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ON THE LOWEST ORDER ELECTROWEAK CORRECTIONS TO SPIN 1/2 FERMION SCATTERING.

I. The One-Loop Diagrammar

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I. INTRODUCTION

Interest to gauge models of particle interactions is still increasing [1]. The parent version of such models, SU(2) x U(1) model by Weinberg and Salam [2], has received recently several important experimental confirmations [3] and now pretends to be a theory unifying weak and electromagnetic interactions of leptons and photons. Further development of gauge theories is connected essentially with constructing grand symmetries containing the SU(2) x U(1) symmetry as a subgroup.

The most important feature of gauge models is their renormalizability which permits us to calculate unambiguously higher order effects. So far, however, a decisive test was not performed to check the higher order predictions of gauge theories, like $g_\ell - 2$ test showing with the excellent accuracy the validity of QED.

It is obvious that any calculation of radiative corrections (RC) within gauge theories aimed at studying experimental possibilities to check theoretical predictions for higher order effects seems to be an urgent problem. From this point of view RC to μ-decay were considered thoroughly [4]. But the result turned out to be negative - higher order effects cannot be observed through μ-decay, as their contribution to measured distributions is negligible. Other calculations of the radiative corrections, available in the literature, were carried out in general with methodical purposes, e.g., to $W^{\pm}$- and Z^{o}-decays [5], to elastic $\nu\nu$ - (ref. [6]), $\nu_\mu e$ - (ref. [7]), and $\mu\mu$- (ref. [8]) - scatterings.

Scattering processes are probably more promising for measuring higher order effects than decay processes. In the near future those transfer momenta will be reached where weak and electromagnetic interactions are of the equal strength and one may hope that higher order effects due to both interactions will also be approximately of the same magnitude, at least, in some parts of the phase space of a reaction and/or in some special experimental conditions. The calculations [6-8] were earlier performed in the approximation with the invariants of an amplitude s , t , u much smaller than vector boson masses squared, i.e., they are inapplicable in the high energy region of interest.

In this paper we present the total set of formulae for all diagrams involved in calculating the one-loop corrections to scattering of any two fermions with spin 1/2 not restricted by the above-mentioned approximation. The calculations are performed by the dimensional regularization method [9]. The pole terms, i.e., the coefficients of $1/(n-4)$ (where n is the space-time dimensionality) are found without any approximations, finite parts are derived under the only restriction

$$s,\ t,\ u,\ M_V^2,\ M_\chi^2 \gg m_i^2, \tag{1.1}$$

where m_i is any fermion mass, M_V is any heavy vector boson mass, and M_χ is any scalar boson mass. Inequality (1.1) simplifies finite parts essentially but does not restrict the range of applicability of resulting expressions at high energies.

All calculations are carried out in the unitary gauge, therefore we deal only with physical particles - spin 1/2 fermions ψ, charged $W^\pm$ and neutral Z° heavy vector bosons, γ-quanta, and neutral scalar fields χ. The Lagrangian of interaction of fermion fields with vector and scalar fields is chosen in the form

$$L = -i f^V \cdot \bar\psi_1 \cdot \gamma_\mu (a+\gamma_5)\psi_2 \cdot V_\mu - i f^q e \bar\psi \gamma_\mu \psi \cdot A_\mu - f^\chi \bar\psi_1 (1+b\gamma_5)\psi_2 \chi, \tag{1.2}$$

with $V_\mu = W_\mu$ or Z°_μ. Representation (1.2) of the part of the Lagrangian allows us not to specify the type of fermion (ℓ, ν,...) and of heavy vector boson ($W^\pm$, Z°,...). Weak fermion currents in (1.2) are arbitrary linear combinations of V and A (S and P) currents. To calculate the one-loop approximation for the fermion scattering it is necessary to specify also the three-linear vector (3V), bilinear vector - scalar (2VS), and three-linear scalar (3S) parts of the Lagrangian. The first part is chosen in the form

$$L_{3V} = -i f^{3V} [V^\circ_\mu V^+_\nu (\partial_\mu V^-_\nu - \partial_\nu V^-_\mu) + V^-_\mu V^\circ_\nu (\partial_\mu V^+_\nu - \partial_\nu V^+_\mu) + \\ + V^+_\mu V^-_\nu (\partial_\mu V^\circ_\nu - \partial_\nu V^\circ_\mu)], \tag{1.3}$$

(with the neutral vector field $V^\circ_\mu = A_\mu$ or Z°_μ) typical for gauge models [10], and other parts in the simplest form

$$L_{\chi} = - f^{V\chi} \cdot |V|^2 \cdot \chi - f^{3\chi} \chi_1 \cdot \chi_2 \cdot \chi_3 . \qquad (1.4)$$

The one-loop approximation of the amplitude is contributed also by the diagrams generated by 4V-, 2V2S- and 4S-interactions (bubbles) and by self-interaction of scalar fields (tadpoles). However, the latter diagrams give only constant contributions to the corresponding self-energy operators which do not change physical (renormalized) values of these operators. For this reason the bubbles and tadpoles are not considered here.

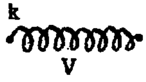
It is obvious that the formulae derived here are not related directly with a given gauge model. The set of presented formulae is exhaustive in the sense that any diagram, involved in the one-loop approximation for spin 1/2 fermion scattering in any theory dealing with ψ , V , A and χ fields interacting through eqs. (1.2)-(1.4), enters into this set. For the physical treating of the lowest order RC it should be supplemented only by photon bremsstrahlung diagrams. Their contribution to RC depends on concrete experimental conditions. The contribution of bremsstrahlung diagrams to RC for specific experiments is calculated exactly in our paper [11].

