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QUANTUM NONLOCAL CONSERVED CHARGES FOR GENERALIZED NONLINEAR SIGMA MODELS

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There exist some two-dimensional classical field theoretical models possessing an infinite number of conserved quantities. If these classical quantities survive in the case of quantization, this provides factorization of the S-matrix and the absence of particle production[1,2]. This is the case of the Sine-Gordon theory [3] and of the O(N) nonlinear sigma model in which there are both local[4] and nonlocal[5,6] conserved currents. The factorization property and the absence of particle production made it possible to compute explicitly the S-matrix[2] for these theories.

For the classical generalized nonlinear sigma models there exist infinite sets of local[7] as well as nonlocal[8,9] conserved currents. In ref.[7] it has been shown that for the $O(N) \otimes O(N)$ and $U(N) \otimes U(N)$ models the classical local conserved charges survive in the case of quantization. However, for CP^{N-1} model it is not the case; because of the anomalies, breacking the conservation laws, arising in the quantum case[7].

In the present article it is shown that the classical nonlocal charges[8,9] for the generalized nonlinear sigma models survive in the case of quantization. Consequently, quantum nonlocal charges and asymptotical states for these models result in the absence of particle production and the factorization of the S-matrix [6]. However for the CP^{N-1} and the Grassmann theories, for which nonlocal charges also exist, the existence of asymptotical free states is not established because of long-range forces[10]. For the latter two models, quantum local charges do not exist[7], but the absence of particle production may be checked up to the forth-order perturbation term in coupling constant of the S-matrix[11].

For the classical generalized nonlinear sigma models the infinite set of conserved currents has the following form[9]

$$J^{(k)}_{\mu}(x) = [A_{\mu}(x), \chi^{(k)}(x)] + \epsilon_{\mu\nu} [A^{\nu}(x), \chi^{(k-1)}(x)], \quad (k = 1,2,\ldots), \qquad (1)$$

where the functions $\chi^{(k)}$ are determined from the equation

$$\partial_\mu \chi^{(k)}(x) = \epsilon_{\mu\nu} \{ \partial^\nu \chi^{(k)}(x) - i[A^\nu(x), \chi^{(k-1)}(x)] \} . \tag{2}$$

Here $[A, B]$ denotes the matrix commutator and $\epsilon_{01} = \epsilon_{10} = 1$. The solutions of eqs. (2) are given by

$$\chi^{(k)}(x) = -\int_{-\infty}^{x_1} dy_1 \{ \partial_0 \chi^{(k-1)}(x_0, y_1) - i[A_0, \chi^{(k-1)}](x_0, y_1) \}. \tag{3}$$

Starting from $\chi^{(1)} = C$ and $\chi^{(0)} = 0$, we have one infinite series of functions (3) and corresponding nonlocal currents (1). From these currents we have the conserved charges

$$Q^{(k)}(x_0) = \int_{-\infty}^{\infty} dx_1 J_0^{(k)}(x_0, x_1), \quad (k=1,2,\ldots.). \tag{4}$$

The explicit form of the second conserved current is given by

$$Q^{(2)} = \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{x_1} dy_1 [A_0(x_0, x_1), [A_0(x_0, y_1), C]] + i \int_{-\infty}^{\infty} dx_1 [A_1(x), C], \tag{5}$$

where C is an $N \times N$ Hermitean constant matrix. For particular models $A_\mu(x)$ are given by:

$$A_\mu^{jk}(x) = ig^{j\ell}(x)\, \partial_\mu g^{\ell k}(x) = 2i(n_j \partial_\mu n_k - \partial_\mu n_j\, n_k) \tag{6}$$

for the O(N) model,

$$A_\mu^{jk}(x) = i g^{j\ell} \partial_\mu g^{\ell k}(x) = 2i(\bar{z}_j \partial_\mu z_k(x) - \partial_\mu \bar{z}_j z_k(x)) \tag{7}$$

