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ANOMALOUS DIMENSION QUARK COUNTING AT LARGE TRANSFERRED MOMENTA IN QCD

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Recently a considerable progress has been made in studying the quantum chromodynamic corrections, that multiply the nominal point-like power asymptotics of the large energy and transferred momenta processes [1,2].

In this note we give a general formula called in the following the anomalous dimension quark counting rule. It determines the leading log's corrections to the canonic point-like asymptotics [3] of an arbitrary particle reaction at large transferred momenta in terms of active and passive quarks of hadrons participating in the relevant reaction.

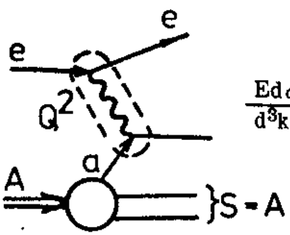
For an element of differential inclusive cross-section we have

$$d\sigma = d\sigma_0 \frac{(1-x)^{s-1}}{\Gamma(s)}\left[(\log Q^2)^{\gamma(s)+r\log(1-x)}\right]^{H}, \qquad (1)$$

$d\sigma_0$ is the corresponding point-like x-section, $r=16/25$ (for $n_f=4$), and $s=2n_{\text{passive}} = \sum_{i\text{-hadrons}} 2(n_i-1)$ is the doubled total number of passive quarks (spectators) belonging to the participating in reaction hadrons; H is the total number of active quarks in hard scattering regime, which coincides with the total number of hadrons in reaction.

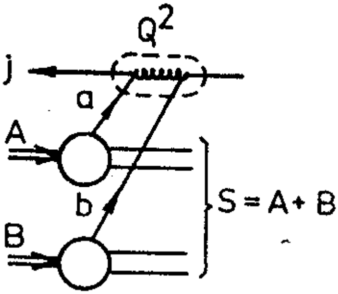
The quantity $\gamma(n) = -\frac{r}{4}\left[1-\frac{2}{n(n+1)}+4\sum_{k=2}^{n}\frac{1}{k}\right]$ is the standard anomalous dimension (in one-loop approximation) corresponding to the nonsinglet part of the quark distribution (decay) function.

Let us consider now some illustrative examples of this formula $(Q^2, p_T^2 \gg m^2)$:



$$\frac{E\,d\sigma}{d^3k} \propto \left(\frac{\alpha}{Q^2}\right)^2 \frac{(1-x)^{A-1}}{\Gamma(A)}\left[(\log Q^2)^{\gamma(A)+r\log(1-x)}\right]^{1}.$$

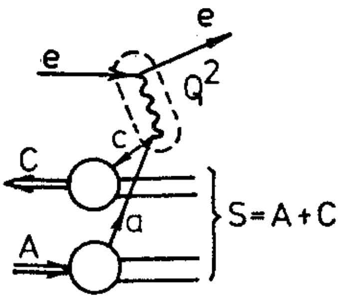
Fig.1. Deep inelastic scattering $e + A \to e + \ldots$; $S = A = 2(n_A - 1)$; $H = 1$.



$$\frac{E d\sigma}{d^3p} \propto \left(\frac{\alpha_s(p_T^2)^2}{p_T^2}\right) \cdot \frac{(1-x)^{A+B-1}}{\Gamma(A+B)} \times$$

$$\times [(\log \hat{Q}^2)^{\gamma(A+B)+r\log(1-x)}]^2.$$

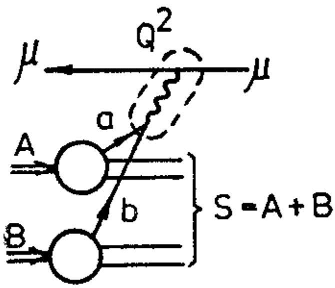
Fig.2a. Inclusive jet production $A+B \to j+\ldots$ at large p_T. $S = A + B = 2(n_A+n_B-2)$; $H = 2$.



$$\frac{E d\sigma}{d^3k} \propto \left(\frac{\alpha}{Q^2}\right)^2 \cdot \frac{(1-x)^{A+C-1}}{\Gamma(A+C)} \times$$

$$\times [(\log Q^2)^{\gamma(A+C)+r\log(1-x)}]^2.$$

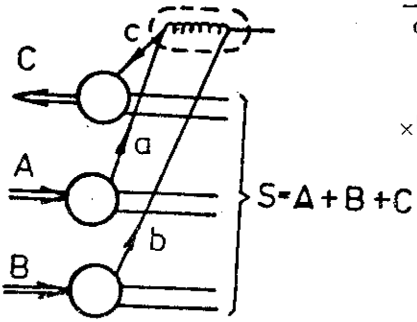
Fig.2b. Deep inelastic semi-inclusive hadron electroproduction $e+A \to C+\ldots$; $S = A+C = 2(n_A+n_C-2)$; $H=2$.