Some of the results presented here have been published earlier in ref.[12] . Since then, one-loop formulae were already used in two investigations of RC to deep inelastic neutrino scattering [13] and RC to P-odd asymmetries in deep inelastic scattering of charged polarized leptons on nucleons [14]. During two-year work with these formulae they were checked many times and the form of their representation got simpler and more convenient for applications. Furthermore, here we generalize the formulae of ref.[12] to the time-like transfer momenta region,what permits us to apply this one-loop diagrammar both to the scattering reactions and to annihilation processes.

This paper is organized as follows. In the next section all possible self-energy diagrams are given and in Sec. 3 some vertices are presented. Other vertices and all two-particle exchange diagrams will be given in the next part of this paper.

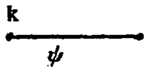
As we deal with a very large amount of formulae we are forsed to use as minimum words as possible. We shall keep the following scheme of exposition of results: give a picture of a diagram which is also the tittle of the corresponding item, then give the initial integral corresponding to the diagram and finally present the result of calculation of this diagram followed by the explanation of a new notation.

We use the Pauli metric. Feynman rules for propagators are as follows:

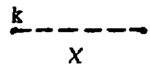


$\frac{-i}{(2\pi)^4} \cdot \frac{\delta_{\alpha\beta} + k_\alpha k_\beta / M_V^2}{k^2 + M_V^2}$

$\frac{-i}{(2\pi)^4} \cdot \frac{\delta_{\alpha\beta}}{k^2}$



$\frac{-1}{(2\pi)^4} \cdot \frac{\hat{k} + im}{k^2 + m^2}$



$\frac{-i}{(2\pi)^4} \cdot \frac{1}{k^2 + M_\chi^2}$

The Feynman rules for vertices are derived straightforwardly from Lagrangian (1.2)-(1.4). Each initial integral is multiplied by a factor $i^k (2\pi)^{-4}$, where k is the corresponding perturbation order. Factors due to external lines are not included into the initial integral.

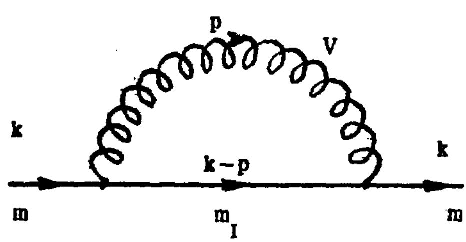
II. SELF-ENERGY DIAGRAMS

Self-Energy Fermion Diagrams

Restricting ourselves to the one-loop approximation for the amplitude of spin 1/2 fermion scattering we need self-energy fermion operator $\Sigma(k)$ only for fixing some renormalization constants [7]. For this reason only the first three coefficients $\Sigma(im)$, B and A in the Taylor expansion of $\Sigma(k)$ are required

$$\Sigma(k) = \Sigma(im) + B(\hat{k} - im) + C \cdot (\hat{k} - im)^2 + \ldots + \hat{A} \cdot \gamma_5 + \hat{D} \gamma_5 (k^2 + m^2) + \ldots \quad (2.1)$$

1.



$$\Sigma_V(k) = i(f^V)^2 \cdot \int \frac{d^n p}{(2\pi)^n} \frac{\gamma_\alpha (a+\gamma_5)(\hat{k}-\hat{p}+im_I)\gamma_\beta (a+\gamma_5)}{(p^2+M_V^2)[(p-k)^2+m_I^2]} \cdot (\delta_{\alpha\beta} + \frac{p_\alpha p_\beta}{M_V^2}) \quad (2.2)$$

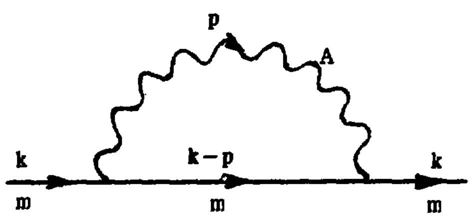
$$\Sigma_V(im) = im \frac{(f^V)^2(a^2+1)}{16\pi^2} \cdot (\frac{3m_I^2 - m^2}{M_V^2} \cdot P - \frac{3}{2}) + im_I \cdot \frac{(f^V)^2(a^2-1)}{16\pi^2} [(6 - 2\frac{m_I^2}{M_V^2}) \cdot P - -1 + 3\ln\frac{M_V^2}{M_W^2}], \quad (2.3)$$

$$\hat{A}_V = \frac{2(f^V)^2 \cdot a}{16\pi^2} (\frac{3m_I^2 - m^2}{M_V^2} P - \frac{3}{2}) \cdot \hat{k}, \quad B_V = \frac{(f^V)^2(a^2+1)}{16\pi^2} (3\frac{m_I^2 - m^2}{M_V^2} \cdot P - \frac{3}{2}), \quad (2.4)$$

$$P = \frac{1}{n-4} + \frac{1}{2}\gamma + \ln\frac{M_W}{\eta \cdot 2 \cdot \sqrt{\pi}}, \quad (2.5)$$

where P is a typical pole term representating the ultraviolet divergences at $n = 4$, η is an arbitrary parameter with the mass dimensionality, γ is the Euler constant.

2.