for the CP^{N-1} model

$$A_{\mu}^{jk}(x) = ig^{j\ell}\,\partial_{\mu} g^{\ell k}(x) = 2i(\bar{V}_{ja}\,\partial_{\mu} V_{ak}(x) - \partial_{\mu}\bar{V}_{ja} V_{ak}(x)) \tag{8}$$

for the Grassmannian models and

$$A_{\mu}^{jk}(x) = i\, g^{-1}_{j\ell}(x)\,\partial_{\mu} g_{\ell k}(x) \tag{9}$$

for the principal chiral fields. From the equations of motion it follows that (6)-(9) are conserved currents, i.e., $\partial^{\mu}A_{\mu}=0$. For our purpose it is convenient to use the Hermitian currents $\partial^{\mu}A_{\mu}=0$. Because of hermiticity of (6)-(9) and C it follows that the charge (5) also is a hermitean matrix.

It can be pointed that from (2) or (3) it follows that all $\chi^{(k)}(x)$ $(k=1,2\dots)$ have the same scale dimensions. In our case this dimension is zero. The conserved charges (4) also have the zero scale dimensions.

In the quantum case the currents (1) as well as the equations for the generating functions (2), when $A_{\mu}(x)$ and $\chi^{(k)}$ are operator-valued functions, are needed to be correctly determined. For these purpose the short-distance expansion of the product of two operators is used (see Appendix A). Then we define

$$\begin{aligned} J_{\mu}^{(k)}(x_1,x_2) = {} & [A_{\mu}(x_1),\chi_{\delta}^{(k)}(x_2)] + \epsilon_{\mu\nu}[A^{\nu}(x_1),\chi_{\delta}^{(k-1)}(x_2)] \\ & - \frac{(N-2)\,x_{\mu}^{12}}{2\pi x_{12}^2}\,\chi_{\delta}^{(k)}(x_2) - \frac{(N-2)\,x_{\mu}^{12}}{2\pi x_{12}^2}\,\tilde{\chi}_{\delta}^{(k-1)}(x_2) \\ & - \epsilon_{\mu\nu}\frac{(N-2)\,x^{\nu}_{12}}{2\pi x_{12}^2}\,\tilde{\chi}_{\delta}^{(k)}(x_2) - \epsilon_{\mu\nu}\frac{(N-2)\,x^{\nu}_{12}}{2\pi x_{12}^2}\,\chi_{\delta}^{(k-1)}(x_2), \end{aligned} \tag{10}$$

and

$$\partial_\mu \chi_\delta^{(k)}(x_1) = \epsilon_{\mu\nu}\partial^\nu \chi_\delta^{(k-1)}(x_1) + \epsilon_{\mu\nu}[A^\nu(x_1), \chi_\delta^{(k-1)}(x_2)]$$

$$- \epsilon_{\mu\nu}\frac{(N-2)\, x_{12}^\nu}{2\pi x_{12}^2}\chi_\delta^{(k-1)}(x_2) - \frac{(N-2)\, x_\mu^{12}}{2\pi x_{12}^2}\tilde{\chi}_\delta^{(k-1)}(x_2)\,. \tag{11}$$

Here $\tilde{\chi}_\delta^{(k)}(x)$ are suitable functions providing the limit

$$\lim_{\substack{x_2 \to x_1 = x,\\ x_{12}^2 = (x_1 - x_2)^2 = -\delta < 0.}} Q_\delta^{(k)}(x_1, x_2) = \lim_{x_2 \to x_1 = x} \int_{-\infty}^{\infty} dx_1\, J_0^{(k)}(x_1, x_2) \tag{12}$$

For the second conserved charge we have

$$Q_\delta^{(2)} = \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{x_1-\delta} dy_1\, [A_0(x_0, x_1), [A_0(x_0, y_1), C]]$$

$$- \frac{N-2}{2\pi}\ln\mu\delta \int_{-\infty}^{\infty} dx_1 [A_1(x_0, x_1), C]\,. \tag{13}$$

