ln(\frac{xyQ^2}{\mu_R^2})+\phi(x,y)-2(C_F-\frac{C_A}{2})\ln(xy)]\},$$

where $\phi(x,y)$ is regular for $x=y=0$ and does not depend on μ, μ_R and Q.

The SD-amplitude $E(x,y)$ depends in particular on momentum invariants xQ^2, xyQ^2, yQ^2 containing x,y and that is why there appear $\ln x$ and $\ln y$ terms. Mass and renormgroup logs differ in their origin, so it makes sense to consider them separately.

The most simple structure have the corrections to the gluon propagator (see diagram 13, Table 1). They depend only on xyQ^2 The gluon propagator renormalizes multiplicatively, and this means that the corresponding log corrections $\ln(xyQ^2/\mu_R^2)$ will exponentiate in higher orders. Thus, if the appropriate choice of μ_R reduces considerably the magnitude of the 1-loop correction, then the higher-order corrections also become smaller. Note that in an Abelian theory (e.g. in QED) the behaviour of $\bar{g}(\mu_R)$ is completely determined by that of the renormalized propagator of vector particles, and the most natural choice for μ_R in this case is $\mu_R^2 = xyQ^2$ (in a physical scheme). The average value of μ_R^2 for such a choice may be easily estimated if we define the 1-loop correction due to the $\ln(xyQ^2/\mu_R^2)$ -term to vanish for $\mu_R^2 = \langle\mu_R^2\rangle$. This gives

$$\langle\mu_R^2\rangle = \langle x\rangle\langle y\rangle Q^2 , \tag{3.6}$$

where

$$\langle x\rangle = \exp(\langle\ln x\rangle) \equiv \exp\{(\int_0^1 \frac{dx}{x}\ln x\,\phi(x))(\int_0^1 \frac{dx}{x}\phi(x))^{-1}\} \tag{3.7}$$

For small r we have $\langle x\rangle = \exp(-1/r)$, e.g., $\langle x\rangle = 5\cdot 10^{-5}$ $(7\cdot 10^{-3})$ for $r = 0.1$ (0.2).

In QCD, the effective coupling constant $\bar{g}(\mu_R)$ depends both on the renormalization constant of the quark-gluon vertex and of the quark propagator. Thus, to find $\langle\mu_R^2\rangle$ in this case we take the sum of all terms $\ln(xyQ^2/\mu_R^2)$, $\ln(xQ^2/\mu_R^2)$, $\ln(yQ^2/\mu_R^2)$ present in the UV-divergent diagrams 3,7-9, 11-13 (Table 1). A simple calculation shows that for small r this sum vanishes if we take

$$\mu_R^2 = \langle\mu_R^2\rangle = Q^2\exp[-\frac{1}{r}\,\frac{64-4N_f}{41-2N_f}]. \tag{3.8}$$

If $N_f \leq 6$, the exponent in eq. (3.8) is close to $(-3/2r)$. In other words, these corrections are minimal for $\mu_R^2 = \langle x\rangle^{3/2} Q^2$, i.e., when $\langle\mu_R^2\rangle$ is equal to the geometric average of quark and gluon virtualnesses. It is easy to note, however, that the contribution of the RG-logs into C rises only as $1/r$ for small r. Hence, the main contribution $(O(1/r^2))$ into C is given by mass logarithms. The largest contribution, equal to

$$E^{(0)}(x,y)\,\frac{\alpha_s(\mu_R)}{4\pi}\,\frac{C_A}{2}(\ln^2(xy) + 2\ln(xy)\ln\frac{Q^2}{\mu^2}) \tag{3.9}$$

is given by the diagrams 9,10 (Table 1). Similar terms appear also in the "nonplanar" diagrams 3-6 (Table 1), but they have a colour factor $(C_F - C_A/2)$ which is 9 times $(9 \simeq N_c^2)$ smaller than that of the diagrams 9,10, in accordance with the rules of the $1/N_c$ -expansion [20].