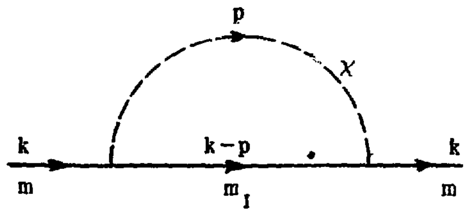
$$\Sigma_A(k) = i(f^q)^2 e^2 \int \frac{d^n p}{(2\pi)^n} \cdot \frac{(2-n)(\hat{k}-\hat{p}) + imn}{p^2[(p-k)^2 + m^2]}. \quad (2.6)$$

$$\Sigma_A(im) = im \frac{(f^q)^2 e^2}{16\pi^2} (6P - 4 + 3\ln\frac{m^2}{M_W^2}), \quad \hat{A}_A = 0, \quad (2.7)$$

$$B_A = \frac{(f^q)^2 e^2}{16\pi^2} \cdot (-2P - 4P_{IR} + 4 - 3\ln \frac{m^2}{M_W^2}), \qquad (2.8)$$

where $P_{IR} = P$ and subscript IR denotes the infrared origin of this pole term.

3.



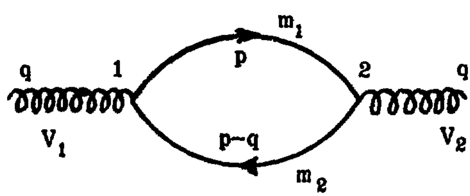
$$\Sigma_\chi(k) = -i(f^\chi)^2 \cdot \int \frac{d^n p}{(2\pi)^n} \cdot \frac{(\hat{k}-\hat{p})(1-b^2) + im_I(1+b^2+2b\gamma_5)}{(p^2+M_\chi^2)[(p-k)^2+m_I^2]} \cdot \qquad (2.9)$$

$$\Sigma_\chi(im) = im\frac{(f^\chi)^2(1-b^2)}{16\pi^2}(-P + \frac{1}{4} - \frac{1}{2}\ln\frac{M_\chi^2}{M_W^2}) + im_I\frac{(f^\chi)^2(1+b^2)}{16\pi^2}(-2P + 1 - \ln\frac{M_\chi^2}{M_W^2}), \qquad (2.10)$$

$$\hat{A}_\chi = im_I\frac{2b(f^\chi)^2}{16\pi^2}(-2P + 1 - \ln\frac{M_\chi^2}{M_W^2}), \quad B_\chi = (f^\chi)^2 \cdot \frac{(1-b^2)}{16\pi^2}(-P + \frac{1}{4} - \frac{1}{2}\ln\frac{M_\chi^2}{M_W^2}). \qquad (2.11)$$

Self-Energy Vector Boson Diagrams

1.



$$\Pi_{\alpha\beta}(q) = -f_1^V \cdot f_2^V \int \frac{d^n p}{(2\pi)^n} \cdot \mathrm{Sp}\, \gamma_\alpha (a_2+\gamma_5) \frac{\hat{p}+im_1}{p^2+m_1^2} \gamma_\beta (a_1+\gamma_5) \frac{\hat{p}-\hat{q}+im_2}{(p-q)^2+m_2^2} =$$

$$= 4i \frac{f_1^V f_2^V}{16\pi^2} \{ [\frac{2}{3}(1+a_1a_2)(q^2\delta_{\alpha\beta}-q_\alpha q_\beta)+(m_1^2+m_2^2)(1+a_1a_2)\delta_{\alpha\beta}+ \quad (2.12)$$

$$+2m_1m_2(1-a_1a_2)\delta_{\alpha\beta}] \cdot P + 2(1+a_1a_2)(q^2\delta_{\alpha\beta}-q_\alpha q_\beta) \cdot I_1(q^2, m_1^2, m_2^2) \}.$$

Here and in the following

$$I_1(q^2, m_1^2, m_2^2) = \int_0^1 x(1-x) \ln[x(1-x) \frac{q^2}{M_W^2} + x \frac{m_1^2}{M_W^2} + (1-x) \frac{m_2^2}{M_W^2}] dx =$$

$$= -\frac{5}{18} + \frac{1}{3} \cdot \frac{m_1^2+m_2^2}{q^2} + \frac{1}{3} \cdot \frac{(m_1^2-m_2^2)^2}{q^4} + \frac{1}{6} \ln \frac{m_1 m_2}{M_W^2} + (\frac{1}{4} \cdot \frac{m_1^4-m_2^4}{q^4} + \quad (2.13)$$

$$+ \frac{1}{6} \cdot \frac{(m_1^2-m_2^2)^3}{q^6}) \ln \frac{m_1^2}{m_2^2} + (\frac{1}{12} - \frac{1}{12} \cdot \frac{m_1^2+m_2^2}{q^2} - \frac{1}{6} \cdot \frac{(m_1^2-m_2^2)^2}{q^4}) \frac{1}{q^2} L(q^2, m_1^2, m_2^2).$$

$$L(q^2, m_1^2, m_2^2) = \lambda(q^2, -m_1^2, -m_2^2) \cdot J(q^2, m_1^2, m_2^2), \quad (2.14)$$

$$\lambda(q^2, -m_1^2, -m_2^2) = (q^2+m_1^2+m_2^2)^2 - 4m_1^2 m_2^2, \quad (2.15)$$

$$J(q^2, m_1^2, m_2^2) = \int_0^1 \frac{dx}{x(1-x)q^2 + x m_1^2 + (1-x) m_2^2} =$$

$$(2.16)$$

$$= \frac{1}{\sqrt{\lambda(q^2, -m_1^2, -m_2^2)}} \ln \left| \frac{q^2+m_1^2+m_2^2+\sqrt{\lambda(q^2,-m_1^2,-m_2^2)}}{q^2+m_1^2+m_2^2-\sqrt{\lambda(q^2,-m_1^2,-m_2^2)}} \right| \quad \text{for} \quad \begin{matrix} -(m_1-m_2)^2 \le q^2 < \infty \\ -\infty < q^2 \le -(m_1+m_2)^2 \end{matrix}$$