It can be checked that the charge (13) indeed is conserved, i.e.,

$$\frac{dQ_\delta^{(2)}}{dx_0} = \int_{-\infty}^{\infty} dx_1 \{[A_0(x_0, x_1), [A_1(x_0, x_1 - \delta), C]]$$

$$- [A_1(x_0), x_1 + \delta), [A_0(x_0, x_1), C]] - \frac{N-2}{2\pi}\ln\mu\delta\,\partial_0 A_1(x)\} = \tag{14}$$

$$= \int dx_1 \frac{N-2}{2\pi} \ln\mu\delta\, \partial_0 A_1 - \frac{N-2}{2\pi}(2+\ln\mu\delta)\, \partial_1 A_0 - \frac{N-2}{2\pi} \ln\mu\delta\, \partial_0 A_1 \} = 0, \tag{14}$$

where the short-distance expansion of the product of two conserved currents given in ref.[6] is used.

Taking the limit $\delta \to 0$ from (13) we have

$$Q^{(2)}(x_0) = \int_{-\infty}^{\infty} dx \{ \int_{-\infty}^{x_1} dy_1 [A_0(x_0 x_1), [A_0(x_0, y_1), C]] - Z [A_1(x_0, x_0), C] \}, \tag{15}$$

where Z is the *infinite renormalization* constant.

The higher conserved charges ($k > 2$) can be constructed from (10) or from the commutation relations for $Q^{(2)}$ given by (15).

From the existence of the higher quantum nonlocal conserved charges and the asymptotical states of the generalized nonlinear sigma models there follow the factorization of the S-matrix and the absence of the particle production [6].

APPENDIX A

Consider the short-distance expansion of the product of two local field operators $A(x)$ and $B(x)$, i.e.,

$$A(x)\, B(y) = \sum_{\chi} C_{\chi}(x-y)\, O_{\chi}(y), \tag{A. 1}$$

where $C(x)$ are c-number functions and O_{χ} are composite operators*. The functions $C_{\chi}(x)$ have the following properties

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* If A and B have isotopic indices, i.e., if they are transformed under some internal symmetry group G, then $C_{\chi}(x)$ are scalars with respect to G.

$$C_\chi(\lambda x) = \lambda^{d_\chi - d_A - d_B} C_\chi(x) \tag{A.2}$$

with respect to the scale transformations. Consequently, the singular terms in (A.1) are given by the following condition

$$d_\chi \le d_A + d_B \tag{A.3}$$

for the scale dimensions of the fields $A(x)$, $B(x)$ and $O_\chi(x)$. Consider the following, interesting for our purpose, cases:

a) $A(x)$ and $B(x)$ are scalar fields with zero scale dimensions. Then from (A.3) it follows that there is only one singular term

$$A(x)\, B\,(0) \sim c \ln(\mu^2 x^2 + i\epsilon x_0)\, O_0(x), \tag{A.4}$$

where c is normalization constant, μ is a parameter with mass dimension and O_0 is an operator with zero scale dimension.

b) One conserved current $J_\mu(x)$ and one scalar field $\chi(x)$ with zero scale dimension which is odd with respect to the space reflections, i.e., $\chi(x_0, -x_1) = -\chi(x_0, x_1)$. Then from (A.3) taking into account Lorentz and space reflection invariances and current conservation, we obtain the following singular terms

$$J_\mu(x)\,\chi(0) \sim \frac{(N-2)\,x_\mu}{2\pi x^2}\chi(x) - \frac{(N-2)\,\epsilon_{\mu\nu}\,x^\nu}{2\pi x^2}\tilde{\chi}(x), \tag{A.5}$$

where the normalization given in paper /6/ is used and

$$\chi(x) = \int_{-\infty}^{x_1} dy_1 J_0(x_0, y_1), \tag{A.6}$$

$$\tilde{\chi}(x) = \int_{-\infty}^{x_1} dy_1 J_1(x_0, y_1).$$

Here $\tilde{\chi}$ is an even operator with respect to the space reflections.

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