$$\frac{E d\sigma}{d^3Q} \propto \left(\frac{\alpha}{Q^2}\right)^2 \cdot \frac{(1-x)^{A+B-1}}{\Gamma(A+B)} \times$$

$$\times [(\log Q^2)^{\gamma(A+B)+r\log(1-x)}]^2.$$

Fig.2c. Prompt lepton prodiction $A+B \to \mu+\ldots$; $S = A+B = 2(n_A+n_B-2)$; $H = 2$.



$$\frac{Ed\sigma}{d^3p} \propto \left(\frac{\alpha_s(p_T^2)^2}{p_T^2}\right) \cdot \frac{(1-x)^{A+B+C-1}}{\Gamma(A+B+C)} \times$$

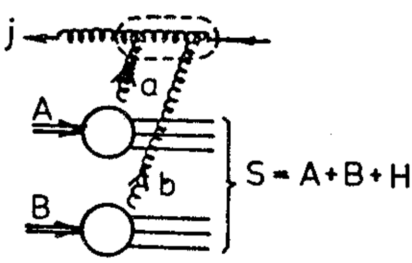
$$\times\left[(\log p_T^2)^{\gamma(A+B+C)+r\log(1-x)}\right]^3 .$$

Fig.3. Inclusive hadron production at large p_T in $A+B\to C+\dots$; $S=A+B+C=2(n_A+n_B+n_C-3)$; $H=3$.

Note, in the case of the gluonic hard scatterers, the corresponding inclusive cross-sections can be obtained from eq. (1) by the following change: $S\to S+H$, $r\to r'=36/25$

$$\gamma(n)\to\gamma'(n) = -\frac{r'}{4}\left[\frac{1}{3}-\frac{4}{n(n-1)}-\frac{4}{(n+1)(n+2)}+4\sum_2^n\frac{1}{k}\right].$$

For example, the gluon jet production cross-section in hadron collisions at large p_T $A+B\to J_G+\dots$ reads (cr. Fig.2a):



$$\frac{Ed\sigma}{d^3p} \propto \left(\frac{\alpha_s(p_T^2)^2}{p_T^2}\right) \cdot \frac{(1-x)^{A+B+1}}{\Gamma(A+B+2)} \times$$

$$\times\left[(\log\hat{Q}^2)^{\gamma'(A+B+2)+r'\log(1-x)}\right]^2 .$$

Fig.4. Inclusive gluon jet production $A+B\to J_G+\dots$; at large Q_T; $S=A+B+1=2(n_A+n_B+1)$; $H=2$.

In spite of the model independence of the presented anomalous dimension quark counting formula, the most simple illustration of eq. (1) provides its derivation in hard scattering model of the hadron constituents [4].

Indeed, in the particular case of the inclusive jet produnction at large p_T in hadronic collision $A+B\to J+\dots$ the hard scattering Ansatz reads

$$E\frac{d\sigma}{d^3p}(A+B\to \text{jet}+\dots)=\frac{1}{\pi}\sum_{a,b,c,\dots}\iint dx_a dx_b F_{a/A}(x_a,Q^2)F_{b/B}(x_b,Q^2)\times$$
$$\times\, d\hat\sigma(ab)/d\hat t\cdot \hat s\,\delta(\hat s+\hat t+\hat u)\,, \tag{2}$$

where (see Fig.2a):

$$\hat s=(p_a+p_b)^2\simeq x_a x_b s;\quad \hat t=(p_a-p_c)^2\simeq x_a t\,;\qquad \hat u=(p_b-p_c)^2\simeq x_b u$$

and $d\hat\sigma(ab)/d\hat t$ is the constituent hard scattering cross-section determined by the corresponding QCD Born diagrams.

This equation can be calculated by using as an input the quark distributions inside the hadrons A and B. Keeping in mind the QCD result in the leading log's approximation [5]

$$F_{a/A}(x_a,Q^2)\propto\frac{(1-x_a)^{\bar A-1}}{\Gamma(\bar A)}\exp c\xi,\quad \bar A=2(n_A-1)+r\,\xi, \tag{3}$$

where $\xi=\log(\log Q^2/\log Q_0^2)$, $r=16/25$ and $c=r(\ln 2-½)$ and similarly the $F_{b/B}$-function, we find

$$Ed\sigma/d^3p(A+B\to \text{jet}+\dots)=\alpha_S^2(p_T^2)\cdot p_T^{-4}\cdot J(x_1x_2;p_T^2)\,,$$