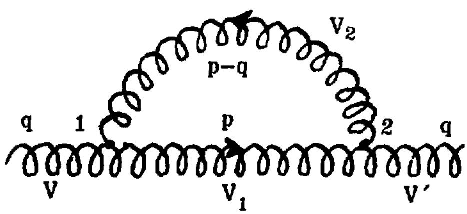
$$= \frac{2}{\sqrt{-\lambda(q^2, -m_1^2, -m_2^2)}} \operatorname{arctg} \frac{\sqrt{-\lambda(q^2, -m_1^2, -m_2^2)}}{q^2 + m_1^2 + m_2^2} \quad \text{for} \quad -(m_1^2 + m_2^2) \leq q^2 \leq -(m_1 - m_2)^2.$$

$$= \frac{2}{\sqrt{-\lambda(q^2, -m_1^2, -m_2^2)}} \left[\operatorname{arctg} \frac{\sqrt{-\lambda(q^2, -m_1^2, -m_2^2)}}{q^2 + m_1^2 + m_2^2} + \pi \right] \quad \text{for} \quad -(m_1 + m_2)^2 < q^2 \leq -(m_1^2 + m_2^2).$$

Having in mind ineq. (1.1) we take for $I_1(q^2, m_1^2, m_2^2)$ its approximate value

$$I_1(q^2, m_1^2, m_2^2) \approx -\frac{5}{18} + \frac{1}{6} \ln \frac{|q^2|}{M_W^2} . \qquad (2.17)$$

2.



$$\Pi_{\alpha\beta}(q) = f_1^{3V} f_2^{3V} \int \frac{d^n p}{(2\pi)^n} \cdot \frac{\delta_{\mu\nu} + p_\mu p_\nu / M_1^2}{p^2 + M_1^2} \cdot \frac{\delta_{\rho\sigma} + \epsilon (p-q)_\rho (p-q)_\sigma / M_2^2}{(p-q)^2 + \epsilon M_2^2} \times$$

$$\times V_{\alpha\mu\rho} \cdot V_{\beta\nu\sigma} = \delta_{\alpha\beta} \Pi(q^2) + q_\alpha q_\beta \cdot \Theta(q^2). \qquad (2.18)$$

$$V_{\alpha\mu\rho} = (2p-q)_\alpha \delta_{\mu\rho} - (p-2q)_\mu \delta_{\alpha\rho} - (p+q)_\rho \delta_{\alpha\mu} , \qquad (2.19)$$

$$\Pi(q^2) = i\frac{f_1^{3V} f_2^{3V}}{16\pi^2}\{[\frac{\epsilon}{6}\cdot\frac{q^6}{M_1^2M_2^2} - (\frac{5}{3} - \frac{\epsilon}{2})\frac{q^4}{M_1^2} - \frac{7}{6}\epsilon\frac{q^4}{M_2^2} - (\frac{22}{3} + \epsilon)q^2 -$$

$$- \frac{17}{6}\epsilon q^2(\frac{M_1^2}{M_2^2} + \frac{M_2^2}{M_1^2}) + (\frac{15}{2} - \frac{3}{2}\epsilon)M_1^2 + 6\epsilon M_2^2 - \frac{3}{2}\epsilon(\frac{M_1^4}{M_2^2} + \frac{M_2^4}{M_1^2})]\cdot P -$$

$$-(q^2 + \epsilon M_2^2 - \frac{1}{4}M_1^2)\ln\frac{M_1^2}{M_W^2} - \epsilon\cdot(q^2 + M_1^2 - \frac{1}{4}M_2^2)\ln\frac{M_2^2}{M_W^2} - \frac{\epsilon}{12}\cdot\frac{q^6}{M_1^2M_2^2} -$$

$$-(\frac{1}{6} + \frac{\epsilon}{4})\frac{q^4}{M_1^2} - \frac{5}{12}\epsilon\cdot\frac{q^4}{M_2^2} + (\frac{1}{3} + \frac{\epsilon}{2})q^2 - \frac{7}{12}\epsilon q^2(\frac{M_1^2}{M_2^2} + \frac{M_2^2}{M_1^2}) - (\frac{7}{8} - \frac{3}{4}\epsilon)M_1^2 -$$

$$- \frac{\epsilon}{8}M_2^2 - \frac{\epsilon}{4}(\frac{M_1^4}{M_2^2} + \frac{M_2^4}{M_1^2}) + [\frac{\epsilon}{2}\cdot\frac{q^4}{M_1^2M_2^2} + \frac{q^2}{M_1^2} + \epsilon\frac{q^2}{M_2^2} + 5 + \qquad (2.20)$$

$$+ \frac{\epsilon}{2}\cdot(\frac{M_1^2}{M_2^2} + \frac{M_2^2}{M_1^2})]\cdot I_2(q^2, M_1^2, \epsilon M_2^2) + [-\frac{q^4}{M_1^2} - \epsilon\frac{q^4}{M_2^2} - 4q^2 - 2\epsilon q^2(\frac{M_1^2}{M_2^2} + \frac{M_2^2}{M_1^2}) +$$