It is easy to calculate that the contribution (3.9) vanishes for $\mu^2 = \sqrt{xy}\,Q^2$. The average value of μ^2 in this case is given by

$$\langle\mu^2\rangle \simeq Q^2 \exp(-3/2r). \qquad (3.10)$$

So far we have analyzed only the μ- and μ_R-dependence of the ℓ-loop corrections to E for fixed wave functions. In fact, these also depend on μ and μ_R, e.g., in the leading logarithm approximation it is possible to show that for $\mu = \mu_R$ and x close to 0

$$\phi(x,\mu^2)|_{x\sim 0} \simeq \phi(x,\mu_0^2)\exp\{\ln x \times \times\left(\frac{2C_F}{b_0}\ln\frac{\ln(\mu^2/\Lambda^2)}{\ln(\mu_0^2/\Lambda^2)} + O(\alpha_s(\mu^2) - \alpha_s(\mu_0^2))\right\}. \qquad (3.11)$$

In other words, the exponent $r(\mu^2)$ in the parameterization $\phi(x,\mu^2) \sim x^{r(\mu^2)}$ grows with growing μ^2. On the other hand, $\langle x\rangle$ and $\langle\mu^2\rangle$ (see eqs. (3.6), (3.7)) depend on r, i.e., on the form of the w.f. Taking $\mu^2 = \langle\mu^2\rangle$ in eq. (3.11) we obtain a system of two equations. Solving it we obtain the Q^2-dependence of $\langle\mu^2\rangle$ and this enables us to estimate the magnitude of $\langle k^2\rangle$ (the average virtualness of lines inside the SD-subprocess) for various Q^2.

If $|\langle k^2\rangle| \lesssim 1$ GeV^2 then PT usually does not work. In particular, if $\Lambda_{PH} \gtrsim 0.5$ GeV then the coupling constant $\alpha_s(-\langle k^2\rangle)$ is rather large in this region and the expansion over $\alpha_s(-\langle k^2\rangle)$ is meaningless. However, even if $\Lambda \lesssim 0.1$ GeV and the perturbative expansion converges rapidly enough, this expansion is spoiled by power corrections $O(M^2/\langle k^2\rangle)$ which cannot be neglected for $-\langle k^2\rangle \lesssim 1$ GeV^2 (M as usual, is a typical hadronic scale: $M \sim 1/R_{conf} \sim 500$ MeV).

To find the explicit dependence of $\langle\mu^2\rangle$ on Q^2 one must specify the parameter Λ in eq. (3.11) and $\phi(x,\mu_0^2)$ at some μ_0^2. Note that from eqs. (1.7) and (3.11) it follows that if $f(1-x,M^2) \sim \phi^2(x,M^2)$ then for $x \sim 0$ this connection is not affected by scaling violations. That is why we take $r(\mu^2 = 30\ \text{GeV}^2) \simeq 0.5$ (see the discussion after eq. (1.6)). If we choose $\Lambda_{PH} = 0.1$ GeV then $\langle\mu^2\rangle = 1$ GeV^2 for $Q^2 \simeq 100$ GeV^2. If we choose a larger value for Λ_{PH} then $\langle\mu^2\rangle = 1$ GeV^2 is satisfied at larger Q^2 (at $Q^2 = 200$ GeV^2 for $\Lambda = 0.2$ GeV and at $Q^2 = 3000$ GeV^2 for $\Lambda = 0.4$ GeV). If Λ_{PH} is smaller than 0.1 GeV then $\langle\mu^2\rangle = 1$ GeV^2 is satisfied at smaller Q^2 (at $Q^2 = 60$ GeV^2 for $\Lambda = 50$ MeV, at $Q^2 = 50$ GeV^2 for $\Lambda = 30$ MeV and at $Q^2 = 40$ GeV^2 for $\Lambda = 10$ MeV).

Thus, for any reasonable choice of Λ_{PH} a simple-minded PT can be used for calculating $F_\pi(Q)$ in QCD only in the region $Q^2 \gtrsim 100$ GeV^2. This value is much larger than the largest now accessible momentum transfer $Q^2 \simeq 4$ GeV^2. If the parameter Λ_{PH} is sufficiently small ($\Lambda_{PH} \le 0.1$ GeV, say) then PT is spoiled only by large power corrections, which reflect essentially the fact that the pion has a finite size. There are also power corrections due to quark confinement and other nonperturbative effects. Hence, to understand the behaviour of $F_\pi(Q)$ for moderately large Q^2 one must take into account the power corrections in some way (e.g., phenomenologically). A preliminary estimate made by one of the authors (A.R.) shows that even making the simplest assumptions about the structure of power corrections gives for $F_\pi(Q)$ a curve which is in a satisfactory agreement with experimental data in the region $Q^2 = 1 \div 4$ GeV^2 *. It is worth noting here that if one simply extrapolates the asymptotic formula (1.3) for $Q^2 F_\pi(Q)$ in the region $Q^2 \le 4$ GeV^2, then, for any choice of the wave function, the resulting curve crosses the experimental one ($Q^2 F_\pi^{exp}(Q) \simeq 0.4$ GeV^2) at roughly a right angle (see ref. /7/). But, as we have seen in the present paper, there is no justification for such an extrapolation, and the drastic disagreement mentioned above should not be treated as an evidence against QCD.