$$J=\frac{(1-x_1)^{\bar A-1}}{\Gamma(\bar A)}\,\frac{(1-x_2)^{\bar B-1}}{\Gamma(\bar B)}\int_0^1\!\!\int_0^1\frac{du\,dv\,u^{\bar A-1}\,v^{\bar B-1}}{[1-u(1-x_1)]^2[1-v(1-x_2)]^2}\times$$
$$\times(1-x_1x_2)\,\delta[uv(1-x_1-x_2)-(1-u-v)]\,, \tag{4}$$

where the used variables are

$$x_1 = -t/u + s = x_R \frac{\sin^2\theta/2}{1 - x_R \cos^2\theta/2}$$

$$x_2 = -u/t + s = x_R \frac{\cos^2\theta/2}{1 - x_R \sin^2\theta/2}$$

$$x_R = -(u+t)/s = 1 - M^2/s.$$

Thus, for the x_R approaching 1 eqs. (2,4) are determined by the initial constituent distributions near unity, and

$$J = \frac{(1-x_R)^{A+B-1}}{\Gamma(A+B)} \cdot \frac{[(\log Q^2)^{\gamma(A+B)+r\log(1-x)}]^2}{(\cos^2\theta/2)^A (\sin^2\theta/2)^B}, \tag{5}$$

where

$$\gamma(n) = -r\psi(n+1) + C \simeq -\frac{r}{4}[1 - \frac{2}{n(n+1)} + 4\sum_2^n \frac{1}{k}],$$

$C = r(\ln 2 - ½) \simeq 0.12...$ and $\Psi(z)$ digamma function.

Note, that near the phase-space boundary, where $(1-x_R) \propto O(\frac{m^2}{Q^2})$ eq. (5) as well as the hard scattering formula (2) will be modified by the double-log factors (quark form factors) /2/.

We should like to emphasize an interesting viewpoint on the outgoing hadron momentum fraction x_R-dependence of the invariant $A+B \to J+...$ cross section. This dependence enters the cross section like the structure distribution function of the big dihadron compound system $(A+B)=D$, which collects the constituents of the initial hadrons A and B,

$$F_{a+b/A+B}(x_R) \propto \frac{(1-x_R)^{A+B-1}}{\Gamma[2(n_A+n_B-2)]}$$

and corresponds to the picking out the pair of active quarks <u>a</u>, <u>b</u> with resulting momentum fraction x_R.

Finally, we stress that the main result can be expressed by the formula in which the logarithmic exponents are determined by the anomalous dimensions of the moments of constituent distribution and decay functions of the participating hadrons. The number of the corresponding moments (indices of $\gamma(n)$)) are related to the number of valence quarks constituting these hadrons (see the Table).

Table

k	Process $A+B\to C+\ldots;\ \ell = e^{\pm}, \mu^{\pm}, \nu^{\pm}$	$\gamma(s)$, $s = 2k$	$H\cdot\gamma(s)$
1.	$\ell + \pi \to \ell + \ldots$	$\gamma[2(2-1)]$	$1\cdot\gamma(2)$
	$\ell + \bar{\ell} \to \pi + \ldots$	$\gamma[2(2-1)]$	$1\cdot\gamma(2)$
2.	$\ell + p \to \ell + \ldots$	$\gamma[2(3-1)]$	$1\cdot\gamma(4)$
	$\ell + \pi \to \ell + \pi + \ldots$	$\gamma[2(4-2)]$	$2\cdot\gamma(4)$
3.	$\ell + p \to \ell + \pi + \ldots$	$\gamma[2(5-2)]$	$2\cdot\gamma(6)$
	$\pi + p \to$ jet, $\ell, \ell\bar{\ell} + \ldots$	$\gamma[2(5-2)]$	$2\cdot\gamma(6)$
4.	$\ell + p \to \ell + p + \ldots$	$\gamma[2(6-2)]$	$2\cdot\gamma(8)$
	$p + p \to$ jet, $\ell, \ell\bar{\ell} + \ldots$	$\gamma[2(6-2)]$	$2\cdot\gamma(8)$
	$\pi + p \to \pi + \ldots$	$\gamma[2(7-3)]$	$3\cdot\gamma(8)$
5.	$p + p \to \pi + \ldots$	$\gamma[2(8-3)]$	$3\cdot\gamma(10)$
	$\pi + p \to \pi + \pi + \ldots$	$\gamma[2(9-4)]$	$4\cdot\gamma(10)$
6.	$p + p \to p + \ldots$	$\gamma[2(9-3)]$	$3\cdot\gamma(12)$
	$\pi + p \to \pi + p + \ldots$	$\gamma[2(10-4)]$	$4\cdot\gamma(12)$
	$p + p \to \pi + \pi + \ldots$	$\gamma[2(10-4)]$	$4\cdot\gamma(12)$
	etc.		

The more detailed justification of the presented counting rules and their relation with the hard scattering mechanism in QCD as well as asymptotic exclusive-inclusive connection will be presented elsewhere.

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