$$+ M_1^2 + \epsilon M_2^2 - \epsilon\cdot(\frac{M_1^4}{M_2^2} + \frac{M_2^4}{M_1^2})]\cdot I_0(q^2, M_1^2, \epsilon M_2^2)\},$$

$$\Theta(q^2) = i\frac{f_1^{3V} f_2^{3V}}{16\pi^2}\cdot[-\frac{\epsilon}{6}\cdot\frac{q^4}{M_1^2M_2^2} + (\frac{5}{3} - \frac{\epsilon}{2})\frac{q^2}{M_1^2} + \frac{7}{6}\epsilon\cdot\frac{q^2}{M_2^2} + \frac{25}{3} +$$

$$+ \epsilon + \frac{7}{3}\epsilon(\frac{M_1^2}{M_2^2} + \frac{M_2^2}{M_1^2})]\cdot P, ^{*} \qquad (2.21)$$

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* The contribution of the second term in (2.18) to the amplitude is proportional to m^2, because of $q_\alpha q_\beta$ factor, so according to (1.1), the finite part of (2.21) may be neglected.

$$I_0(q^2, M_1^2, \epsilon M_2^2) = \int_0^1 \ln\left[x(1-x)\frac{q^2}{M_W^2} + x\frac{M_1^2}{M_W^2} + (1-x)\frac{\epsilon M_2^2}{M_W^2}\right] dx =$$

$$= -2 + \ln\frac{M_1 \cdot (\epsilon M_2)}{M_W^2} + \frac{1}{2}\cdot\frac{M_1^2-\epsilon M_2^2}{q^2}\ln\frac{\epsilon M_2^2}{M_1^2} + \frac{1}{2}\cdot\frac{1}{q^2}\cdot L(q^2, M_1^2, \epsilon M_2^2), \tag{2.22}$$

$$I_2(q^2, M_1^2, \epsilon M_2^2) = \int_0^1 [x(1-x)q^2 + xM_1^2 + (1-x)\epsilon M_2^2]\ln[x(1-x)\frac{q^2}{M_W^2} + x\frac{M_1^2}{M_W^2} +$$

$$+(1-x)\frac{\epsilon M_2^2}{M_W^2}] = -\frac{5}{18}q^2 - \frac{2}{3}M_1^2 - \frac{2}{3}\epsilon M_2^2 - \frac{1}{6}\cdot\frac{(M_1^2-\epsilon M_2^2)^2}{q^2} + (\frac{q^2}{6} + \frac{1}{2}\cdot M_1^2 +$$

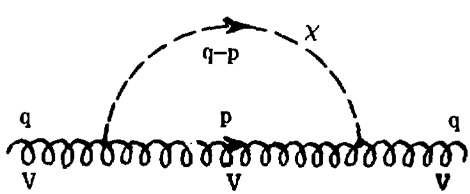
$$+ \frac{\epsilon}{2}M_2^2)\ln\frac{M_1(\epsilon M_2)}{M_W^2} + (\frac{M_1^4-\epsilon M_2^4}{4q^2} + \frac{(M_1^2-\epsilon M_2^2)^3}{12q^4})\ln\frac{\epsilon M_2^2}{M_1^2} + \tag{2.23}$$

$$+ \frac{\lambda(q^2, -M_1^2, -\epsilon M_2^2)}{12q^4} L(q^2, M_1^2, \epsilon M_2^2).$$

Here $\epsilon=1$ if V_2 is a heavy vector boson and $\epsilon=0$ if V_2 is the photon. In the latter case the derivative of $\Pi(q^2)$ with respect to q^2 on the mass shell reads

$$\frac{\partial\Pi(q^2)}{\partial q^2}\Big|_{q^2=-M_1^2} = i\frac{f_1^{3V} f_2^{3V}}{16\pi^2}(-4\cdot P + 4\cdot P_{IR}). \tag{2.24}$$

3.



$$\Pi_{\alpha\beta}(q) = (f^{V\chi})^2 \cdot \int \frac{d^np}{(2\pi)^n}\cdot\frac{\delta_{\alpha\beta} + p_\alpha p_\beta / M_V^2}{(p^2+M_V^2)[(p-q)^2+M_\chi^2]}. \tag{2.25}$$

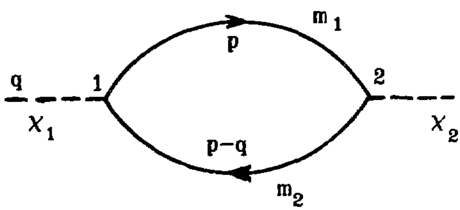
$$\Pi(q^2) = i\frac{(f^{V\chi})^2}{16\pi^2}\{[\frac{1}{6}\cdot\frac{q^2}{M_V^2}+\frac{1}{2}\cdot\frac{M_\chi^2}{M_V^2}-\frac{3}{2}]\cdot P-\frac{1}{12}\cdot\frac{q^2}{M_V^2}-\frac{1}{4}\cdot\frac{M_\chi^2}{M_V^2}-\frac{1}{4}+$$

$$+\frac{1}{2}\cdot\frac{1}{M_V^2}\cdot I_2(q^2,M_\chi^2,M_V^2)-I_0(q^2,M_\chi^2,M_V^2)\}, \qquad (2.26)$$

$$\Theta(q^2)= i\,\frac{(f^{V\chi})^2}{16\pi^2}\cdot(-\frac{2}{3}\cdot\frac{1}{M_V^2})P \quad . \qquad (2.27)$$

Self-Energy Diagrams of a Scalar Boson

1.