4. STRUCTURE OF HIGHER-ORDER CORRECTIONS

The minimization procedure for the 1-loop correction described in the preceding section makes sense only if the terms most singular at $x = y = 0$ in higher orders of PT are completely determined by those of the 1-loop approximation. Otherwise decreasing the 1-loop correction (which, as we have seen, can always be made zero by an appropriate choice of μ) may not result in reducing the multi-loop corrections. So, now we want to discuss the structure of $E(x, y, \mu, Q)$ in higher orders of PT.

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*A detailed discussion of this problem will be given elsewhere.

In particular, the μ-dependence of E can be derived from eq. (3.11) using the fact that the product $\phi \otimes E \otimes \phi$ (see eq. (1.1)) does not depend on μ. Note that in deriving eq.(3.11) we have also taken into account the RG-logs, i.e., the dependence of α_s on μ_R. If this dependence is ignored, then

$$\phi(x,\mu^2)|_{x\sim 0} \sim \exp\{\frac{\alpha_s}{2\pi} C_F \ln x \ln\frac{\mu^2}{\mu_0^2} + O(\alpha_s^2)\}\phi(x,\mu_0^2). \qquad (4.1)$$

This means that the 1-loop term $C_F\frac{\alpha_s}{2\pi}\ln(xy)\ln\frac{Q^2}{\mu^2}$ from eq. (3.5) exponentiates in higher orders. Thus, the choice $\mu^2=\sqrt{x}yQ^2$ reduces the higher-order corrections only if the 1-loop term $(\alpha_s C_F \ln^2(xy))/4\pi$ from eq. (3.5) also exponentiates.

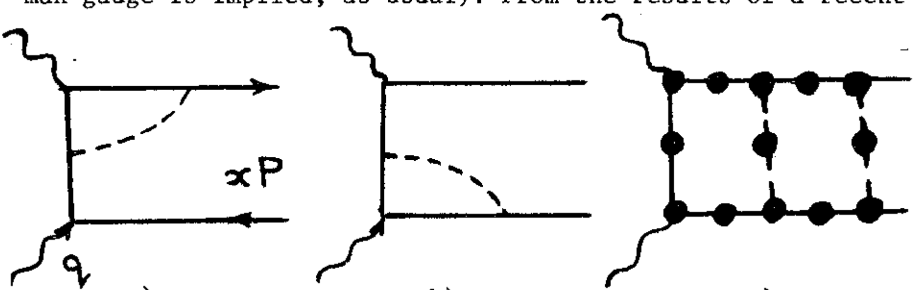
A similar problem arises also in the analysis of T(q,p), the forward Compton amplitude (as is well-known, T(q,p) can be related to the deep inelastic cross-section). For T(q,p) one has a representation similar to eq. (1.1):

$$T(P,q) \equiv \tilde{T}(\omega, Q^2) = \int_0^1 \frac{dx}{x} E(x\omega, Q, \mu, \mu_R, g)\, f(x,\mu,\mu_R,g). \qquad (4.2)$$

We recall that $q^2=-Q^2$, $\omega=2(Pq)/Q^2$ and the region $|\omega|<1$ is analysed. In the lowest order $E=(1-x\omega)^{-1}$ The region $x\omega\sim 1$ in this case is an analogue of the region $x,y\sim 0$ for pion form factor. In the next-to-leading order the main contribution, equal to

$$2\frac{1}{1-x\omega}\frac{\alpha_s}{4\pi}C_F(\ln^2\frac{(1-x\omega)Q^2}{\mu^2} - \ln^2\frac{Q^2}{\mu^2}), \qquad (4.3)$$

is given for $x\omega\sim 1$ by the diagrams shown in fig.3a,b (Feynman gauge is implied, as usual). From the results of a recent



a) b) c)

Fig.3

paper [21], it follows that this contribution exponentiates in higher orders. To simplify the analysis, the authors of ref. [21] have used the axial gauge (see, e.g., ref. [10]). In this gauge, the most singular terms (in particular, that given by eq. (4.3)) appear only in the generalized ladder graphs (fig.3c).