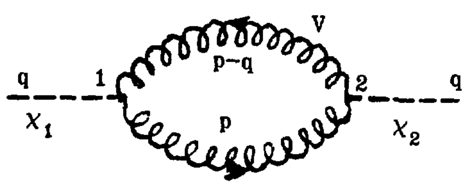


$$\Pi(q^2) = f_1^\chi\cdot f_2^\chi \int \frac{d^np}{(2\pi)^n}\, Sp(1+b_2\gamma_5)\,\frac{\hat{p}+im_1}{p^2+m_1^2}\,(1+b_1\gamma_5)\,\frac{\hat{p}-\hat{q}+im_2}{(p-q)^2+m_2^2} = \qquad (2.28)$$

$$=4i\,\frac{f_1^\chi f_2^\chi}{16\pi^2}\{[(1-b_1b_2)(q^2+2m_1^2+2m_2^2)+2m_1m_2(1+b_1b_2)]P+$$

$$+(1-b_1b_2)(-1+\frac{1}{2}\ln\frac{|q^2|}{M_W^2})q^2\}.$$

2.



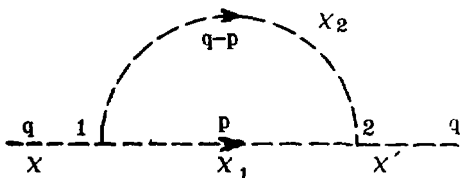
$$\Pi(q^2)=N\cdot f_1^{V\chi} f_2^{V\chi} \int \frac{d^n p}{(2\pi)^n}\cdot \frac{\delta_{\alpha\beta}+p_\alpha p_\beta/M_V^2}{p^2+M_V^2}\cdot\frac{\delta_{\alpha\beta}+(p-q)_\alpha(p-q)_\beta/M_V^2}{(p-q)^2+M_V^2}=$$

$$=iN\frac{f_1^{V\chi}f_2^{V\chi}}{16\pi^2}\{[-\frac{1}{2}\cdot\frac{q^4}{M_V^4}-3\frac{q^2}{M_V^2}-6]\cdot P-2+\frac{q^2}{2M_V^2}-\frac{q^2}{2M_V^2}\ln\frac{M_V^2}{M_W^2}-$$

$$-(\frac{q^4}{4M_V^2}+\frac{q^2}{M_V^2}+3)I_0(q^2,M_V^2,M_V^2)\}. \quad (2.29)$$

Here N is a combinatorial factor equal to 1 for charged vector bosons in the loop and to 2 for neutral vector bosons.

3.



$$\Pi(q^2)=Nf_1^{3\chi}f_2^{3\chi}\cdot\int\frac{d^n p}{(2\pi)^n}\cdot\frac{1}{(p^2+M_{\chi_1}^2)[(p-q)^2+M_{\chi_2}^2]}=$$

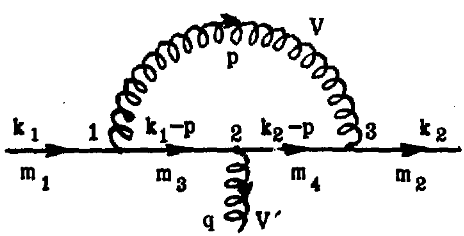
$$=iN\frac{f_1^{3\chi}f_2^{3\chi}}{16\pi^2}\cdot[-2P-I_0(q^2,M_{\chi_1}^2,M_{\chi_2}^2)]. \quad (2.30)$$

Here N is also some combinatorial factor.

III. VERTEX DIAGRAMS

At first we shall present a set of one-loop vertices with internal boson lines coupled to the fermion line. These diagrams are characterized by q^2 -independent pole terms.

1.



$$\Gamma_\rho(q^2) = -i f_1^V f_2^V f_3^V \int \frac{d^n p}{(2\pi)^n} \cdot \gamma_\alpha (a_3+\gamma_5) \frac{\hat{k}_2 - \hat{p} + i m_4}{(k_2-p)^2 + m_4^2} \gamma_\rho (a_2+\gamma_5) \frac{\hat{k}_1 - \hat{p} + i m_3}{(k_1-p)^2 + m_3^2} \times$$

$$\times \gamma_\beta (a_1+\gamma_5) \cdot \frac{\delta_{\alpha\beta} + p_\alpha p_\beta / M_V^2}{p^2 + M_V^2} = \frac{f_1^V f_2^V f_3^V}{16\pi^2} \{ [\frac{2m_3^2 + 2m_4^2 - m_1^2 - m_2^2}{M_V^2} \gamma_\rho (A_1 + B_1\gamma_5) - \quad (3.1)$$

$$- \frac{m_1 m_2}{M_V^2} \gamma_\rho (A_1 - B_1\gamma_5) + \frac{m_2 m_3}{M_V^2} \gamma_\rho (A_2 + B_2\gamma_5) - \frac{m_1 m_3}{M_V^2} \gamma_\rho (A_2 - B_2\gamma_5) -$$

$$- \frac{m_2 m_4}{M_V^2} \gamma_\rho (A_3 + B_3\gamma_5) + \frac{m_1 m_4}{M_V^2} \gamma_\rho (A_3 - B_3\gamma_5) - \frac{m_3 m_4}{M_V^2} \gamma_\rho (A_4 + B_4\gamma_5)] \cdot P +$$

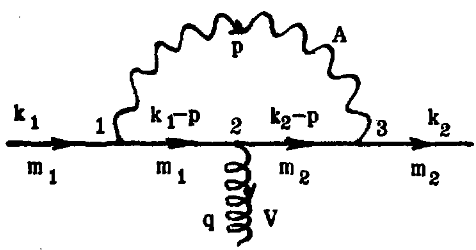
$$+ [-5 + 2\frac{M_V^2}{q^2} + (3 - 2\frac{M_V^2}{q^2}) \ln \frac{|q^2|}{M_V^2} - 2(1 - \frac{M_V^2}{q^2})^2 (\Phi(1) - \Phi(1 - \frac{q^2}{M_V^2}))] \gamma_\rho (A_1 + B_1\gamma_5) \},$$

where $\Phi(x) = -\int_0^x t^{-1} \ln|1-t|\, dt$ is the Spence function, and coefficients A_i and B_i are defined as

$$A = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = \begin{pmatrix} a_1 a_2 a_3 + a_1 + a_2 + a_3 \\ a_1 a_2 a_3 + a_1 - a_2 - a_3 \\ a_1 a_2 a_3 - a_1 - a_2 + a_3 \\ a_1 a_2 a_3 - a_1 + a_2 - a_3 \end{pmatrix}, \quad B = \begin{pmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{pmatrix} = \begin{pmatrix} a_2 a_3 + a_1 a_2 + a_1 a_3 + 1 \\ a_2 a_3 - a_1 a_2 - a_1 a_3 + 1 \\ a_2 a_3 - a_1 a_2 + a_1 a_3 - 1 \\ a_2 a_3 + a_1 a_2 - a_1 a_3 - 1 \end{pmatrix}$$

(3.2)

2.



$$\Gamma_\rho(q^2) = -i f_1^q f_3^q f_2^V e^2 \int \frac{d^n p}{(2\pi)^n} \cdot \gamma_\alpha \cdot \frac{\hat{k}_2 - \hat{p} + im_2}{(k_2-p)^2 + m_2^2} \gamma_\rho (a_2 + \gamma_5) \times$$

$$\times \frac{\hat{k}_1 - \hat{p} + im_1}{(k_1-p)^2 + m_1^2} \gamma_\alpha \cdot \frac{1}{p^2} = \frac{f_1^q f_3^q f_2^V}{16\pi^2} e^2 \cdot [-2P - 2(q^2 + m_1^2 + m_2^2) \times \quad (3.3)$$

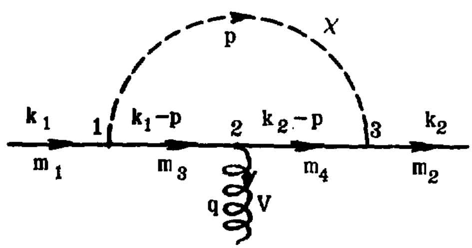
$$\times J(q^2, m_1^2, m_2^2) P_{IR} - \ln \frac{|q^2|}{M_W^2} + 4 \ln \frac{|q^2|}{m_1 m_2} - q^2 F(q^2, m_1^2, m_2^2)] \gamma_\rho (a_2 + \gamma_5).$$

$$K(q^2, m_1^2, m_2^2) = \int_0^1 \frac{dx}{-k_x^2} \ln \left| \frac{-k_x^2}{M_W^2} \right|, \quad k_x = x k_1 + (1-x) k_2 ,$$

$$= \frac{1}{q^2} \cdot [\ln^2 \frac{q^2}{M_W^2} - \frac{1}{2} \ln^2 \frac{m_1^2}{M_W^2} - \frac{1}{2} \ln^2 \frac{m_2^2}{M_W^2} - 2\Phi(1)] \quad \text{for } m_1^2, m_2^2 \ll q^2 < \infty \quad (3.4)$$

$$= \frac{1}{q^2} \cdot [\ln^2 \frac{|q^2|}{M_W^2} - \frac{1}{2} \ln^2 \frac{m_1^2}{M_W^2} - \frac{1}{2} \ln^2 \frac{m_2^2}{M_W^2} - 2\Phi(1) - \pi^2] \quad \text{for } -\infty < q^2 \ll -(m_1+m_2)^2$$

3.



$$\Gamma_\rho(q^2) = i f_1^\chi f_2^V f_3^\chi \int \frac{d^n p}{(2\pi)^n} \cdot (1 + b_3 \gamma_5) \frac{\hat{k}_2 - \hat{p} + im_4}{(k_2-p)^2 + m_4^2} \gamma_\rho (a_2 + \gamma_5) \times$$

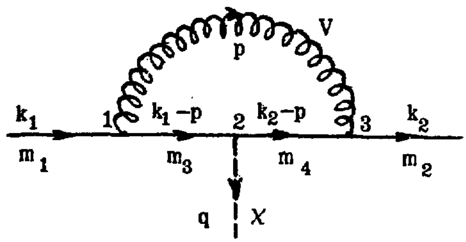
$$\times \frac{\hat{k}_1 - \hat{p} + im_3}{(k_1-p)^2 + m_3^2} (1 + b_1 \gamma_5) \frac{1}{p^2 + M_\chi^2} = \frac{f_1^\chi f_2^V f_3^\chi}{16\pi^2} \{ -P + \frac{1}{2} + \frac{M_\chi^2}{q^2} - \quad (3.5)$$

$$-\frac{1}{2}\ln\frac{|q^2|}{M_W^2}-\frac{M_\chi^2}{q^2}\ln\frac{|q^2|}{M_\chi^2}-\frac{M_\chi^4}{q^4}[\Phi(1)-\Phi(1-\frac{q^2}{M_\chi^2})]\}\gamma_\rho(A+B\gamma_5).$$