It is easy to note that the contributions (4.3) and (3.9) as well as the diagrams where they appear (fig.3a,b and diagrams 9,10 of Table 1, respectively) have a similar structure. Furthermore, a straightforward calculation shows that in the axial gauge the terms having structure of eq. (3.9) appear only in the ladder diagrams 1,2 (Table 1) and just with the right colour factor C_F. Thus, there exists a full analogy between the two problems. This practically guarantees the exponentiation of the $O(\alpha_s \ln^2(xy))$ -term and, hence, the decrease of higher order corrections if the choice $\mu^2 = \sqrt{xy}\,Q^2$ is made.

However, even for $\mu^2 = \sqrt{xy}\,Q^2$ the 1-loop contribution contains less singular terms $O(\alpha_s^2 \ln x/(xy))$, $O(\alpha_s^2 \ln y/(xy))$ which give $O(1/r)$-contribution into B. Of course, taking $\mu^2 = a\sqrt{xy}\,Q^2$ with an appropriately chosen a it is possible to remove from B the term $O(1/r)$ also. But it should be remembered that the 2-loop correction to anomalous dimensions of the composite operators (the $O(\alpha_s(\mu^2))$)-term in eq. (3.11) gives a contribution of the same order into $F_\pi(Q)$. Furthermore, it is well-known that both the 1-loop corrections into E and the 2-loop correction to anomalous dimensions depend on the chosen renormalization scheme (see ref. [14]). This dependence disappears only if one takes into account both corrections simultaneously. Note that in eq. (3.11) the $O(\alpha_s(\mu^2))$-correction term stays in the exponent. This suggest that the $O(\alpha_s \ln xy)$ -factor in E should also exponentiate. Otherwise it is very hard to understand how the scheme dependence is cancelled.

5. SUMMARY AND CONCLUSIONS

Summarizing this paper, we may conclude that our analysis of the 1-loop correction shows that the large corrections to $F_\pi(Q)$ are mainly due to an unproper choice of the parameters μ and μ_R. The 1-loop correction is considerably reduced if we make the physically most natural choice $\mu^2, \mu_R^2 \sim \langle k^2 \rangle$, where $\langle k^2 \rangle$ is a typical virtualness inside the SD-subprocess. A preliminary analysis of the general structure of the perturbative expansion indicates that this choice reduces the higher-order corrections as well. A very important problem is to complete this analysis, i.e., to show that all $\ln x$, $\ln y$ -factors really exponentiate in higher orders. As a first step one must sum up the double logs $(\alpha_s \ln^2 xy)^N$. Due to the results of ref.[21] there is no doubt about the exponentiation of these terms into the Sudakov exponent. However, a considerable effort probably should be made to generalize this result for all non-leading logarithms.

After the removal of the most singular $O((\ln^2 xy)/(xy))$ - term from the 1-loop correction to the SD-amplitude $E(x,y)$ (this is achieved by taking $\mu^2 = \sqrt{xy}\, Q^2$), there remain $O(\frac{1}{xy} \ln(xy))$ -terms in $E^1(x,y)$. These less singular terms give a contribution into $F_\pi(Q)$ of the same order as the 2-loop corrections to anomalous dimensions of the corresponding composite operators. Thus, a problem of primary importance is the calculation of these corrections.

The analysis performed in the present paper shows also that for accessible momentum transfers the average virtualness of the exchanged gluon is very small compared to the typical hadronic scale $M \sim 0.5$ GeV. In other words, for $Q^2 \lesssim 100$ GeV^2 the pion form factor is not a truly short-distance problem and to understand the behaviour of $F_\pi(Q)$ for moderately large Q^2 (in particular, to clarify the true nature of the quark counting rules proposed in refs.[22,23]) one should develop methods of taking into account the effects usually referred to as power (or higher-twist) corrections.

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