Here

$$A=a_2-b_1+b_3-a_2b_1b_3,\qquad B=a_2b_1-a_2b_3+b_1b_3-1. \tag{3.6}$$

4.



$$\Gamma(q^2)=-f_1^Vf_2^\chi f_3^V\int\frac{d^np}{(2\pi)^n}\gamma_\alpha(a_3+\gamma_5)\frac{\hat{k}_2-\hat{p}+im_4}{(k_2-p)^2+m_4^2}(1+b_2\gamma_5)\frac{\hat{k}_1-\hat{p}+im_3}{(k_1-p)^2+m_3^2}\times$$

$$\times\gamma_\beta(a_1+\gamma_5)\frac{\delta_{\alpha\beta}+p_\alpha p_\beta/M_V^2}{p^2+M_V^2}=i\frac{f_1^Vf_2^\chi f_3^V}{16\pi^2}\{[(6+\frac{m_2^2+m_1^2-2m_3^2-2m_4^2}{M_V^2})\times$$

$$\times(A_1+B_1\gamma_5)-\frac{2m_1m_2}{M_V^2}(A_1-B_1\gamma_5)+\frac{2m_2m_3}{M_V^2}(A_2+B_2\gamma_5)+ \tag{3.7}$$

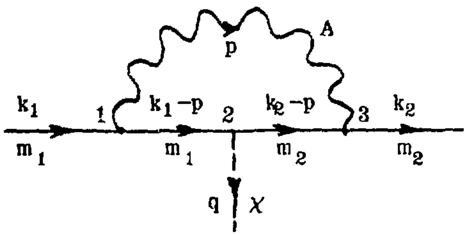
$$+\frac{m_1m_3}{M_V^2}(A_2-B_2\gamma_5)+\frac{m_2m_4}{M_V^2}(A_3+B_3\gamma_5)+\frac{2m_1m_4}{M_V^2}(A_3-B_3\gamma_5)-$$

$$-\frac{2m_3m_4}{M_V^2}(A_4+B_4\gamma_5)]\cdot P+[-1+3\ln\frac{M_V^2}{M_W^2}+2(\Phi(1)-\Phi(1-\frac{q^2}{M_V^2}))](A_1+B_1\gamma_5)\}.$$

Here

$$A=\begin{pmatrix}A_1\\A_2\\A_3\\A_4\end{pmatrix}=\begin{pmatrix}a_1a_3+a_3b_2-a_1b_2-1\\a_1a_3-a_3b_2-a_1b_2+1\\a_1a_3+a_3b_2+a_1b_2+1\\a_1a_3-a_3b_2+a_1b_2-1\end{pmatrix},\quad B=\begin{pmatrix}B_1\\B_2\\B_3\\B\end{pmatrix}=\begin{pmatrix}a_3-a_1-b_2+a_1a_3b_2\\a_3+a_1-b_2-a_1a_3b_2\\a_3+a_1+b_2+a_1a_3b_2\\a_3-a_1+b_2-a_1a_3b_2\end{pmatrix}\tag{3.8}$$

5.

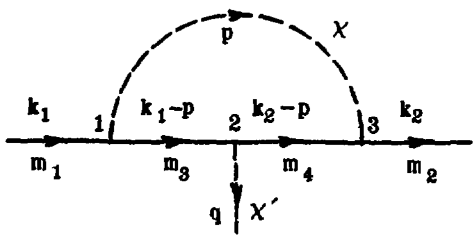


$$\Gamma(q^2)=-f_1^q f_3^q f_2^{\chi} e^2\int\frac{d^np}{(2\pi)^n}\gamma_\alpha\frac{\hat{k}_2-\hat{p}+im_2}{(k_2-p)^2+m_2^2}(1+b_2\gamma_5)\frac{\hat{k}_1-\hat{p}+im_1}{(k_1-p)^2+m_1^2}\gamma_\alpha\cdot\frac{1}{p^2}=$$

$$=i\frac{f_1^q f_3^q f_2^{\chi}}{16\pi^2}e^2\,[8P+2(q^2+m_1^2+m_2^2)J(q^2,m_1^2,m_2^2)P_{IR}-6+\tag{3.9}$$

$$+4\ln\frac{m_1m_2}{M_W^2}+q^2\cdot K(q^2,m_1^2,m_2^2)]\cdot(1+b_2\gamma_5).$$

6.



$$\Gamma(q^2)=f_1^{\chi} f_2^{\chi} f_3^{\chi} \int \frac{d^n p}{(2\pi)^n}(1+b_3\gamma_5)\frac{\hat{k}_2-\hat{p}+im_4}{(k_2-p)^2+m_4^2}(1+b_2\gamma_5)\frac{\hat{k}_1-\hat{p}+im_3}{(k_1-p)^2+m_3^2}\times$$

$$\times(1+b_1\gamma_5)\frac{1}{p^2+M_\chi^2}=i\frac{f_1^{\chi} f_2^{\chi} f_3^{\chi}}{16\pi^2}\{-2P+2-\ln\frac{|q^2|}{M_W^2}- \quad (3.10)$$

$$-\frac{M_\chi^2}{q^2}[\Phi(1)-\Phi(1-\frac{q^2}{M_\chi^2})]\}(A+B\gamma_5).$$

Here

$$A=b_1b_3-b_1b_2-b_2b_3+1, \qquad B=b_1-b_2+b_3-b_1b_2b_3. \quad (3.11)$$

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